THE NONSINGLET SPIN STRUCTURE FUNCTION g_1 AT SMALL x

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The perturbative QCD predictions for the small x behaviour of the nucleon spin structure functions is discussed. The role of the resummation of the $\ln^2 1/x$ terms is emphasized. Predictions for the nonsinglet structure function g_1 in case of a flat as well as a dynamical input are given.

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1. Introduction

Since 1988, when the famous EMC experiment [1] provided surprising results, the polarized deep inelastic lepton–nucleon scattering (DIS) became very interesting from experimental as well as theoretical point of view. This experiment, in which longitudinally polarized muons scattered on longitudinally polarized protons, brought the conclusion that quarks are carrying only a small part of the proton spin projection in the polarized proton. This result called as 'spin crisis' is still a challenge for theoretical and experimental research. The main questions to answer are: how the nucleon spin is distributed among its constituents: quarks and gluons and how the dynamics of these constituent interactions depend on spin. Solutions to these problems may be found within perturbative QCD because they involve hard and semihard (short-distance) processes. Recently the experimental data have allowed to investigate the nucleon spin structure in the large range of the kinematical variables: Bjorken x and Q^2. The most interesting, both theoretically and phenomenologically, is the region of small x. Theoretical understanding of the small x ($x \sim 10^{-3}$ and less) behaviour of the polarized nucleon structure function enables the correct estimation of Γ_1 momenta in the sum rules. It is very important because present experimental data do not cover the whole very small x region and the only way (at present) to know the nucleon spin structure completely is extrapolation of large and medium x results into the small x region through the theoretical QCD analysis. On the other hand, future polarized experiments in HERA [2] will enable spin DIS investigations in the very small x region: $x \sim 10^{-4}$ and less. Then theoretical predictions would be verified by the experiment. These future spin experiments would be a crucial test of theoretical analysis. Description of the nucleon spin structure function g_1 within perturbative QCD for small x can be done in different frames (in LO, NLO, $\ln 1/x$, $\ln^2 1/x$ *etc.* approximations) giving different results for g_1 in this region. Thus the future comparison of theoretical and experimental results could be definitive. In the next section we shall discuss the polarized structure functions of nucleon in the small Bjorken x region. We shall emphasize the $\ln^2 1/x$ resummation which is significant in this region. In point 3 the nonsinglet $g_1^{\rm NS}(x,Q^2)$ predictions are presented. We show LO and unified LO $+ \ln^2 1/x$ resummation results in case of a flat (nondynamical) and a dynamical input parametrization as well. We compare our numerical results with recent SMC data. Finally in conclusions we shall briefly discuss future experimental hopes and possible scenario of solving the spin crisis problem.

2. Spin structure functions in the small Bjorken x region

Determination of the nucleon spin structure functions in the small Bjorken x region is very important from both theoretical and experimental point of view. Because of technical limit, present experiments do not give any information about small x region ($x \sim 10^{-4}, 10^{-5}$) and therefore, there are still uncertainties in the determination of parton distribution functions (in particular gluons) in this region. Theoretical analyses, based on the perturbative QCD, allow to calculate the nucleon structure function within some approximations (Q^2LO, Q^2NLO, $\ln 1/x$ *etc.*). The choice of some particular approximation depends of course on the region of its application and the basic criterion is the agreement of theoretical predictions with experimental data. Thus the small x behaviour of the nucleon spin structure functions implied by QCD can be tested experimentally via the sum rules (BSR, EJSR) [5]. Moreover, the aim of the QCD analysis is to yield an adequate, compact description of the nucleon structure functions in the whole range of x. The small x region is also a challenge for QCD analysis, because theoretical predictions of the structure function $g_1^p(x,Q^2)$ at low x are relevant for the future polarized HERA measurements [6].

The small value of x ($x \to 0$) corresponds by definition to the Regge limit and therefore the small x behaviour of structure functions can be described using the Regge pole exchange model [2]. The Regge theory predicts, that spin dependent structure functions $g_1^{p,n,d}$ in the small x region behave as

$$g_1^{p,n,d} \sim x^{-\alpha}\,, \tag{2.1}$$

where α denotes the axial vector meson trajectory and lies in the limits:

$$-0.5 \leq \alpha \leq 0\,. \tag{2.2}$$

The experimental data from HERA confirm such a Regge behaviour of structure functions (2.1) but only in the low Q^2 region $Q^2 \leq \Lambda^2$ ($\Lambda^2 \approx 200$ MeV) *i.e.* in the region, where the perturbative methods are not applicable. At larger Q^2, because of parton interaction, the structure functions undergo the GLAP Q^2 evolution [3, 7, 10] and their behaviour, implied by perturbative QCD is more singular than that, coming from the Regge picture. This fact is also in agreement with experiments of unpolarized as well as polarized DIS [4, 11]. It is well known at present, that for $x \rightarrow 0$ the Regge behaviour $x^{-\alpha}$ ($-0.5 \leq \alpha \leq 0$) is less singular than the perturbative QCD predictions for all of parton distributions except unpolarized, nonsinglet (valence) quarks $q_{\rm NS}$. It has been lately noticed [12, 16, 17] that the spin dependent structure function g_1 in the small x region is dominated by $\ln^2(1/x)$ terms. These contributions correspond to the ladder diagrams with quark and gluon exchanges along the ladder — *cf.* Fig. 1. The contribution of non-ladder diagrams to the nonsinglet spin dependent structure function is negligible. Thus the behaviour of the spin dependent nucleon structure functions at small x is expected to be governed by leading double logarithmic terms of

