APPLYING THE ELASTIC MODEL FOR VARIOUS NUCLEUS–NUCLEUS FUSION

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The Elastic Model of two free parameters $m\,,d$ given by Scalia has been used for wider energy regions to fit the available experimental data for potential barriers and cross sections. In order to generalize Scalia's formula in both sub- and above-barrier regions, we calculated $m\,,d$ for pairs rather than those given by Scalia and compared the calculated cross sections with the experimental data. This makes a generalization of the Elastic Model in describing fusion process. On the other hand, Scalia's range of interacting systems was $24 \leq A \leq 194$ where A is the compound nucleus mass number. Our extension of that model includes an example of the pairs of A larger than his final limit aiming to make it as a general formula for any type of reactants: light, intermediate or heavy systems. A significant point is the comparison of Elastic Model calculations with the well known methods studying complete fusion and compound nucleus formation, namely with the resultants of using Proximity potential with either Sharp or Smooth cut-off approximations.

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1. Introduction

Fusion occurs when two nuclei come together with sufficient kinetic energy to overcome their mutual electrostatic repulsion and form a new nucleus having charge and baryon numbers equal to the sum of those of the reactants [1]. Study of heavy-ion fusion has an increasing interest as a greater variety of heavy-ion beams are becoming available. Large accumulated data and much theoretical activity is devoted to understand the basic terms for describing the process of nuclear fusion of heavy-ion (barrier heights, fusion radii, critical angular momenta, *etc.*) [2]. Depending upon the available kinetic energy, fusion will take place either by passing over (above-barrier fusion) or by quantum mechanical tunnelling (sub-barrier fusion) through the Coulomb barrier generated by the electrostatic repulsion [1]. Cross sections for near- and sub-barrier heavy-ion fusion can be some orders of magnitude larger than the predictions of traditional models that are quite successful above the barrier [3]. Low energy fusion of two colliding ions depends on the possibility to overcome the repulsive potential barrier between them. At classical level, the transmission coefficient as a function of bombarding energy changes suddenly from zero to one at $E = V_B$. Quantal effects smooth out this transition. The presence of couplings to other degrees of freedom modifies substantially the penetration and enhances the sub-barrier fusion cross sections [4].

2. Theory

In order to calculate fusion cross section, Scalia deduced the following formula within his successive work on the elastic model [5]

$$\sigma_{\rm fus} = \pi \left(\frac{2\eta}{k}\right)^2 \exp\Big(-\exp\left(\exp[mE+d]\right)\Big)\Big(1+\exp\left(-\exp(\exp[mE+d])\right)\Big)\,, \tag{1}$$

where η is the Coulomb parameter, k is the wave number, and m, d are the two free parameters, m is expressed in (MeV^{-1}), d is dimensionless. These parameters are energy-independent and are different for different reactions. This formula is valid for the sub-barrier fusion systems with $24 \leq A \leq 194$ as A is of the compound nucleus. Scalia tabulated the two free parameters for 53 interacting systems within the above considered range.

In order to expand Scalia's range of interacting systems, we calculate the m, d parameters and cross sections for new systems, and compare our cross sections with those obtained by the well known analytical form of the reaction cross section [6]

$$\sigma_{\rm rec} = \pi\lambda^2 \sum_{l=0}^{l}(2l+1)P_l(E)T_l(E)\,, \tag{2}$$

where $T_l(E)$ is the transmission coefficient and $P_l(E)$ is the probability of the specified process to take place.

For fusion, we have $P_l(E) = 1$. The transmission coefficient in sharp cut-off approximation is given by [6]

$$T_l(E) = \begin{cases} 1\,, & \text{for } l \leq l_{\max} \\ 0\,, & \text{for } l > l_{\max} \end{cases}\,, \tag{3}$$

where $l_{\rm max}$ is the maximum angular momentum, and so the cross section in sharp cut-off approximation will be

$$\sigma_{\rm fus} = \pi\lambda^2 \sum_{l=0}^{l_{\rm max}} (2l+1)\,. \tag{4}$$

On the other hand, a smooth cut-off approximation [7] gives the value of fusion cross section as:

$$\sigma_{\rm fus} = \pi\lambda^2 \sum_{0}^{l_{\rm max}} \frac{2l+1}{1+\exp\left[\frac{2\pi(V-E_{cm})}{\hbar\omega}\right]}\,. \tag{5}$$

The barrier potential, V, contains a nuclear part taken from the proximity formalism [18]. The present study gives a wide range of comparisons between the calculated $\sigma_{\rm fus}$ due to Eq. (1) and in parallel due to Eqs. (4) and (5) to fit the available experimental data.

