Vol. 31 (2000) ACTA PHYSICA POLONICA B No 8

THE ELEMENTARY METHOD IN PAIRING ENERGY III. NEUTRON–PROTON VERSUS LIKE-NUCLEON CORRELATIONS*

S. Szpikowski

Institute of Physics, M. Curie-Skłodowska University
Radziszewskiego 10, 20-031 Lublin, Poland
and ECT Villa Tambosi, Strada delle Tabarelle 286
38050 Vilazzano (Trento) Italy

(Received June 9, 2000)

The neutron–proton pairing correlations have been analysed in comparison with like-nucleon correlations by means of the elementary method based on the group theory treatment. Analytical formulae allow to compare the contribution of the neutron–proton interaction only, which is in several cases the main ingredient of the total pairing interaction. The obtained formulae have also been applied to the binding energy (the congruence energy).

PACS numbers: 21.60.Cs

1. Introduction

Pairing interactions as a model short-range residual interaction have a very long history. Racah [1] gave an exact formula for the electron pairing energy in the degenerated (one level) case in the L–S coupling scheme. Racah was also the first who introduced the quantum *seniority* number as the number of non-paired particles. The so-called seniority scheme in L–S coupling was generalized by Flowers [2] as well as Edmonds and Flowers [3] for a nuclear j–j coupling with the similar notion of the seniority quantum number. The problem was considered with the help of orthogonal, in L–S coupling, and symplectic, in j–j coupling, groups of dimensions $2l+1$ or $2j+1$ depending on the space of a considered energy level. That was, however, the limitation of a practical use of the group theory, because every shell was connected with a different dimension group, which did not allow for treatment of more than one energy level. Since 1958 the situation has been

* Parts I and II are given in [23,24].

improved by the BCS theory of an approximate treatment of the pairing interaction in the solid state physics [4]. The BCS theory was soon applied to nuclear physics by Mottelson [5] and Belaev [6]. The BCS theory, at that time, was applied only for treatment of like-particles. Further progress was made, when the so-called quasi-spin method was introduced. Wada, Takano and Fokuda [7] were the first who oserved simple and closed commutation relations of the second order creation and annihilation operators which were recognised as characteristic relations of the Lie algebra for the spin SU(2) group and contributed to the name of the quasi-spin method. The symmetry quasi-spin groups were found to be complementary to the known unitary groups of Racah and Flowers. The most important feature of the quasi-spin symmetry was provided by the same dimension symmetry groups for any l- and j-levels which allowed for an exact group theory treatment of many level problem. In such a way the quasi-spin method was used as a probe of the BCS approximation.

Kerman [8] and Kerman, Lawson and Mac Farlane [9] applied the quasi-spin method for any set of like-nucleon energy levels. Almost at the same time (1961) Helmers [10] pointed out that the bilinear products of proton and neutron creation and annihilation operators generate transformations of the orthogonal group SO(5) (or Sp(4)) for the j–j coupling scheme. Later several authors from different nuclear centers almost simultaneously but independently developed the quasi-spin method for both: j–j coupling with the symmetry SO(5), and L–S coupling, with the symmetry SO(8) [11–18]. Since 1964 there have appeared hundreds of papers devoted to the quasi-spin method not only in the nucleon pairing correlations but also in quarks and interacting bosons.

Although the quasi-spin method allowed for accurate treatment of nn, pp and np pairing correlations on the same footing, the neutron–proton part had been for a long time neglected because of a great difference in the energy between the proton and neutron single particle shells of heavy nuclei. Recently, the np correlations have again attracted attention for light as well as heavy nuclei with $N \approx Z$, where neutrons and protons are placed on the same single-particle energy level. The attention has been paid to the former papers from the sixties, where the accurate solution of the pairing interaction was obtained with the help of the orthogonal groups SO(5) and SO(8). For example, in the paper by Engel *et al.* [19] the formulae for the np part of the pairing Hamiltonian have been given for the special cases of $T = T_0$ and $T = T_0 + 1$ and then the exact results have been compared with the Fock–Bogoliubov approximate treatment. The interplay between like-particles and neutron–proton isovector correlations in the nuclei near $N = Z$ has also been discussed. However, one needs to develop the isospin broken formula for $T > T_0 + 1$ to draw general conclusions. In another paper by

Engel *et al.* [20] the symmetry SO(8) was employed to consider not only the isovector but also the isoscalar part of the broken proton–neutron pairing correlations. The paper by Civitarese *et al.* [21] makes an attempt to prove that the isospin symmetric pairing Hamiltonian treated in the Bogolyubov transformation fails to describe the physical nuclear states. Instead, one has to consider the broken pairing Hamiltonian and hence, once again the algebraic formulae for *nn*, *pp* and *np* parts of pairing interactions are needed to draw a proper conclusion. However, those formulae have been obtained only for special cases, which either prevent from generalized conclusions or even lead to improper generalized conclusions. The neutron–proton pairing interaction by means of the same model Hamiltonian as in the presented paper has been recently considered for some deformed nuclei [28].