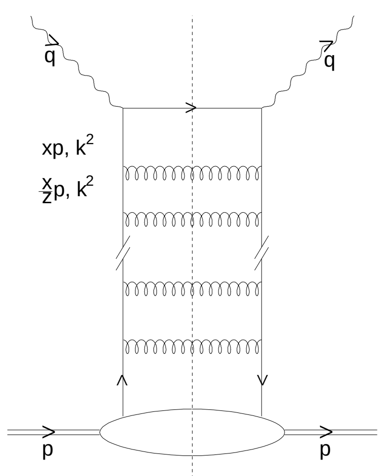

Fig. 1. A ladder diagram generating double logarithmic $\ln^2(1/x)$ terms in the nonsingled spin structure function g_1.

type $\alpha_s^n \ln^{2n}(x)$. These terms must be resummed in the coefficients and splitting functions $P_{ij}(x, \alpha_s^2)$. Combining the standard LO GLAP approach with the double $\ln^2 x$ resummation, it is possible on one hand to guarantee an agreement of QCD predictions with experimental data in the large and moderately small x and on the other hand to generate the singular small x shape of polarized structure functions, governed by $\ln^2 x$ terms. In this way one can obtain system of equations, containing both LO GLAP evolution and the double logarithmic $\ln^2 x$ effects at small x. Analyses of such unified GLAP LO + $\ln^2 x$ approach are presented in [12]. On the basis of this interesting method we give in the next chapter the predictions for the $g_1^{\rm NS}$ function in the case of nonsingular as well as singular input parametrization $g_1^{\rm NS}(x, Q_0^2)$.

The small x behaviour of both nonsinglet and singlet spin dependent structure functions $g_1^{\rm NS}(x, Q^2)$ and $g_1^S(x, Q^2)$ is governed by the double logarithmic terms $\alpha_s^n \ln^{2n}(x)$ [12,16,17]. But in contrast to the singlet polarized function, for the nonsinglet one the contribution of nonladder diagrams is negligible. Thus we should consider only ladder diagrams with quark (antiquark) exchange, Fig. 1. Hence the nonsinglet part of the polarized structure function g_1 has a form:

$$g_1^{\rm NS}(x, Q^2) = g_1^p(x, Q^2) - g_1^n(x, Q^2)\,, \tag{2.3}$$

where g_1^p and g_1^n are spin dependent structure functions of proton and neutron respectively. Let us remind the meaning of g_1. In the Bjorken limit

$$g_1(x) \;=\; \tfrac{1}{2} \sum_{i=u,d,s,\ldots} e_i^2 \Delta q_i(x)\,, \tag{2.4}$$

$$\Delta q_i(x) \;=\; q_{i+}(x) - q_{i-}(x)\,, \tag{2.5}$$

where e_i is a charge of the i-flavour quark, $q_{i+}(x)$ ($q_{i-}(x)$) is the density distribution function of the i-quark with the spin parallel (antiparallel) to the parent nucleon. The function $g_1(x, Q^2)$ is related to the helicity of the nucleon (*i.e.* spin projection on the momentum direction). Thus the integral

$$\langle \Delta q_i \rangle = \int_0^1 \Delta q_i(x) dx \tag{2.6}$$

is simply a part of the nucleon helicity, carried by a quark of i-flavour ($i = u, d, s, \ldots$). Polarized distribution functions of quarks are defined as:

$$\Delta q = \Delta q_{\mathrm{val}} + \Delta q_{\mathrm{sea}} \,, \tag{2.7}$$
$$\Delta q_{\mathrm{sea}} = \Delta \bar{q}_{\mathrm{sea}} \equiv \Delta \bar{q} \tag{2.8}$$

hence

$$\Delta q_{\mathrm{val}} = \Delta q - \Delta \bar{q} \,. \tag{2.9}$$

According to (2.4)–(2.9) one can obtain for the colour number $N_c = 3$:

$$g_1^p = \frac{2}{9}\Delta u + \frac{1}{18}\Delta d + \frac{5}{18}\Delta \bar{u} + \frac{1}{9}\Delta \bar{s} \,, \tag{2.10}$$

$$g_1^n = \frac{1}{18}\Delta u + \frac{2}{9}\Delta d + \frac{5}{18}\Delta \bar{u} + \frac{1}{9}\Delta \bar{s} \,, \tag{2.11}$$

and hence

$$g_1^{\mathrm{NS}} = \frac{1}{6}(\Delta u_{\mathrm{val}} - \Delta d_{\mathrm{val}}) = \frac{1}{6}(\Delta u - \Delta d) \,. \tag{2.12}$$