3. Results and discussion

To apply Scalia's formula for 18 new pairs systems, we calculated m, d parameters using a non-linear least square program [19] for these pairs and they were used to recover both of the sub- and above barrier fusion regions. Results are shown in Table I. Inserting these values into Eq. (1), to

TABLE I

Values of m, d parameters due to present work for different systems

Systems	m	d	Systems	m	d
^{30}Si + ^{24} Mg	0.163843	4.340716	^{27}Al + ^{70} Ge	0.087249	5.173701
^{28}Si + ^{26} Mg	0.158695	4.295850	^{19}F + ^{93} Nb	0.097973	5.104471
^{28}Si + ^{100} Mo	0.063515	5.059009	^{28}Si + ^{93} Nb	0.076786	6.043992
^{48}Ti + ^{58} Ni	0.064455	5.500944	^{35}Cl + ^{116} Sn	0.042273	4.785914
^{50}Ti + ^{60} Ni	0.066500	5.541179	^{35}Cl + ^{124} Sn	0.044458	5.078492
^{64}Ti + ^{64} Ni	0.062261	5.129971	^{16}O + ^{144} Sm	0.081789	5.422129
^{27}Al + ^{74} Ge	0.080216	4.723900	^{17}O + ^{144} Sm	0.077014	5.051195
^{27}Al + ^{73} Ge	0.080219	4.768351	^{4}He + ^{154} Sm	0.284878	4.771994
^{27}Al + ^{72} Ge	0.086942	5.141291	^{16}O + ^{186} W	0.067235	5.019201

calculate fusion cross sections, we compared the results with those calculated by Eqs. (4) and (5). To show the ability of recovering wider energy range, we calculate fusion cross sections of the undertaken pairs and compare these values with the available measured data. It is clear that Eq. (1) can be successfully used to fit the available data as largely as Eq. (4) and even in comparison with the most accurate form (5). Results are shown on Figs. 1–10.

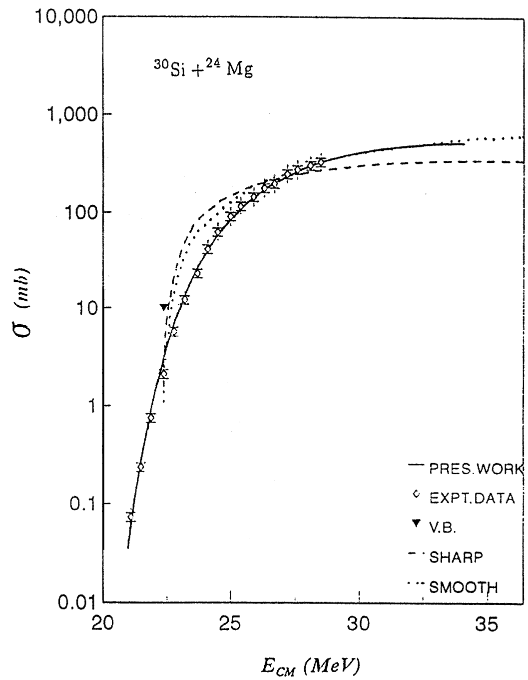
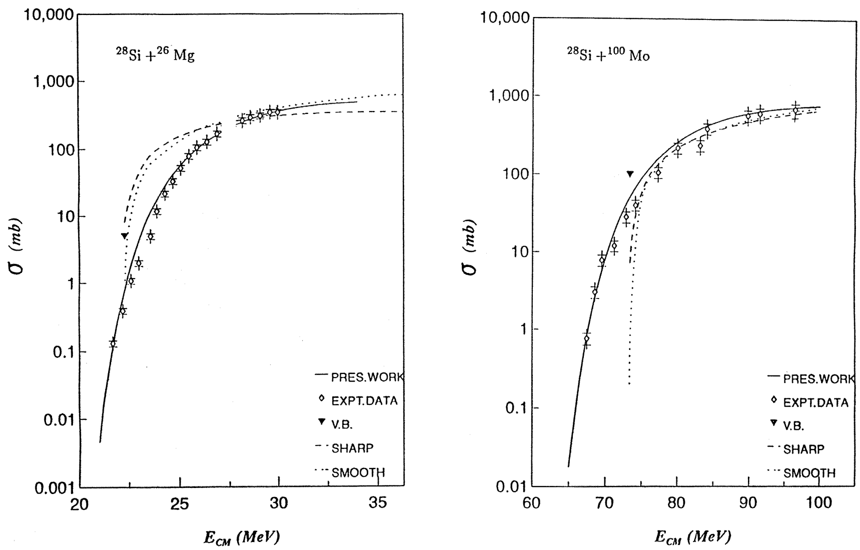
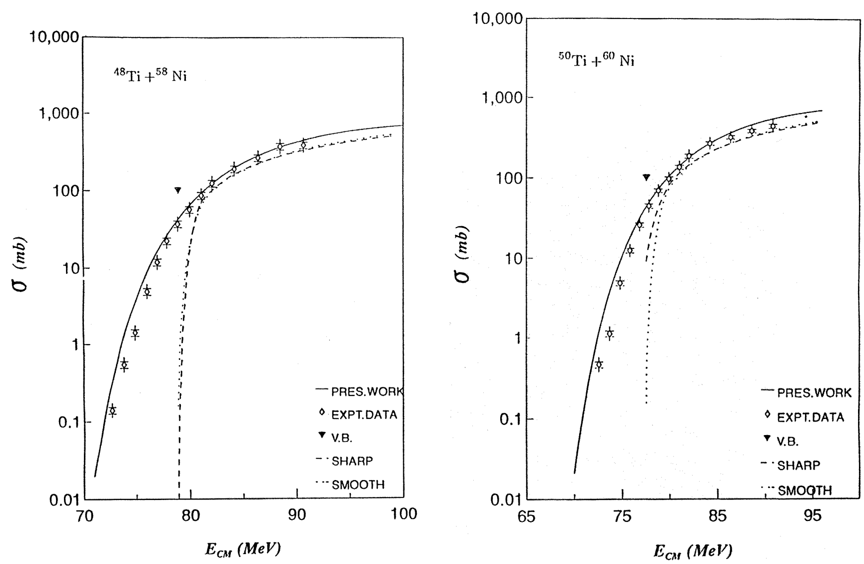