It is justified to make an attempt to obtain general algebraic forms for the three parts of the broken in $\mathrm{SU}_T(2)$ pairing Hamiltonian. The formulae can constitute the basis for the discussion of the interplay between like-particles and *np* pairing interactions. For this purpose we have employed the elementary method in pairing interactions [22, 23], which will be generalized to obtain the T_0 dependence of energy formulae.

The next section will present a short review of the elementary pairing method and then we will extend it to treat separately *nn*, *pp* and *np* pairing correlations, which allows to introduce the non-equal strength factors for these interactions. The section will also present general conclusions about the competition of these pairing parts. The last section includes examples of algebraic pairing formulae.

2. The elementary method

At the beginning of this section we will follow the presentation of the method and notation given in [23]. Let us assume a configuration of nucleons on the j-level with $T = T_0$, for the neutrons — the higher part, and the protons — the lower part of figure 1.

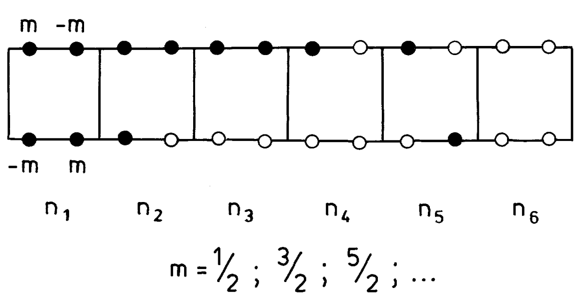

Fig. 1. Schematic configuration of neutrons (upper line) and protons (lower line) in the state $|\nu t; nT = T_0\rangle$.

The numbers n_i denote the numbers of the same four-state structures with different $m > 0$. The physical quantum numbers are given by n_i:

$$\begin{aligned}
&\sum_i n_i = \frac{1}{2}(2j+1) \equiv \Omega\,,\\
&4n_1 + 3n_2 + 2n_3 + n_4 + 2n_5 = n\,,\\
&\frac{1}{2}n_2 + n_3 + \frac{1}{2}n_4 = T\,,\\
&\frac{1}{2}n_2 + \frac{1}{2}n_4 = t\,,\\
&n_2 + n_4 + 2n_5 = \nu
\end{aligned} \tag{1}$$

or

$$\begin{aligned}
&n_1 + \frac{1}{2}n_2 = \frac{n}{4} - \frac{\nu}{4} - \frac{T}{2} + \frac{t}{2}\,,\\
&n_3 = T - t\,,\\
&n_4 = 2t - n_2\,,\\
&n_5 = \frac{\nu}{2} - t\,,\\
&n_6 = \Omega - T - \frac{\nu}{2} - n_1\,,
\end{aligned} \tag{2}$$

where n is the nucleon number, ν — the seniority number, t — the reduced isotopic spin (the isotopic spin of unpaired nucleons). The structure n_5 involves np pairs with the same m — hence it is a symmetric configuration in a m-space and in an isotopic space it must be an antisymmetric one with a zero isospin contribution. The symmetric in $\mathrm{SU}_T(2)$ pairing Hamiltonian, H_{pair}, is constructed in terms of the pair creation and annihilation operators for nn, pp and np pairs

$$\frac{H_{\mathrm{pair}}}{-G} \equiv H'_{\mathrm{pair}} = S_+^n S_-^n + S_+^p S_-^p + \frac{1}{2} S_+^{np} S_-^{np}\,, \tag{3}$$

where G is the strength of the pairing interaction.

The elementary method is based on the following rule: the H'_{pair} annihilates a pair of nucleons coupled to $J = 0$ and $T = 1$ and creates such a pair either in the same place or in the other two-particle empty space. The number of annihilation–creation actions is the pair energy in $(-G)$ units. This simple rule can be complemented with two additional remarks:

(i) the number $\frac{1}{2}$ in front of the np part of (3) is a square of the Clebsch–Gordan coefficient of coupling a neutron–proton pair to a total two-particle $T = 1$. That factor $\frac{1}{2}$ must be put in front of the neutron–proton annihilation–creation actions.

(ii) If an annihilation–creation action changes the structure of the four state blocks in figure 1, then an additional structure factor w_1 must be put in front of nn or pp actions and the factor w_2 — in front of np actions.