The simple form of g_1^{NS} (2.12) results from the assumption of SU(3) flavour symmetry:

$$\Delta \bar{u} = \Delta \bar{d} \tag{2.13}$$

and hence all of gluon and sea quark contributions from the proton and the neutron structure function cancel mutually. This feature that the small x behaviour of the spin dependent nonsinglet structure function is governed by the double logarithmic terms $\alpha_s^n \ln^{2n}(x)$ is very important from the point of view of small x QCD analysis. This is different from the case of unpolarized nonsinglet structure functions F_2^{NS}, where the small x behaviour of F_2, generated by the $\alpha_s^n \ln^{2n}(x)$ terms, is dominated by the nonperturbative contribution of A_2 Regge pole. For g_1^{NS} the relevant A_1 Regge pole has low intercept $\alpha_{\mathrm{NS}}(0) \leq 0$ and for small x in the Regge limit one has:

$$g_1^{\mathrm{NS}}(x, Q^2) \sim x^{-\alpha_{\mathrm{NS}}(0)} \,. \tag{2.14}$$

Thus the Regge behaviour of the spin dependent structure functions is unstable against the resummation of the $\ln^2 x$ terms, which generate more singular x shape than relation (2.14) with $\alpha_{\mathrm{NS}}(0) \leq 0$. Therefore the measurement of the nonsinglet spin dependent structure function can be a very important test of the QCD perturbative analyses in the small x region. In our numerical analysis we follow [12] and [17]. Solving the unified equation incorporating GLAP Q^2 evolution and the $\ln^2 x$ resummation we get the results for the nonsinglet polarized structure function $g_1^{\mathrm{NS}}(x, Q^2)$ in the perturbative region $Q^2 > Q_0^2$ for different values of $x \in (0;1)$. This equation taking into account both GLAP evolution and $\ln^2 x$ effects for $g_1^{\rm NS}$ function has the form [12, 17]:

$$f(x,k^2) = f^{(0)}(x,k^2) + \frac{2\alpha_s(k^2)}{3\pi}\int\limits_x^1 \frac{dz}{z}\int\limits_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2} f\left(\frac{x}{z},k'^2\right)$$
$$+\frac{\alpha_s(k^2)}{2\pi}\int\limits_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\left[\frac{4}{3}\int\limits_x^1\frac{dz}{z}\frac{(z+z^2)f(x/z,k'^2)-2zf(x,k'^2)}{1-z}\right.$$
$$\left.+\left(\frac{1}{2}+\frac{8}{3}\ln(1-x)\right)f(x,k'^2)\right], \tag{2.15}$$

where

$$f^{(0)}(x,k^2) = \frac{\alpha_s(k^2)}{2\pi}\left[\frac{4}{3}\int\limits_x^1\frac{dz}{z}\frac{(1+z^2)g_1^{(0)}(x/z)-2zg_1^{(0)}(x)}{1-z}\right.$$
$$\left.+\left(\frac{1}{2}+\frac{8}{3}\ln(1-x)\right)g_1^{(0)}(x)\right]. \tag{2.16}$$

The unintegrated distribution f in the equation (2.15) are related to the $g_1(x,Q^2)$ via

$$g_1(x,Q^2) = g_1^{(0)}(x) + \int\limits_{k_0^2}^{Q^2(1/x-1)}\frac{dk^2}{k^2} f\left(x(1+\frac{k^2}{Q^2}),k^2\right), \tag{2.17}$$

where

$$g_1^{(0)}(x) = \int\limits_0^{k_0^2}\frac{dk^2}{k^2} f(x,k^2). \tag{2.18}$$

3. Predictions for the nonsinglet spin structure function g_1

We solve Eq. (2.15) using different parametrizations of $g_1^{\rm NS(0)}(x)$: the simple one, implied by Regge behaviour of $g_1^{\rm NS}$ in nonperturbative region

$$g_1^{\rm NS(0)}(x) \equiv g_1^{\rm NS}(x,Q_0^2) = N(1-x)^3 \tag{3.1}$$

and two dynamical inputs: GRSV (Glück, Reya, Stratmann, Vogelsang) [14] and GS (Gehrmann, Stirling) [15]. The nonsinglet spin dependent structure function must satisfy the Bjorken sum rule, which can be written as:

$$\int_0^1 g_1^{\rm NS}(x,Q_0^2)dx = \int_0^1 (g_1^p - g_1^n)(x,Q_0^2)dx = \frac{1}{6}g_A \tag{3.2}$$

independently of the value of Q^2. This means that for any Q^2, the first moment of $g_1^{\rm NS}$ must be equal to $1/6\; g_A$ similarly to the case of the low scale Q_0^2 (3.2):

$$\langle g_1^{\rm NS}(x,Q^2)\rangle \equiv \int_0^1 g_1^{\rm NS}(x,Q^2)dx = \int_0^1 (g_1^p - g_1^n)(x,Q^2)dx = \frac{1}{6}g_A = 0.2095\,. \tag{3.3}$$