Fig. 1. Calculated fusion cross section for the reaction ^{30}Si $+^{24}$ Mg compared to experimental data [10]. The solid curve is calculated from Eq. (1), the long dashed curve from Eq. (4) and the dotted curve from Eq. (5). The sign (▼) indicates the value of fusion barrier.



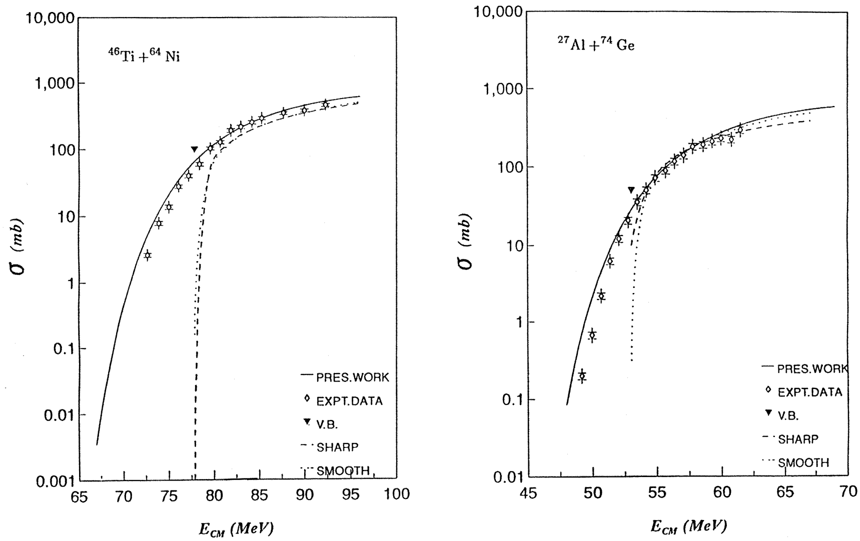
(a) (b)

Fig. 2. (a) — Same as for Fig. 1 for the reaction ^{28}Si +^{26} Mg [10], (b)— Same as for Fig. 1 for the reaction ^{28}Si +^{100} Mo [11].



