

CONSIDERATIONS ON ANOMALOUS VECTOR BOSON COUPLINGS

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We discuss the meaning of anomalous vector boson self couplings. Implications of present experimental constraints for future colliders are discussed. Results for triple vector boson production at the LHC are given.

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1. Introduction

The standard model is well established by the experiments at LEP and the Tevatron. Any deviations of the standard model can therefore be introduced only with care. Changes to the standard model come with different forms of severity. In order to see at what level anomalous vector boson couplings can be reasonably discussed one has to consider these cases separately. Changes to the gauge structure of the theory, that do not violate the renormalizability of the theory, *i.e.* the introduction of extra fermions or possible extensions of the gauge group are the least severe. They will typically generate small corrections to vector boson couplings via loop effects. In this case also radiative effects will be generated at lower energies. For the LHC the important thing in this case is not to measure the anomalous couplings precisely, but to look for the extra particles. This subject belongs naturally to the “extensions of the standard model” working group. We will not discuss it further. In the other case, a more fundamental role is expected for the anomalous couplings, implying strong interactions. In this case one has to ask oneself whether one should study a model with or without a fundamental Higgs boson.

Simply removing the Higgs boson from the standard model is a relatively mild change. The model becomes nonrenormalizable, but the radiative effects grow only logarithmically with the cut-off. The question is whether this scenario is ruled out by the LEP1 precision data. The LEP1 data appear to be in agreement with the standard model, with a preferred low Higgs mass. One is sensitive to the Higgs mass in three parameters, known as S,T,U or $\varepsilon_1, \varepsilon_2, \varepsilon_3$. These receive corrections of the form $g^2 \log(m_H/m_W) + \text{constant}$, where the constants are of order one. The logarithmic enhancement is universal and would also appear in models without a Higgs as $\log(\Lambda)$, where Λ is the cut-off, where new interactions should appear. Only when one can determine the three different constants independently, can one say that one has established the standard model. At present the data do not suffice to do this to great enough precision.

A much more severe change to the standard model is the introduction of non-gauge vector boson couplings. These new couplings violate renormalizability much more severely than simply removing the Higgs boson. Typically quadratically and quartically divergent corrections would appear to physical observables. It is therefore questionable, if one should study models with a fundamental Higgs boson, but with extra anomalous vector boson couplings. It is hard to imagine a form of dynamics that could do this. If the vector bosons become strongly interacting the Higgs probably would at most exist in an “effective” way. The most natural way is therefore to study anomalous vector boson couplings in models without a fundamental Higgs. Actually when one removes the Higgs boson the standard model becomes a gauged non-linear sigma-model. The non linear sigma-model is well known to describe low-energy pion physics. The “pions” correspond to the longitudinal degrees of freedom of the vector bosons. To f_π corresponds the vacuum expectation value of the Higgs field. Within this description the standard model corresponds to the lowest order term quadratic in the momenta, anomalous couplings to higher derivative terms. The systematic expansion in terms of momenta is known as chiral perturbation theory and is extensively used in meson physics.

Writing down the most general non-linear chiral Lagrangian containing up to four derivatives gives rise to a large number of terms, which are too general to be studied effectively. One therefore has to look for dynamical principles that can limit the number of terms. Of particular importance are approximate symmetry principles. In the first place one expects CP-violation to be small. We limit ourselves therefore to CP-preserving terms. In order to see what this means in practice it is advantageous to describe the couplings in a manifestly gauge-invariant way, using the Stückelberg formalism [1]. One needs the following definitions:

$$F_{\mu\nu} = \frac{i\tau_i}{2}(\partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\epsilon^{ijk}W_\mu^j W_\nu^k) \quad (1)$$

is the SU(2) field strength.

$$D_\mu U = \partial_\mu U + \frac{ig}{2}\tau_i W_\mu^i U - \frac{igtg\theta_w}{2}U\tau_3 B_\mu \quad (2)$$

is the gauge covariant derivative of the SU(2) valued field U , that describes the longitudinal degrees of freedom of the vector fields in a gauge invariant way.