This condition implies the proper normalisation constants N in all of input parametrizations. Thus the constant N in (3.1), found from the Bjorken sum rule is equal to $2/3\; g_A = 0.838$ (we set the axial vector coupling $g_A = 1.257$) and the Regge nonsingular input (3.1) takes the form:

$$\text{REGGE}: \quad g_1^{\rm NS}(x,Q_0^2) = \frac{2}{3}g_A(1-x)^3 = 0.838(1-x)^3\,. \tag{3.4}$$

The Regge behaviour of structure functions at small x, as it was mentioned above, has been confirmed by HERA experiments in the low Q^2 region ($Q^2 < 1$ GeV^2). Therefore the choice of the Regge input allows to unify the nonperturbative origin with QCD perturbative analysis starting at $Q_0^2 \sim 1$ GeV^2. In this way, assuming the Regge (flat, nonsingular) behaviour of structure functions at low Q^2 scale *i.e.* $Q_0^2 = 1$ GeV^2, we expect that the singular small x behaviour of polarized structure functions is completely generated by QCD evolution, involving NLO or even (as in our case) GLAP+$\ln^2 x$ approach. This analysis, based on the Regge input (3.4), is however one of two main possible scenarios, describing the small x behaviour of spin structure functions. The second is to allow steeper (more singular) inputs of structure functions at Q_0^2, what intensifies more the growth of structure functions (with $x \to 0$) implied by QCD. The only constraint on these two scenarios is consistency of their predictions with experimental data. In our analysis of the $g_1^{\rm NS}$ structure function we consider dynamical inputs proposed by GRSV [14] and GS [15]. These inputs result from a global analysis of all recently available deep inelastic polarized structure function data [8]. Our calculations incorporating both GLAP evolution and resummation of the $\ln^2 x$ terms are based on the LO fitted inputs. In such a way the spin dependent nonsinglet structure function $g_1^{\rm NS}$ (2.12) has an input form:

$$\begin{aligned} {\rm GRSV}\ :\ & g_1^{\rm NS}\left(x,Q_0^2=1\ {\rm GeV}^2\right)=0.327x^{-0.267} \\ & \times(1-0.583x^{0.175}+1.723x+3.436x^{3/2})(1-x)^{3.486} \\ & +0.027x^{-0.624}(1+1.195x^{0.529}+6.164x+2.726x^{3/2})(1-x)^{4.215}\,. \end{aligned} \tag{3.5}$$

$$\begin{aligned} {\rm GS}\ :\ & g_1^{\rm NS}(x,Q_0^2=4\ {\rm GeV}^2)=0.29x^{-0.422}(1+9.38x-4.26\sqrt{x}) \\ & \times(1-x)^{3.73}+0.196x^{-0.334}(1+10.46x-5.10\sqrt{x})(1-x)^{4.73} \end{aligned} \tag{3.6}$$

(for details see Appendix A). All numerical calculations have been performed in C code on PC computer under LINUX system. Our numerical results for $g_1^{\rm NS}$ based on Regge (3.4), GRSV (3.5) and GS (3.6) input parametrizations are presented in Figs. 2–6. In Fig. 2 we plot different input parametrizations $g_1^{\rm NS}(x,Q_0^2)$. Figs. 3,4 show the nonsinglet function $g_1^{\rm NS}$ after evolution to $Q^2=10$ GeV^2 for these different parametrizations (Regge, GRSV, GS) and Figs. 5,6 present the function $6xg_1^{\rm NS}=x(\Delta u_{\rm val}-\Delta d_{\rm val})$ at $Q^2=10$ GeV^2 also for different inputs $g_1^{\rm NS}(x,Q_0^2)$. In all of Figs. 3–6 pure GLAP evolution is compared with double logarithmic $\ln^2 x$ effects at small x. Additionally, in Figs. 4–6 we compare our numerical results with recent SMC (1997) data [8]. Contributions $6\langle g_1^{\rm NS}\rangle$ (3.3) and $6\Delta I(x_a,x_b,Q^2)$,

$$\Delta I(x_a,x_b,Q^2)\equiv\int\limits_{x_a}^{x_b} g_1^{\rm NS}(x,Q^2)dx\,, \tag{3.7}$$

to the Bjorken sum rule at $Q^2=10$ GeV^2 together with experimental SMC values are presented in Table I. From Figs. 3–6 one can read that the double logarithmic $\ln^2 x$ effects are very significant for $x\leq 10^{-2}$. Besides, as it has been expected, the growth of the nonsinglet proton spin structure function $g_1^{\rm NS}$ in the very small x region is much steeper for dynamical parametrizations (GRSV or GS) than for the Regge one. The comparison of our theoretical model with experimental data in Table I and Figs. 4–6 yields the conclusion that all of the theoretical predictions for different parametrizations (Regge, GRSV, GS) and incorporating pure LO GLAP QCD evolution as well as LO GLAP evolution with $\ln^2 x$ effects are in a good agreement with experimental data within statistical errors. Unfortunately, the most interesting x region is still not available for experiment. So the problem, which QCD approach is the most adequate for the description of small x physics in the polarized deep-inelastic scattering of particles remains unsolved.