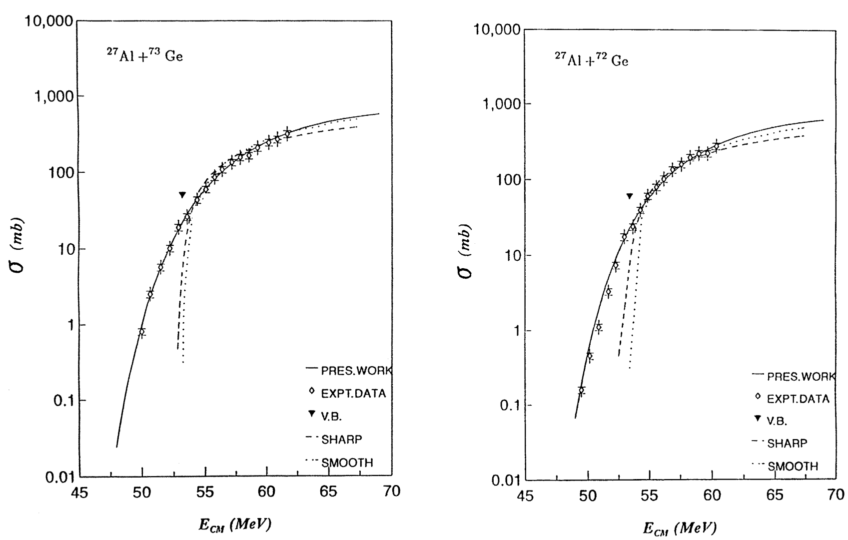
(a) (b)

Fig. 3. (a) — Same as for Fig. 1 for the reaction ^{48}Ti +^{58} Ni [12], (b) — Same as for Fig. 1 for the reaction ^{50}Ti +^{60} Ni [13].



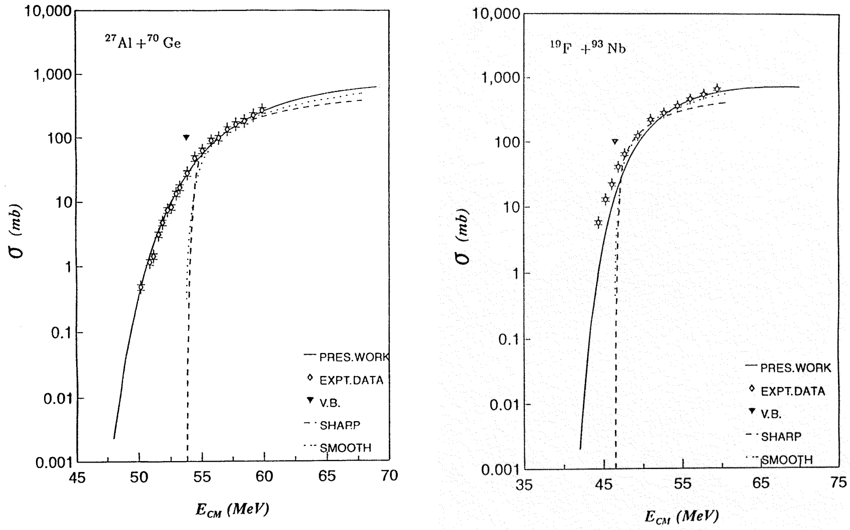
(a) (b)

Fig. 4. (a) — Same as for Fig. 1 for the reaction $^{46}\mathrm{Ti} +^{64}$ Ni [13], (b) — Same as for Fig. 1 for the reaction $^{27}\mathrm{Al} +^{74}$ Ge [14].



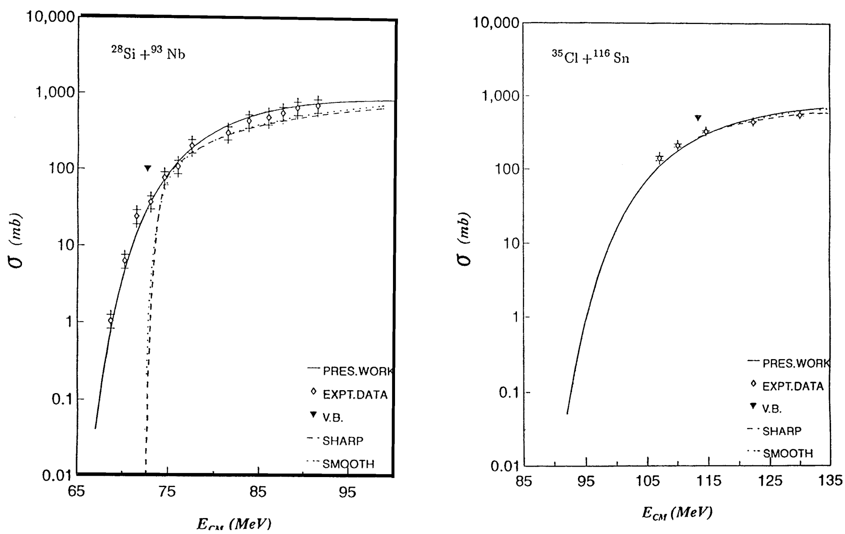
(a) (b)

Fig. 5. (a) — Same as for Fig. 1 for the reaction $^{27}\mathrm{Al} +^{73}$ Ge, (b) — Same as for Fig. 1 for the reaction $^{27}\mathrm{Al} +^{72}$ Ge [14].