Application of these rules to the initial state in figure 1 gives:

$$E_n = n_1(1 + w_1 n_6) + n_2(1 + w_1 n_6) + n_3(1 + n_6)\,, \tag{4}$$

$$E_p = n_1(1 + n_3 + w_1 n_4 + w_1 n_6)\,, \tag{5}$$

$$E_{np} = 2 \cdot \frac{1}{2} n_1(1 + w_2 n_4 + 2w_2 n_6) + \frac{1}{2} n_2(1 + n_4 + 2w_2 n_6) \tag{6}$$

and

$$\begin{aligned} E_{\text{pair}} = E_n + E_p + E_{np} \; &= \; \left(n_1 + \frac{1}{2} n_2\right) [3 + n_4 + 2(w_1 + w_2) n_6] \\ &+ \; n_3(1 + n_1 + n_6) + n_1 n_4 (w_1 + w_2 - 1)\,. \end{aligned} \tag{7}$$

If we put

$$w_1 + w_2 = 1 \tag{8}$$

and introduce (2) to (7), we get

$$E_{\text{pair}} = \frac{1}{4}(n - \nu)\left(2\Omega + 3 - \frac{n}{2} - \frac{\nu}{2}\right) - \frac{1}{2} T(T+1) + \frac{1}{2} t(t+1)\,, \tag{9}$$

which is an exact formula for the symmetric $\mathrm{SU}_T(2)$ pairing energy of the system with neutrons and protons [11].

The E_{pair} does not depend on the T_0 because of the $\mathrm{SU}_T(2)$ symmetry and hence, in our starting state construction, figure 1, we could take any T_0, in our case $T_0 = T$. However, the separate parts of E_{pair}, namely E_n, E_p and E_{np} are not the simultaneous eigenvalues of the respective parts of the H_{pair} (3) because they have broken the $\mathrm{SU}_T(2)$ symmetry. To interpret these values (4)–(6) we should introduce the basis for irreducible representations of the group SO(5) and then calculate the mean values of $\langle S_+ S_- \rangle$ for three parts of (3). The IR basis vectors read

$$|(\alpha_1 \alpha_2) n T T_0 \beta\rangle\,,$$

where

$$\alpha_1 = \frac{1}{2}(\Omega - \nu)\,, \qquad\qquad \alpha_2 = t \tag{10}$$

label the IR of SO(5) and the four other quantum numbers identify the states within a given IR, β is the fourth non-physical quantum number without any known physical operator attached to this number. However, the most practical IR's in the physical application of the SO(5) do not need this fourth quantum number. There are three classes of such representations, namely $(\alpha, 0)$, $\left(\alpha, \frac{1}{2}\right)$ and (α, α). Next the representation $(\alpha, 0)$ will be taken with seniority $\nu = 0$ and hence $\alpha_1 = \frac{1}{2}\Omega$ and $\alpha_2 = 0$. The vector basis now reads $|\Omega; nTT_0\rangle$. The matrix elements of the pair creation and pair annihilation operators were calculated in several old pairing papers. We take the formulae from [24] and then, the calculated three mean values $\langle S_+ S_-\rangle$ in the basis $|\Omega; nTT_0\rangle$ are compared with three elementary pairing formulae (4)–(6) which gives

$$w_1 = \frac{2T+2}{2T+3}\,, \qquad\qquad w_2 = \frac{1}{2T+3}\quad {}^{1}. \tag{11}$$

At the same time we have checked a very surprising conclusion (8) from our elementary method. We need to remember that the formulae (11) are under the assumptions $T = T_0$ and $\nu = t = 0$. For this case we get $n_2 = n_4 = n_5 = 0$. Hence, the elementary, very handy, pairing formulae in the basis $|\Omega; nT = T_0\rangle$ read

$$E_n \equiv \langle S_+^n S_-^n \rangle = n_1(1 + w_1 n_6) + n_3(1 + n_6)\,, \tag{12}$$
$$E_p \equiv \langle S_+^p S_-^p \rangle = n_1(1 + n_3 + w_1 n_6)\,, \tag{13}$$
$$E_{np} \equiv \frac{1}{2}\langle S_+^{np} S_-^{np} \rangle = n_1(1 + 2w_2 n_6)\,, \tag{14}$$

where

$$n_1 = \frac{n}{4} - \frac{T}{2}\,, \qquad n_3 = T\,, \qquad n_6 = \Omega - \frac{n}{4} - \frac{T}{2} \tag{15}$$

and ω_1, ω_2 are given by (11)