TABLE I

Theoretical contributions $6\Delta I(x_a, x_b, Q^2)$ and their experimental SMC values

PARAMETRIZATION	$6\Delta I$ $(0,1,Q^2)$	$6\Delta I$ $(0,0.003,Q^2)$	$6\Delta I$ $(0.003,0.7,Q^2)$
INPUT	1.257	0.0150	1.232
REGGE LO GLAP	1.255	0.0342	1.219
LO GLAP+$\ln^2 x$	1.249	0.0493	1.198
INPUT	1.257	0.0786	1.194
GRSV LO GLAP	1.249	0.107	1.171
LO GLAP+$\ln^2 x$	1.242	0.119	1.153
INPUT	1.257	0.123	1.160
GS LO GLAP	1.253	0.134	1.151
LO GLAP+$\ln^2 x$	1.247	0.142	1.139
EXPERIMENT	1.29±0.24	* 0.09±0.09	** 1.20±0.24

The mark * means the extrapolation of experimental data to low x and ** is the integral over the measured range of x.

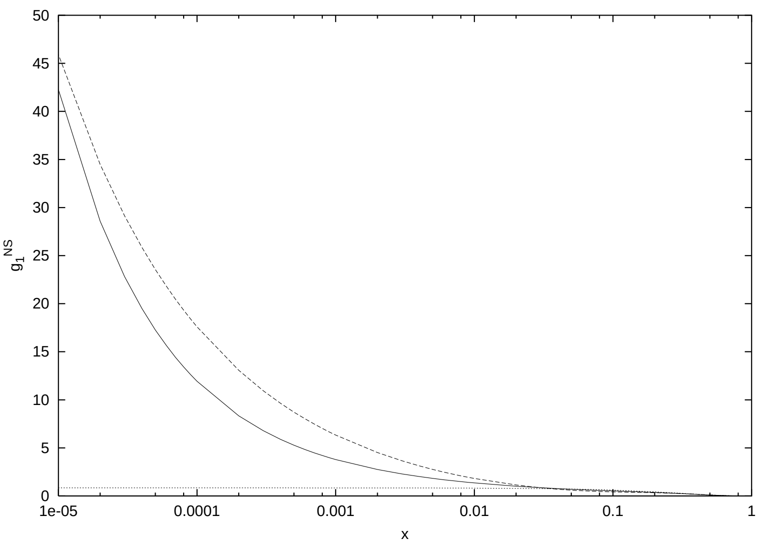

Fig. 2. Input parametrizations of the nonsinglet spin structure function of the proton $g_1^{\rm NS}(x, Q_0^2)$: REGEE (3.4) — dotted line; GRSV (3.5) — solid line; GS (3.6) — dashed line.

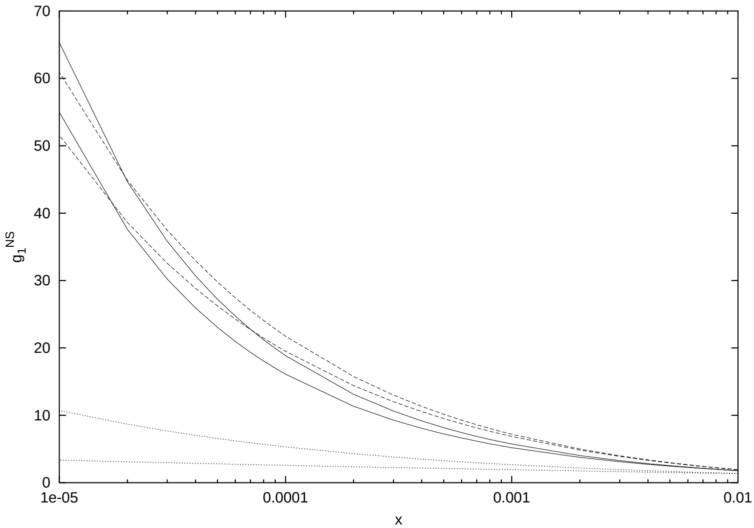

Fig. 3. $g_1^{\rm NS}$ at $Q^2 = 10$ GeV^2 based on inputs: REGGE — dotted, GRSV — solid, GS — dashed. For each pair of lines the pure LO GLAP prediction lies below the LO GLAP+$\ln^2 x$ one.