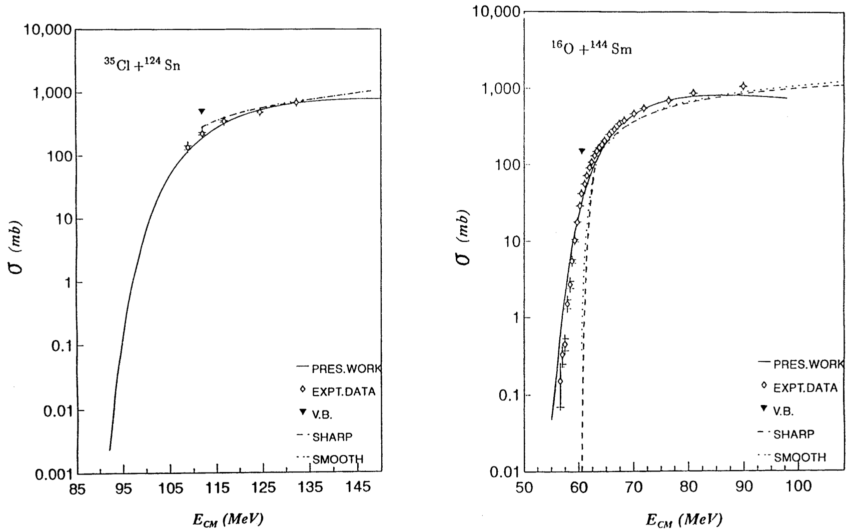
(a) (b)

Fig. 6. (a) — Same as for Fig. 1 for the reaction $^{27}\mathrm{Al} + ^{70}\mathrm{Ge}$ [14], (b) — Same as for Fig. 1 for the reaction $^{19}\mathrm{F} + ^{93}\mathrm{Nb}$ [13].



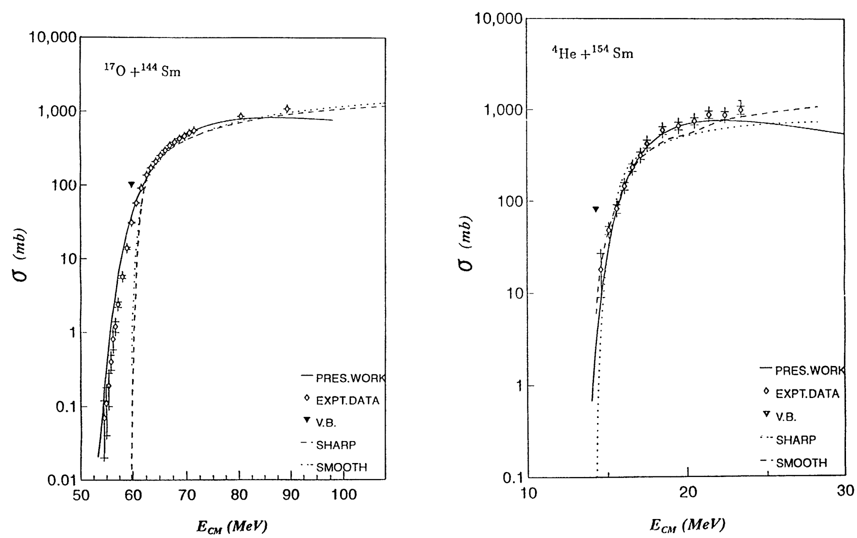
(a) (b)

Fig. 7. (a) — Same as for Fig. 1 for the reaction $^{28}\mathrm{Si} + ^{93}\mathrm{Nb}$ [15], (b) — Same as for Fig. 1 for the reaction $^{35}\mathrm{Cl} + ^{116}\mathrm{Sn}$ [16].



(a) (b)

Fig. 8. (a) — Same as for Fig. 1 for the reaction $^{35}Cl + ^{124}Sn$ [16], (b) — Same as for Fig. 1 for the reaction $^{16}O + ^{144}Sm$ [17].