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (3)$$

is the hypercharge field strength.

$$V_\mu = (D_\mu U)U^\dagger/g, \quad (4)$$

$$T = U\tau_3 U^\dagger \quad (5)$$

are auxiliary quantities having simple transformation properties. Excluding CP violation, the non standard three and four vector boson couplings are described in this formalism by the following set of operators:

$$\mathcal{L}_1 = \text{Tr}(F_{\mu\nu}[V_\mu, V_\nu]), \quad (6)$$

$$\mathcal{L}_2 = i\frac{B_{\mu\nu}}{2}\text{Tr}(T[V_\mu, V_\nu]), \quad (7)$$

$$\mathcal{L}_3 = \text{Tr}(TF_{\mu\nu})\text{Tr}(T[V_\mu, V_\nu]), \quad (8)$$

$$\mathcal{L}_4 = (\text{Tr}[V_\mu V_\nu])^2, \quad (9)$$

$$\mathcal{L}_5 = (\text{Tr}[V_\mu V_\mu])^2, \quad (10)$$

$$\mathcal{L}_6 = \text{Tr}(V_\mu V_\nu)\text{Tr}(TV_\mu)\text{Tr}(TV_\nu), \quad (11)$$

$$\mathcal{L}_7 = \text{Tr}(V_\mu V_\mu)(\text{Tr}[TV_\nu])^2, \quad (12)$$

$$\mathcal{L}_8 = \frac{1}{2}[(\text{Tr}[TV_\mu])(\text{Tr}[TV_\nu])]^2. \quad (13)$$

In the unitary gauge $U = 1$, one has

$$\mathcal{L}_1 = i[(cZ_{\mu\nu} + sF_{\mu\nu})W_\mu^+ W_\nu^- + Z_\nu/c(W_{\mu\nu}^+ W_\mu^- - W_{\mu\nu}^- W_\mu^+)] \quad (14)$$

+ gauge induced four boson vertices

$$\mathcal{L}_2 = i(cF_{\mu\nu} - sZ_{\mu\nu})W_\mu^+ W_\nu^-, \quad (15)$$

$$\mathcal{L}_3 = i(cZ_{\mu\nu} + sF_{\mu\nu})W_\mu^+ W_\nu^- \quad (16)$$

c and s are cosine and sine of the weak mixing angle and $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ and $W_{\mu\nu}^{+,-} = \partial_\mu W_\nu^{+,-} - \partial_\nu W_\mu^{+,-}$.

The standard model without Higgs corresponds to:

$$\mathcal{L}_{\text{EW}} = \frac{1}{2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{g^2 v^2}{4} \text{Tr}(V_\mu V^\mu). \quad (17)$$

2. Dynamical constraints

The list contains terms that give rise to vertices with minimally three or four vector bosons. Already with the present data a number of constraints and/or consistency conditions can be put on the vertices. The most important of these come from the limits on the breaking of the so-called custodial symmetry. If the hypercharge is put to zero the effective Lagrangian has a larger symmetry than $\text{SU}_L(2) \times \text{U}_Y(1)$, *i.e.* it has the symmetry $\text{SU}_L(2) \times \text{SU}_R(2)$. The $\text{SU}_R(2)$ invariance is a global invariance. Within the standard model this invariance is an invariance of the Higgs potential, but not of the full Lagrangian. It is ultimately this invariance that is responsible for the fact that the ρ -parameter, which is the ratio of charged to neutral current strength, is equal to one at the tree level. Some terms in the Lagrangian, *i.e.* the ones containing the hypercharge field explicitly or the terms with T , that project out the third isospin component violate this symmetry explicitly. These terms, when inserted in a loop graph, give rise to quartically divergent contributions to the ρ -parameter. Given the measurements this means that the coefficients of these terms must be extremely small. It is therefore reasonable to limit oneself to a Lagrangian, where hypercharge appears only indirectly via a minimal coupling, so without explicit T . This assumption means physically, that the ultimate dynamics that is responsible for the strong interactions among the vector bosons acts in the non-Abelian sector. Indeed one would normally not expect precisely the hypercharge to become strong. However, we know that there is a strong violation of the custodial symmetry in the form of the top-quark mass. Actually the top-mass almost saturates the existing corrections to the ρ -parameter, leaving no room for violations of the custodial symmetry in the anomalous vector boson couplings. We therefore conclude: *If there really are strong vector boson interactions, the mechanism for mass generation is unlikely to be the same for bosons and fermions.*