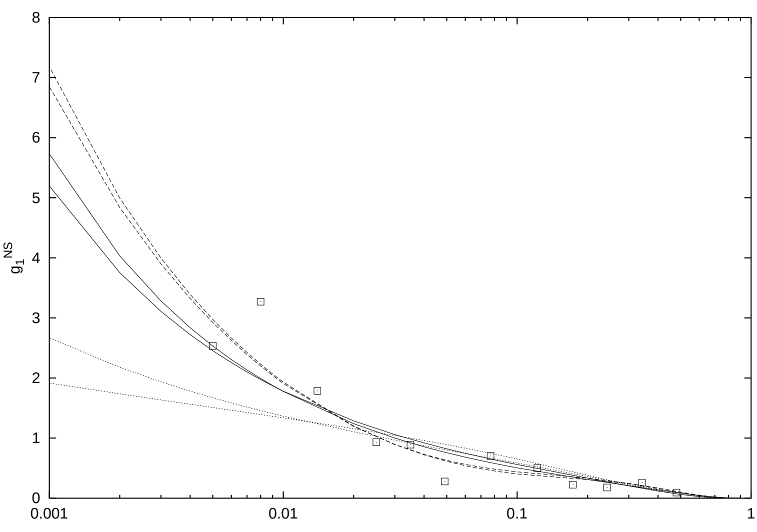

Fig. 4. $g_1^{\rm NS}$ at $Q^2 = 10$ GeV^2; similarly as in Fig. 3 but for the measurable experimentally region of x. Squares show the recent SMC data 1997 [8].

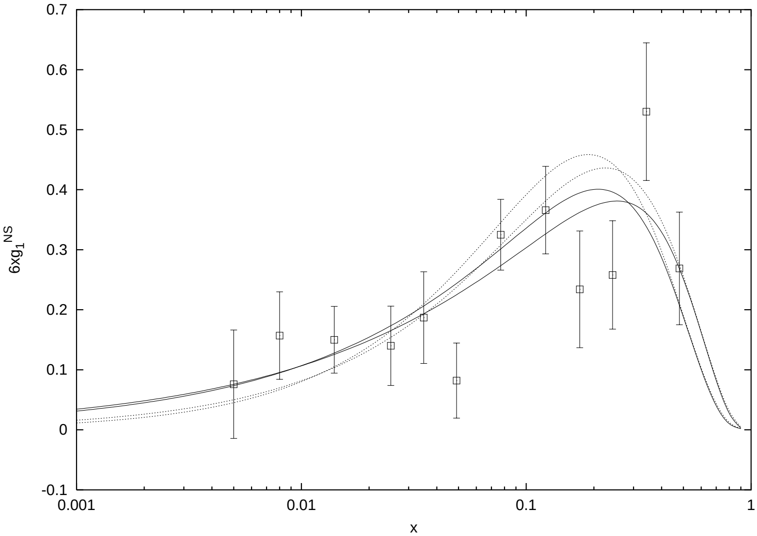

Fig. 5. Function $6xg_1^{\rm NS}$ at $Q^2 = 10$ GeV^2. Predictions based on the REGGE input — dotted and GRSV — solid. LO GLAP above LO GLAP+$\ln^2 x$ at $x = 0.2$. SMC 1997 data with statistical errors are shown.

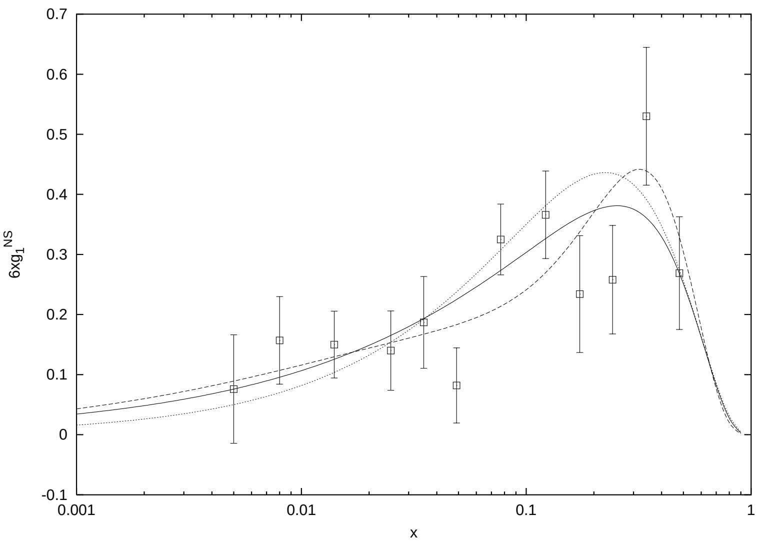

Fig. 6. LO GLAP+$\ln^2 x$ predictions for function $6xg_1^{\rm NS}$ at $Q^2 = 10$ GeV^2, based on input parametrizations: REGGE — dotted, GRSV — solid, GS — dashed. Plots are compared with SMC 1997 data.