Let us now consider the case with $T_0 \neq T$ for the irreducible representation $(\alpha, 0)$ with $\nu = 0$ and basis vectors $|\Omega; nTT_0\rangle$. Now, for $T_0 \neq T$ there are, in our elementary constructions, two fundamentally different structures, figure 2 of the state $|\Omega; nTT_0\rangle$: with

$$\begin{aligned} &\Omega = n_1 + n_2 + n_3 + n_4\,, && \Omega = n_1' + n_2' + n_3' + n_4'\,,\\ &n = 4n_1 + 2n_2 + 2n_3\,, && n = 4'n_1 + 2'n_2 + 2'n_3\,,\\ &T = n_2 + n_3\,, && T = n_2' + n_3'\,,\\ &T_0 = n_2 > 0\,, && T_0 = n_2' - n_3' > 0\,. \end{aligned} \tag{16}$$

[1] In two examples (20; 22) of the paper [23] there is a numerical error in w_1 and w_2. According to the present results (15) the structure factors w_1 and w_2 should be, in both examples, $\frac{2}{3}$ and $\frac{1}{3}$ because in both cases $T = 0$.

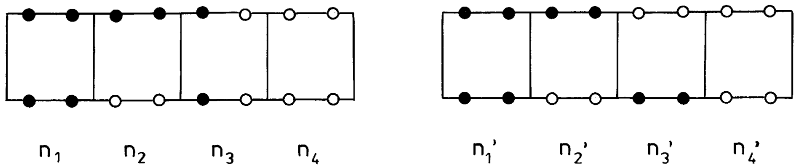

Fig. 2. Two different fundamental schematic configurations for the state $|\Omega\, nTT_0\rangle$ with $\nu = t = 0$.

Comparing the same Ω, n, T and T_0 for both constructions we get

$$n'_1 = n_1 \,, \qquad n'_2 = n_2 + \frac{n_3}{2} \,, \qquad n'_3 = \frac{n_3}{2} \,, \qquad n'_4 = n_4 \,. \tag{17}$$

Let us write down, using the elementary method, the energies (mean values) of the three parts of $H_{\rm pair}$, E_n, E_p and E_{np}. We get
I

$$\begin{aligned} (E_n)_{\rm I} &= n_1 + n_2(1+n_4) + w_1 n_1 n_4 \,, \\ (E_p)_{\rm I} &= n_1(1+n_2) + w_1 n_1 n_4 \,, \\ (E_{np})_{\rm I} &= n_1(1+n_3) + n_3(1+n_4) + 2w_2 n_1 n_2 \end{aligned} \tag{18}$$

II

$$\begin{aligned} (E_n)_{\rm II} &= n'_1(1+n'_3) + n'_2(1+n'_4) + w_1 n'_1 n'_4 \,, \\ &= n_1\left(1+\frac{n_3}{2}\right) + \left(n_2 + \frac{n_3}{2}\right)(1+n_4) + w_1 n_1 n_4 \end{aligned} \tag{19}$$

and similarly

$$\begin{aligned} (E_p)_{\rm II} &= n_1\left(1 + n_2 + \frac{n_3}{2}\right) + \frac{n_3}{2}(1+n_4) + w_1 n_1 n_4 \,, \\ (E_{np})_{\rm II} &= n_1 + 2w_2 n_1 n_4 \,. \end{aligned}$$

Due to the elementary method procedure, these two structures, I and II, are included in the mean value calculation as a linear combination

$$(E_n)_{\rm exact} = \langle S^n_+ S^n_- \rangle = u_1 (E_n)_{\rm I} + u_2 (E_n)_{\rm II} \tag{20}$$

and similarly for $\langle S^p_+ S^p_- \rangle$; $\frac{1}{2}\langle S^{np}_+ S^{np}_- \rangle$, where u_1 and u_2 are the new structure parameters. The numbers of four-state blocks, n_1 and n_4, are the same for both structures and hence, to fix u_1 and u_2 we can put $n_1 = n_4 = 0$

$$\begin{array}{lll} \text{I} & \qquad\qquad & \text{II} \\ E_n = n_2 & & E_n = n_2 + \dfrac{n_3}{2} \\ E_p = 0 & & E_p = \dfrac{n_3}{2} \\ E_{np} = n_3 & & E_{np} = 0 \end{array} \tag{21}$$

Hence, by (20) and (21), we get

$$\begin{aligned} \langle S_+^n S_-^n \rangle &\equiv (E_n)_{\text{exact}} = u_1 n_2 + u_2 \left(n_2 + \frac{n_3}{2} \right) , \\ \langle S_+^p S_-^p \rangle &\equiv (E_p)_{\text{exact}} = u_2 \frac{n_3}{2} , \\ \frac{1}{2} \langle S_+^{np} S_-^{np} \rangle &\equiv (E_{np})_{\text{exact}} = u_1 n_3 . \end{aligned} \tag{22}$$