(a) (b)

Fig. 9. Same as for Fig. 1 for the reaction $^{17}O + ^{144}Sm$ [17], (b) — Same as for Fig. 1 for the reaction $^{4}He + ^{154}Sm$ [18].

Due to all of these figures, we can note that the maximum excitation energy exceeds the barrier heights of the studied pairs. This will display our result that the elastic model can be used to fit measured excitation functions for both below and above barrier fusion channels. In terms of the available experimental data a percentage approaching can be defined as:

$$\alpha = \frac{E_e - V_B}{V_B} \times 100\,, \tag{6}$$

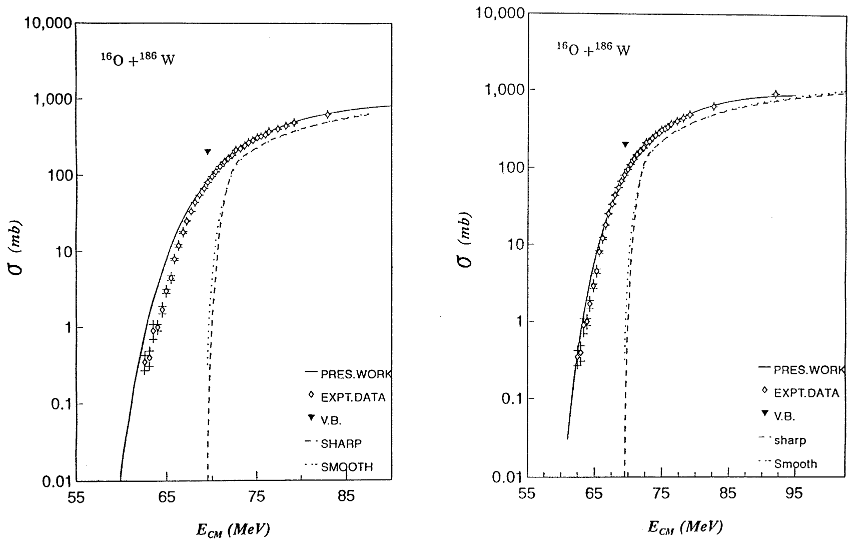
where E_e is the upper limit of an energy region (Table II) within which it is given an accurate fit of the measured cross sections [8–17]

From Table II, it can be seen that the value of α increases as the mass number of the compound nucleus increases.

TABLE II

Percentage approach parameter (α) for different systems

Systems	V_B	$E_{\max}$	$E - V_B$	α
^{30}Si +^{24} Mg	22.39	28.50 [10]	6.11	28
^{28}Si +^{26} Mg	22.31	30.00 [10]	7.69	35
^{28}Si +^{100} Mo	73.40	96.50 [11]	23.10	32
^{48}Ti +^{58} Ni	78.94	90.70 [12]	11.76	15
^{50}Ti +^{60} Ni	77.52	90.80 [13]	13.28	17
^{64}Ti +^{64} Ni	77.84	92.30 [13]	14.46	19
^{27}Al +^{74} Ge	53.05	61.60 [14]	8.55	16
^{27}Al +^{73} Ge	53.23	61.70 [14]	8.47	16
^{27}Al +^{72} Ge	53.42	60.40 [14]	6.98	13
^{27}Al +^{70} Ge	53.82	59.90 [14]	6.08	11
^{19}F +^{93} Nb	46.49	59.40 [13]	12.91	28
^{28}Si +^{93} Nb	72.66	91.40 [15]	18.74	26
^{35}Cl +^{116} Sn	113.42	130.10 [16]	16.68	15
^{35}Cl +^{124} Sn	111.88	132.30 [16]	20.42	18
^{16}O +^{144} Sm	60.58	89.90 [17]	29.32	48
^{17}O +^{144} Sm	59.77	89.30 [17]	29.53	49
^{4}He +^{154} Sm	14.29	23.40 [18]	9.11	64
^{16}O +^{186} W	69.50	92.00 [17,19]	22.50	32



(a) (b)

Fig. 10. (a) — Same as for Fig. 1 for the reaction $^{16}O + ^{186}W$ [19], (b) — Same as for Fig. 1 for the reaction $^{16}O + ^{186}W$ [17]

Another significant note is clear from Fig. 10, where the compound nucleus mass number exceeds effectively the maximum limit given by Scalia, namely $A = 16 + 186 = 202 > 194$. This will guide us to deduce the final result as that the elastic model (1) could be significantly reliable for lighter interacting pairs.

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