Eliminating the custodial symmetry violating interactions we are left with the simplified Lagrangian, containing \mathcal{L}_1 , \mathcal{L}_4 , \mathcal{L}_5 . Besides the vertices there are in principle also propagator corrections. We take the two-point functions without explicit T . Specifically, we add to the theory [2]

$$\mathcal{L}_{\text{hc,tr}} = \frac{-1}{2A_W^2} \text{Tr}[(D_\alpha F_{\mu\nu})(D^\alpha F^{\mu\nu})] + \frac{1}{4A_B^2} (\partial_\alpha B_{\mu\nu})(\partial^\alpha B^{\mu\nu}) \quad (18)$$

for the transverse degrees of freedom of the gauge fields and

$$\mathcal{L}_{\text{hc,lg}} = \frac{g^2 v^2}{4\Lambda_V^2} \text{Tr}[(D^\alpha V^\mu)(D_\alpha V_\mu)] \quad (19)$$

for the longitudinal ones, where the Λ_X parametrize the quadratic divergences and are expected to represent the scales where new physics comes in. In phenomenological applications these contributions give rise to formfactors in the propagators [2, 8]. Introducing such cut-off dependent propagators in the analysis of the vector boson pair production is similar to having s -dependent triple vector boson couplings, which is the way the data are usually analysed.

This effective Lagrangian is very similar to the one in pion-physics. Indeed if one takes the limit, v fixed and gauge couplings to zero, one finds the standard pion Lagrangian. As it stands one can use the LEP1 data to put a limit on the terms in the two point vertices. Using a naive analysis one finds [2] $1/\Lambda_B^2 = 0$. For the other two cut-offs one has:

- A. The case: $\Lambda_V^2 > 0$, $\Lambda_W^2 < 0$, $|A_V| > 0.49 \text{ TeV}$, $|A_W| > 1.3 \text{ TeV}$
- B. The case: $\Lambda_V^2 < 0$, $\Lambda_W^2 > 0$, $|A_V| > 0.74 \text{ TeV}$, $|A_W| > 1.5 \text{ TeV}$

This information is important for further limits at high energy colliders, as it tells us, how one has to cut off off-shell propagators. We notice that the limits on the form factors are different for the transversal, longitudinal and hypercharge formfactors. The precise limits are somewhat qualitative and should be taken as such. However they show that effective cut-off form factors should be taken around 500 GeV. It is certainly not correct to put them at the maximum machine energy. Further information comes from the direct measurements of the three-point couplings at LEP2, which tell us that they are small. Similar limits at the Tevatron have to be taken with some care, as there is a cut-off dependence. As there is no known model that can give large three-point interactions, we assume for the further analysis of the fourpoint vertices, that the three point anomalous couplings are absent. On the remaining two four-point vertices two more constraints can be put. The first comes from consistency of chiral perturbation theory [3]. Not every effective chiral Lagrangian can be generated from a physical underlying theory.

A second condition comes from the ρ -parameter. Even the existing violation of the custodial symmetry, though indirect via the minimal coupling to hypercharge, gives a contribution to the ρ -parameter. It constrains the combination $5g_4 + 2g_5$. The remaining combination $2\mathcal{L}_4 - 5\mathcal{L}_5$ is fully unconstrained by experiment and gives in principle a possibility for very strong interactions to be present. However this particular combination does not

seem to have any natural interpretation from underlying dynamics. Therefore one can presumably conclude that both couplings g_4, g_5 are small. There is a loophole to this conclusion, namely when the anomalous couplings are so large that the one-loop approximation, used to arrive at the limits, is not consistent and resummation has to be performed everywhere. This is a somewhat exotic possibility, that could lead to very low-lying resonances, which ought to be easy to discover at the LHC [4].

3. LHC processes

Given the situation described above one has to ask oneself, what the LHC can do and in which way the data should be analysed. There are essentially three processes that can be used to study vector boson vertices: vector boson pair production, vector boson scattering, triple vector boson production. About the first two we have only a few remarks to make. They were already discussed in detail by several authors.

3.1. Vector boson pair production

Vector boson pair production can be studied in a relatively straightforward way. The reason is that here the Higgs boson does not play a role in the standard model, as we take the incoming quarks to be massless. Therefore naive violations of unitarity can be compensated by the introduction of smooth form-factors.