From (22) we get

$$u_1 = \frac{\frac{1}{2}\langle S_+^{np} S_-^{np} \rangle}{n_3} , \qquad u_2 = \frac{2\langle S_+^p S_-^p \rangle}{n_3} . \tag{23}$$

Calculating from [24] exact values $\langle S_+^{np} S_-^{np} \rangle$ and $\langle S_+^p S_-^p \rangle$ for the IR $(\alpha, 0)$ we get

$$u_1 = \frac{2n_2 + n_3}{2n_2 + 2n_3 - 1} = \frac{T + T_0}{2T - 1} , \qquad u_2 = \frac{n_3 - 1}{2n_2 + 2n_3 - 1} = \frac{T - T_0 - 1}{2T - 1} \tag{24}$$

with the relation

$$u_1 + u_2 = 1 . \tag{25}$$

From the relations (20), (18), (19), (24) and (25) we get

$$w_1 = \frac{\langle S_+^n S_-^n \rangle - n_1 - n_2(1 + n_4) - u_2 \frac{n_3}{2}(1 + n_1 + n_4)}{n_1 n_4} . \tag{26}$$

Calculating, once again, the exact value $\langle S_+^n S_-^n \rangle$ from [24] and for the state $|\Omega\, n T T_0\rangle$ we obtain

$$w_1 = \frac{2T^2 + 2T + 2T_0^2 - 2}{(2T - 1)(2T + 3)}$$

and

$$w_2 = 1 - w_1 = \frac{2T^2 + 2T - 2T_0^2 - 1}{(2T - 1)(2T + 3)} . \tag{27}$$

The same, of course, values for w_1 and w_2 we could obtain taking $\langle S_+^p S_-^p \rangle$ or $\langle S_+^{np} S_-^{np} \rangle$ instead of the $\langle S_+^n S_-^n \rangle$.

In the special case of $T = T_0$ we obtain the formulae (11) from (27).

Now, instead of formulae (12)–(14) for the special case $T = T_0$, we get for any T_0 the general and very handy forms for the mean values:

$$\begin{aligned} E_n &\equiv \langle S_+^n S_-^n \rangle = n_1 + w_1 n_1 n_4 + n_2(1 + n_4) + u_2 \frac{n_3}{2}(1 + n_1 + n_4) , \\ E_p &\equiv \langle S_+^p S_-^p \rangle = n_1(1 + n_2) + w_1 n_1 n_4 + u_2 \cdot \frac{n_3}{2}(1 + n_1 + n_4) , \\ E_{np} &\equiv \frac{1}{2}\langle S_+^{np} S_-^{np} \rangle = n_1 + 2w_2 n_1 n_4 + u_1 n_3(1 + n_1 + n_4) , \end{aligned} \tag{28}$$

where, from (16)

$$n_1 = \frac{n}{4} - \frac{T}{2}\,, \qquad n_2 = T_0\,, \qquad n_3 = T - T_0\,, \qquad n_4 = \Omega - \frac{n}{4} - \frac{T}{2} \tag{29}$$

and

$$\begin{aligned} u_1 &= \frac{T+T_0}{2T-1}\,, & u_2 &= 1 - u_1 = \frac{T-T_0-1}{2T-1}\,, \\ w_1 &= \frac{2T^2+2T+2T_0^2-2}{(2T-1)(2T+3)}\,, & w_2 &= 1 - w_1 = \frac{2T^2+2T-2T_0^2-1}{(2T-1)(2T+3)}\,. \end{aligned} \tag{30}$$

We can check the above formulae taking into account the sum

$$\begin{aligned} &\langle S_+^n S_-^n \rangle + \langle S_+^p S_-^p \rangle + \frac{1}{2}\langle S_+^{np} S_-^{np} \rangle \\ &= n_1(3 + n_2 + n_3 + 2n_4) + (n_2 + n_3)(1 + n_4) \\ &= \frac{n}{4}\left(2\Omega + 3 - \frac{n}{2}\right) - \frac{T}{2}(T+1) = E_{\text{pair}} \end{aligned} \tag{31}$$

which is the exact pairing formula (9) for $\nu = t = 0$.

Now we are in position to consider analytically the neutron–proton pairing interaction and its competition with nn and pp correlations.