4. Summary and conclusions

The results of current experiments are: deviation from the Ellis–Jaffe sum rule and validity of the Bjorken sum rule. This causes that the question "how is the spin of the nucleon made out of partons?" is still open. Experimental results which violating the Ellis–Jaffe sum rule imply that only a very small part of the spin of the proton is carried by quarks are a great puzzle. So where is the nucleon spin? Maybe gluons take a large fraction of the nucleon spin? Or maybe the spin of the proton is "hidden" in orbital angular momentum of quarks and gluons? Maybe at last the solution of the spin problem lies in the small x physics and the lacking spin of the nucleon is hidden in the unmeasured very small x region. The answer to the above questions will be possible thanks to the progress in theoretical and experimental research in the small x physics. Perturbative QCD analysis, based on the GLAP evolution equations is in a good agreement with experimental data. This agreement concern unpolarized and polarized structure functions of the nucleon F_1, F_2, g_1 within NLO approximation in the large and moderately small Bjorken x region. Unfortunately, practically lack of experimental measurements in the very small x region ($x \leq 10^{-3}$) makes satisfactory verification of the theoretical QCD predictions in this region impossible. Knowledge of the behaviour of the nucleon spin structure functions when $x \to 0$ is crucial in the determination of the Bjorken and Ellis–Jaffe sum rules *i.e.* in overcoming the "spin crisis". Understanding of the small x physics in the polarized DIS processes requires taking into account all of these perturbative QCD effects which become significant in the small x region and which could be verified by future experiments. Present QCD analyses, based on the GLAP LO or NLO Q^2 evolutions seem to be incomplete when $x \to 0$. The growth of the unpolarized as well polarized structure functions of the nucleon in the small x region is governed by leading double logarithmic terms of the form $\alpha_s^n \ln^{2n}(x)$, generated by ladder diagrams with quark and gluon exchange. This singular behaviour of the structure functions at low x, implied by $\ln^2 x$ terms, is however better visible in the polarized case. For unpolarized, nonsinglet structure functions of the nucleon the QCD evolution behaviour at small x is screened by the leading Regge contribution. Therefore the spin dependent structure functions of the nucleon are a sensitive test of the perturbative QCD analyses in the low x region. Our numerical analyses incorporating the LO GLAP evolution and the $\ln^2 x$ effects at small x show that the growth of the nonsinglet polarized structure function of the nucleon $g_1^{\rm NS}$, implied by $\ln^2 x$ terms, is significant for $x \leq 10^{-2}$. Our predictions for $g_1^{\rm NS}$ are in a good agreement with the recent SMC data for small x region ($x \sim 10^{-3}$). The contribution from the low x region ($x \leq 0.003$) to the Bjorken sum rule is found to be around 4% (for Regge input $g_1^{\rm NS}(x,Q_0^2)$) and 10% (for dynamical inputs) of the value of the sum. Theoretical predictions for $g_1^{p,n,d}$, taking into account the $\ln^2 x$ resummation effects will be verified experimentally in the future. There are a few hopeful experimental projects of the investigation of the nucleon's spin structure. One of these is the HERMES experiment (started in 1995) located in HERA at DESY with a fixed polarized H,D or ^{3}He target and longitudinally polarized positron beam of 27.5 GeV [18]. The accessible kinematic range is $0.004 < x < 1$ and $0.2 < Q^2 < 20$ GeV^2. The HERMES experiment allows a direct measurement of the polarized quark distributions for individual flavours, also $g_1^{p,n,d}(x,Q^2)$ and even $g_2(x,Q^2)$. The question of the gluon polarization is also addressed experimentally. The polarized gluon distribution $\Delta g(x,Q^2)$ may play a crucial role in understanding of the nucleon spin structure. The measurement of $\Delta g(x,Q^2)$ in the charm production via photon–gluon fusion process $\gamma^* g \to c\bar{c}$ will be possible in the COMPASS experiment at CERN [9]. In this project the polarized muons will be scattered on polarized proton and deuteron targets. The energy of the muon beam will be of 100 GeV and 200 GeV and the Bjorken x region $x > 0.02$. The COMPASS measurements are expected to start in 2000. A very important program which will test many elements of QCD in the perturbative as well as nonperturbative region is the RHIC spin project at Brookhaven [13]. This program with polarized proton–proton collider will start in 2000 and will allow for a measurement of the polarized gluon density via heavy quark production ($gg \to Q\bar{Q}$) or via direct photon production ($gq \to \gamma q$). Finally, a very promising experimental project in high energy spin physics is planned in HERA [6]. The polarization of the proton and electron beams at $\sqrt{s} = 300$ GeV will enable to measure the structure function $g_1(x,Q^2)$ and spin dependent quark distributions $\Delta q_f(x,Q^2)$ at very low x ($x \sim 10^{-5}$). From polarized di-jet production it will be possible to determinate the polarized gluon distribution $\Delta g(x,Q^2)$ in the region $0.002 < x < 0.2$. Additionally in HERA, a program of polarized proton–proton collisions is proposed. This high energy proton–proton scattering will allow, via J/ψ production, for the direct determination of the gluon function $\Delta g(x,Q^2)$. The new HERA projects with polarized experiments and the largely extended kinematical region of x and Q^2 will contribute a lot to our understanding of high energy spin physics. The problem of the spin structure of the nucleon is nowadays one of the most important challenges for theory and experiment.