One produces two vector bosons via normal standard model processes with an anomalous vertex added. The extra anomalous coupling leads to unitarity-violating cross-sections at high energy. As a total energy of 14 TeV is available this is in principle a serious problem. It is cured by introducing a formfactor for the incoming off-shell line connected to the anomalous vertex. Naively this leads to a form-factor dependent limit on the anomalous coupling in question. The LEP1 data gives a lower limit on the cut-off to be used inside the propagator. When one wants an overall limit on the anomalous coupling one should use this value. This is particularly relevant for the Tevatron. Here one typically takes a cut-off of 2 TeV. This might give too strict a limit, as the LEP1 data indicate that the cut-off can be as low as 500 GeV. For practical purposes the analysis at the Tevatron should give limits on anomalous couplings for different values of the cut-off form factors, including low values of the cut-off. For the analysis at the LHC one has much larger statistics. This means, that one can do better and measure limits on the anomalous couplings as a function of the invariant mass of the produced system. This way one measures the anomalous formfactor completely.

3.2. Vector boson scattering

Here the situation is more complicated than in vector boson pair production. The reason is that within the standard model the process cannot be considered without intermediate Higgs contribution. This would violate unitarity. However the incoming vector bosons are basically on-shell and this allows the use of unitarization methods, as are commonly used in chiral perturbation theory in pion physics. These methods tend to give rise to resonances in longitudinal vector boson scattering. The precise details depend on the coupling constants. The unitarization methods are not unique, but generically give rise to large $I = J = 0$ and/or $I = J = 1$ cross-section enhancements. The literature is quite extensive. A good introduction is [5]; a recent review is [6].

3.3. Triple vector boson production

In this case it is not clear how one should consistently approach an analysis of anomalous vector boson couplings. Within the standard model the presence of the Higgs boson is essential in this channel. Leaving it out one has to study the unitarization. This unitarization has to take place not only on the two-to-two scattering subgraphs, as in vector boson scattering, but also on the incoming off-shell vector boson, decaying into three real ones. The analysis here becomes too arbitrary to derive very meaningful results. One cannot confidently calculate anything here without a fully known underlying model of new strong interactions. Also measurable cross sections tend to be small, so that the triple vector boson production is best used as corroboration of results in vector boson scattering. Deviations of standard model cross sections could be seen, but the vector boson scattering would be needed for interpretation.

TABLE I

Total number of events, no cuts, no branching ratios

m_{Higgs}	200	400	600	800
$W^+W^-W^-$	11675	5084	4780	4800
$W^+W^+W^-$	20250	9243	8684	8768
W^+W^-Z	20915	11167	10638	10685
W^-ZZ	2294	1181	1113	1113
W^+ZZ	4084	2243	2108	2165
ZZZ	4883	1332	1087	1085

One therefore needs the standard model results. The total number of vector boson triples is given in Table I. We used an integrated luminosity of 100 fb^{-1} and an energy of 14 TeV throughout.

One sees from this table that a large part of the events comes from associated Higgs production, when the Higgs is light. However for the study of anomalous vector boson couplings, the heavier Higgs results are arguably more relevant. Not all the events can be used for the analysis. If we limit ourselves to events, containing only electrons, muons and neutrinos, assuming just acceptance cuts we find Table II.

TABLE II

Pure leptons, $|\eta| < 3$, $p_T > 20$ GeV, no cuts on neutrinos

m_{Higgs}	200	400	600	800
$W^+W^-W^-$	68	28	25	25
$W^+W^+W^-$	112	49	44	44
W^+W^-Z	32	17	15	15
W^-ZZ	1.00	0.51	0.46	0.45
W^+ZZ	1.70	0.88	0.79	0.79
ZZZ	0.62	0.18	0.13	0.12

We see that very little is left, in particular in the processes with at least two Z -bosons, where the events can be fully reconstructed. In order to see how sensitive we are to anomalous couplings we assumed a $4Z$ coupling with a formfactor cut-off at 2 TeV. We make here no correction for efficiencies *etc.* Using the triple Z -boson production, assuming no events are seen in 100 fb^{-1} , we find a limit $|g_4 + g_5| < 0.09$ at the 95% CL, where g_4 and g_5 are the coefficients multiplying the operators \mathcal{L}_4 and \mathcal{L}_5 . This is to be compared with $-0.15 < 5g_4 + 2g_5 < 0.14$ [7] or $-0.066 < (5g_4 + 2g_5)\Lambda^2(\text{TeV}) < 0.026$ [2, 8]. So the sensitivity is not better than present indirect limits. Better limits exist in vector boson scattering [9] or at a linear collider [10]. In the following tables we present numbers for observable cross sections in different decay modes of the vector bosons. We used the following cuts.

$$\begin{aligned}
 |\eta|_{\text{lepton}} &< 3 \\
 |p_T|_{\text{lepton}} &> 20 \text{ GeV} \\
 |\eta|_{\text{jet}} &< 2.5 \\
 |p_T|_{\text{jet}} &> 40 \text{ GeV} \\
 \Delta R_{\text{jet,lepton}} &> 0.3 \\
 \Delta R_{\text{jet,jet}} &> 0.5 \\
 |p_T|_{2\nu} &> 50 \text{ GeV}
 \end{aligned}$$

(20)

States with more than two neutrinos are not very useful because of the background from two vector boson production. We did not consider final states containing τ -leptons.