3. Applications

Problem No. 1

Let us consider the T_0 dependence of the pairing formulae (28)–(30). After the proper rearrangement we get the following

$$\begin{aligned} E_p &= a\,T_0^2 + b\,T_0 + c\,, \\ E_n &= a\,T_0^2 - b\,T_0 + c\,, \\ E_{np} &= -2a\,T_0^2 - 2c + E_{\text{pair}}\,, \end{aligned} \tag{32}$$

where

$$\begin{aligned} a &= \frac{4n_1n_4 + (1 + n_1 + n_4)(2T+3)}{2(2T-1)(2T+3)}\,, \\ b &= \frac{n_1 - n_4 - 1}{2}\,, \\ c &= n_1 + (1 + n_1 + n_4)\frac{T(T-1)}{2(2T-1)} + 2n_1n_4\frac{T^2+T-1}{(2T-1)(2T+3)} \end{aligned} \tag{33}$$

and E_{pair} is the pairing energy (31).

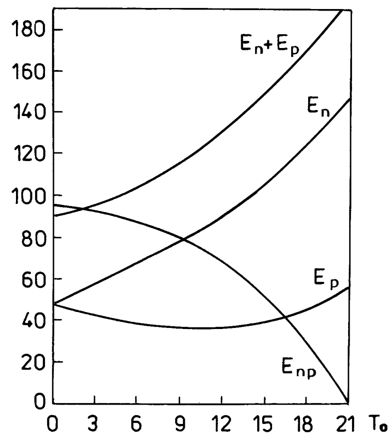

Fig. 3. Mean values of $\langle S^+S^-\rangle$ for *pp*, *nn* and *np* pairing parts of $H_{\rm pair}$ in the state $|\Omega nTT_0\rangle$ versus T_0; $T = 21$.

Figure 3 shows an example of the T_0 dependence for the state $|\Omega nTT_0\rangle$ with $n = 50$ $\Omega = 28$; $T = 21$ and then $n_1 = 2$; $n_4 = 5$. From (33) we get $a = 1.084$; $b = -2$; $c = 47.539$ and $E_{\rm pair} = 194$.

For $T_0 < 0$ we simply change $E_p \rightleftarrows E_n$.

Two interesting conclusions can be drown from figure 3:

(i) The $E_n(E_p)$ reaches the minimum value and that minimum is for the same $|T_0|$. One would rather expect constantly increasing values $E_n(E_p)$.

(ii) The E_{np} takes on more or less the same value as the sum $E_n + E_p$ for T_0 around zero, in our case $-3 \leq T_0 \leq 3$. It means that in this region of the T_0 the neutron–proton pairing is of the greatest importance.

$T = 21$ has been chosen quite arbitrarily. For other T values the ΔT_0 (around $T_0 = 0$), for which the contribution of $n - p$ pairing is larger than the sum of $n - n$ and $p - p$, can be only changed.

Problem No. 2

Let us consider the following problem [19]: let us assume adding to the initial state of a nucleus $|\Omega nT = 0\rangle$ with the mean energies $(E_1)_{nn}$, $(E_1)_{pp}$ and $(E_1)_{np}$ a pair of neutrons keeping the same $\nu = 0$. What is the change in the mean energies $\frac{E_2-E_1}{E_1} \equiv \frac{\Delta E}{E_1}$? The answer, following the formulae (28)–(30), is rather unexpected, namely:

for protons:

$$\left(\frac{\Delta E}{E_1}\right)_{pp} = +0.20$$

for np:

$$\left(\frac{\Delta E}{E_1}\right)_{np} = -0.40$$

for any Ω and any n! Nobody would expect that adding two neutrons will increase the proton correlations ("proton pairs") by 20%. The decrease of the neutron–proton correlation is obvious but the constant value of 40% for such decrease is also unexpected. The more complicated, but also rather non-expected answer is for the neutron correlations. We get

$$\left(\frac{\Delta E}{E_1}\right)_{nn} = \frac{4n\Omega - n^2 - 54n + 120\Omega}{20n\Omega - 5n^2 + 30n}\,. \tag{34}$$

Now the answer depends on the mutual interplay of n and Ω:

1° If we fix $\Omega(2, 3, \ldots)$ then the function (34) lowers beginning with $\frac{17\Omega-29}{10\Omega+5}$ (for $n_{\min} = 4$) and ending with $\frac{5-2\Omega}{5-5\Omega}$ (for $n_{\max} = 4\Omega - 4$).

2° If we fix $n(4, 8, 12, \ldots)$ then the function (34) rises from the initial value $\frac{12-2n}{5n}$ for $\Omega_{\min} = \frac{n}{4} + 1$ to its asymptotic value $\frac{n+30}{5n}$ for $\Omega \to \infty$.

Let us illustrate the results by two examples: for $\Omega = 10$ in the first (Fig. 4), and for $n = 20$ in the second example (Fig. 5).