I would like to thank Jan Kwieciński for a great help and critical remarks during preparing this work. I am also grateful to Andrzej Kotlorz for useful discussions about numerical problems.

Appendix A

Dynamical input parametrizations of the nonsinglet polarized structure function $g_1^{\rm NS}$

In our calculations we adopt GRSV (Glück, Reya, Stratmann, Vogelsang) [14] and GS (Gehrmann, Stirling) [15] parametrizations of polarized valence quarks $\Delta q_{\rm val}$. We assume SU(3) flavour symmetric scenario, where

$$\Delta\bar{u} = \Delta\bar{d}\,. \tag{A.1}$$

This assumption leads to formula (2.12):

$$g_1^{\rm NS} \equiv g_1^p - g_1^n = \frac{1}{6}(\Delta u - \Delta d) = \frac{1}{6}(\Delta u_{\rm val} - \Delta d_{\rm val})\,. \tag{A.2}$$

Input parametrization of $\Delta u_{\rm val}$ and $\Delta d_{\rm val}$ have a general form:

$$\text{GRSV}: \quad \Delta q_{\rm val} = N x^{a_2} x^{a_1 - 1}(1 + Ax^b + Bx + Cx^{3/2})(1-x)^D\,, \tag{A.3}$$

$$\text{GS}: \quad \Delta q_{\rm val} = N' x^{a'-1}(1 + \gamma x + \rho\sqrt{x})(1-x)^{D'}\,, \tag{A.4}$$

where N, N' are normalisation factors, implied by the Bjorken and Ellis–Jaffe sum rules. These sum rules for input scale Q_0^2 can be read as

$$a_3 = \int_0^1 (\Delta u_{\rm val} - \Delta d_{\rm val})dx = 1.257\,, \tag{A.5}$$

$$a_8 = \int_0^1 (\Delta u_{\rm val} + \Delta d_{\rm val})dx = 0.579\,, \tag{A.6}$$

Eqs. (A.5) and (A.6) give immediately

$$\int_0^1 \Delta u_{\rm val} dx = 0.918\,, \tag{A.7}$$

$$\int_0^1 \Delta d_{\rm val} dx = -0.339\,, \tag{A.8}$$

what allows to find N and N' factors. The full set of input parameters for GRSV and GS distributions is as follows:

GRSV:

$Q_0^2 = 1\ \mathrm{GeV}^2$, $\Lambda_{\mathrm{QCD}} = 232$ MeV

$$\begin{aligned} \text{for } \Delta u_{\mathrm{val}}: \ N &= 1.964, \quad a_1 = 0.573,\ a_2 = 0.16, \quad b = 0.175, \\ A &= -0.583,\ B = 1.723,\ C = 3.436,\ D = 3.486, \\ \text{for } \Delta d_{\mathrm{val}}: \ N &= -0.162,\ a_1 = 0.376,\ a_2 = 0, \quad b = 0.529, \\ A &= 1.195, \quad B = 6.164,\ C = 2.726,\ D = 4.215, \end{aligned}$$

GS:

$Q_0^2 = 4\ \mathrm{GeV}^2$, $\Lambda_{\mathrm{QCD}} = 200$ MeV

$$\begin{aligned} \text{for } \Delta u_{\mathrm{val}}: \ N' &= 1.741, \quad a' = 0.578,\ \gamma = 9.38, \ \rho = -4.26,\ D' = 3.73, \\ \text{for } \Delta d_{\mathrm{val}}: \ N' &= -1.176,\ a' = 0.666,\ \gamma = 10.46,\ \rho = -5.10,\ D' = 4.73. \end{aligned}$$

In both GRSV and GS inputs we employ the LO fits. Thus the input parametrizations have final forms:

GRSV:

$$\Delta u_{\mathrm{val}} = 1.964 x^{-0.267}\left(1 - 0.583 x^{0.175} + 1.723 x + 3.436 x^{3/2}\right)(1-x)^{3.486}, \tag{A.9}$$

$$\Delta d_{\mathrm{val}} = -0.162 x^{-0.624}\left(1 + 1.195 x^{0.529} + 6.164 x + 2.726 x^{3/2}\right)(1-x)^{4.215}, \tag{A.10}$$

GS:

$$\Delta u_{\mathrm{val}} = 1.741 x^{-0.422}(1 + 9.38x - 4.26\sqrt{x})(1-x)^{3.73}, \tag{A.11}$$

$$\Delta d_{\mathrm{val}} = -1.176 x^{-0.334}(1 + 10.46x - 5.10\sqrt{x})(1-x)^{4.73}. \tag{A.12}$$

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