TABLE III

<i>ZZZ</i> production in different decay modes					
m_{Higgs}	200	300	400	500	600
6ℓ	0.62	0.29	0.18	0.14	0.13
$4\ell, 2\nu$	5.1	2.5	1.5	1.2	1.1
$4\ell, 2j$	6.6	3.8	2.2	1.7	1.4
$2\ell, 2j, 2\nu$	34	20	12	9.0	7.7
$2\ell, 4j$	24	19	11	7.6	6.0
$2\nu, 4j$	37	34	21	15	11
$6j$	25	31	19	12	8.7

With the given cuts the total number of events to be expected is rather small, in particular since we did not consider the reduction in events due to experimental efficiencies, which may be relatively large, because of the large number of particles in the final state. For the processes containing jets in the final state, there will be large backgrounds due to QCD processes. A final conclusion on the significance of the triple vector boson production for constraining the four vector boson couplings will need more work, involving detector Monte Carlo calculations.

TABLE IV

<i>WWZ</i> production in different decay modes					
m_{Higgs}	200	300	400	500	600
$4\ell, 2\nu$	31	20	17	16	15
$3\ell, 2j, 1\nu$	51	40	31	28	26
$2\ell, 4j$	19	22	17	14	13
$2\nu, 4j$	63	74	60	51	48
$2\ell, 2j, 2\nu$	102	68	54	49	48
$1\ell, 4j, 1\nu$	262	196	140	127	127
$6j$	86	104	78	62	56

However it is probably fair to say from the above results, that no very strong constraints will be found from this process at the LHC, but it is useful as a cross-check with other processes. It may provide complementary information if non-zero anomalous couplings are found.

TABLE V

 ZZW^- (upper) and ZZW^+ (lower) production in different decay modes

m_{Higgs}	200	300	400	500	600
$5\ell, 1\nu$	0.45	1.04	0.63	0.52	0.47
	0.80	1.69	1.08	0.91	0.81
$3\ell, 2j, 1\nu$	3.37	6.89	5.36	4.18	3.73
	5.9	11.5	9.3	7.4	6.5
$1\ell, 4j, 1\nu$	7.6	11.5	12.4	10.0	8.4
	13.3	20.0	21.6	18	15
$4\ell, 2j$	0.29	1.0	0.54	0.38	0.32
	0.49	1.6	0.91	0.65	0.54
$2\ell, 2j, 2\nu$	2.0	6.5	3.5	2.5	2.2
	3.4	10.7	6.1	4.4	3.7
$2\ell, 4j$	2.5	7.4	5.4	3.6	2.9
	4.7	9.5	9.5	6.9	5.6
$4j, 2\nu$	8.9	27	18	12.6	10.4
	195	54	38	28	23
$6j$	5.3	12.3	13.3	8.8	7.4
	9.1	20.7	23	16	12.5

TABLE VI

 $W^-W^+W^+$ production in different decay modes

m_{Higgs}	200	300	400	500	600
$3\ell, 3\nu$	66	44	37	35	33
$\ell^+\ell^+, 2j, 2\nu$	57	43	31	26	24
$\ell^+\ell^-, 2j, 2\nu$	13	7.9	5.3	4.4	4.0
$\ell^+, 4j, 1\nu$	148	129	86	66	58
$\ell^-, 4j, 1\nu$	99	61	36	26	23
$6j$	50	74	46	32	25

TABLE VII

$W^+W^-W^-$ production in different decay mode

m_{Higgs}	200	300	400	500	600
$3\ell, 3\nu$	40	26	22	21	20
$\ell^-\ell^-, 2j, 2\nu$	34	25	17	14	13
$\ell^+\ell^-, 2j, 2\nu$	78	45	30	25	23
$\ell^-, 4j, 1\nu$	90	76	49	37	33
$\ell^+, 4j, 1\nu$	59	35	20	15	13
$6j$	29	43	26	18	14

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