In the first example, for $\Omega = 10$ with a low initial value of nucleons, the added two neutrons increase the E_n by 134% while for the highest number of nucleons, E_n even decreases by 33%. In the second example, for $n = 20$, for a low value of Ω the E_n decreases by 28% while for the asymptotic case $\Omega \to \infty$ it increases by 50%. Boundary limits of the $(\Delta E/E_1)_n$ for any n and Ω are from 170% to –40%. This statement is in contradiction to the conclusion of Engel *et al.* [19].

<u>Problem No. 3</u>

Let us consider even isotopes of a given element with $\nu = 0$ starting with a nucleus of $N = Z$, $T = 0$ and with n_0 nucleons above the magic shell. The other isotopes have $n = n_0 + 2T$ particles, where $T = 1, 2, 3, \ldots$ and obviously $T_0 = T$.

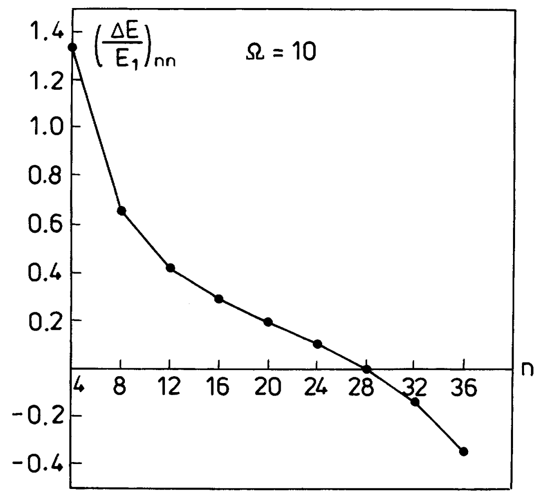

Fig. 4. Change in the neutron correlations measured by $\frac{\Delta E}{E}$ after a pair of neutrons is added, versus n; $\Omega = 10$ is fixed.

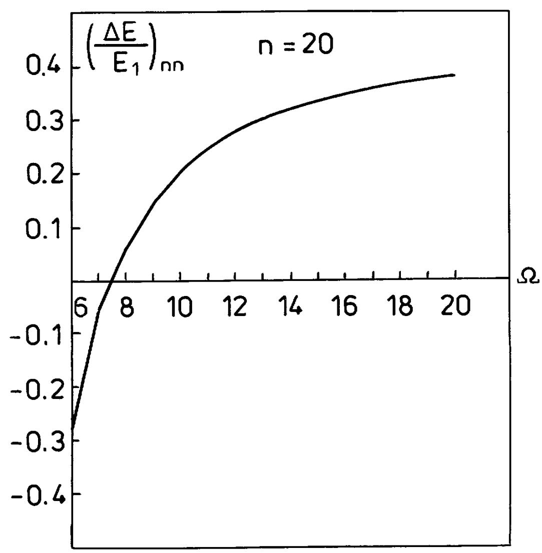

Fig. 5. The same as in figure 4 but versus Ω, n=20 is fixed

Taking into account our formulae (11)–(15) for $T_0 = T$, we get for isotopes under consideration:

$$E_n = \frac{n_0}{4} + \frac{2T+2}{2T+3} \cdot k + T\left(1 + \Omega - \frac{n_0}{4} - T\right) ,$$
$$E_p = \frac{n_0}{4} + \frac{2T+2}{2T+3} \cdot k + \frac{n_0}{4} \cdot T ,$$
$$E_{np} = \frac{n_0}{4} + \frac{2}{2T+3} \cdot k , \qquad (35)$$

where $k = \frac{n_0}{4}\left(\Omega - \frac{n_0}{4} - T\right)$.

We take, as an example, the ^{24}Cr isotopes with $A = 48$–56. Assuming that the valence nucleons are on the degenerated (pf) shell we put in the formulae (35) $n_0 = 8$ and $\Omega = 10$. Figure 6 represents the characteristic dependence of the neutron, proton and neutron–proton correlations measured by the mean values E_n, E_p and E_{np}.

For example, for the isotope $^{24}\mathrm{Cr}_{56}$ for $T = T_0 = 4$ the pair contribution in E_{pair} is 58.5%; 34.5% and 7% for nn; pp and np correlations, respectively.

We should not confuse this conclusion with that in Fig. 3. Here we compare the np pair contribution for a given nucleus with $T = T_0$ while in Fig. 3 we take for comparison the nuclei with the same T but with different T_0.

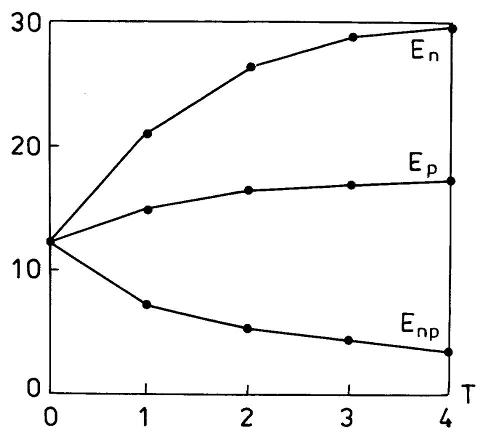

Fig. 6. Neutron, proton, and neutron–proton correlations measured by the mean values E_n, E_p and E_{np} for Cr isotopes with $T = T_0$.

Problem No. 4 — the congruence energy

The so-called "Wigner term" or "congruence energy" in the nuclear mass formula [25, 26] depends on $I = (N - Z)/A$ and is represented by the semi-empirical formula

$$C(I) = -C_0 \exp(-W|I|/C_0)\,, \tag{36}$$

where $C_0 = 10$ MeV and $W = 42$ MeV. The microscopic basis for this term comes, as it was suggested [27], from the neutron–proton pairing except of the constant term of a different origin.

We will compare next, just like in the paper [21], the shape of our E_{np} formula with the congruence energy for the isovector pairing only (36). For this purpose we will consider isotopes of a given element with $T = |T_0|$. Hence, we adopt the formulae (11), (14), (15) which gives

$$\begin{aligned} E_{np} &= -G_{np}(A) n_1 (1 + 2w_2 n_6) \\ &= -G_{np}(A) \left\{ \frac{n-2T}{4} + \frac{n-2T}{4} \cdot \frac{2}{2T+3} \cdot \frac{4\Omega - n - 2T}{4} \right\}, \end{aligned} \tag{37}$$

where n is, as before, the number of valence nucleons and $G_{np}(A)$ is the strength of the neutron–proton pairing interaction, which we assume to be A dependent in the form [21]

$$G_{np} = 1.25 \frac{16}{A + 56}\,. \tag{38}$$

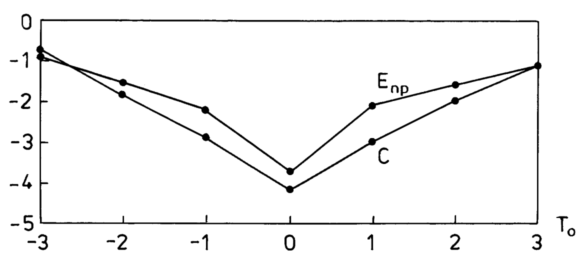

Fig. 7. Comparison of the congruence energy and the mean value E_{np} for Ge isotopes with $T = |T_0|$.

We have taken, for comparison, the isotopes of Ge with A from 58 to 70. In figure 7 we compare two formulae (36) and (37) shifting the congruence energy by 5.81 MeV. It means, that in our comparison the stress is put on the $(N - Z)/A$ dependence of both formulae but not on their absolute values.

It is interesting to note that the conclusion from figure 7, besides the similar dependence of both curves is different from that of the paper [21], where the np pairing contribution decreases faster than the congruence energy. We have to explain that the np pairing contribution has been differently calculated. In the paper [21] it is the eigenvalue of the np pairing part in the state of broken T-symmetry while our formula presents the mean value $\langle S_+^{np} S_-^{np} \rangle$ in the state of a given $T = T_0$.

4. Conclusions

The main part of this presentation is the extended elementary method in the pairing interaction to involve the T_0 dependence for the non diagonal terms S_+S_- and to calculate the mean values of those terms $\langle S_+^n S_-^n \rangle$; $\langle S_+^p S_-^p \rangle$; $\langle S_+^{np} S_-^{np} \rangle$ in the state of the considered irreducible representation of the orthogonal group SO(5). The algebraic, very handy formulae can be and have been, used for analytical solution of different physical problems. Among those problems we have discussed:

(i) the T_0 dependence,

(ii) the change in the pairing contributions following the increase in the number of neutrons by a neutron pair of $T = 1$,

(iii) the change in the pairing contributions in a set of even isotopes,

(iv) the possible microscopical origin of the congruence energy.

We have clarified and generalized several answers to the recently discussed problems stated above.

Although the presented results are based on a simple model, their numerical part are exact. Hence, for the nuclei with the shell model structure similar to the considered nuclei in the paper, the pairing contribution is, at least, of the same qualitative value even in the presence of a more realistic two-body interaction.

The last version of the paper has been prepared during the stay in the Max Planck Institute für Kernphysik in Heidelberg. The author is very grateful to professors David Brink and Renzo Leonardi from ECT in Trento and to professor Hans Weidenmüller from Max Planck Institute in Heidelberg for the invitations, and also to ECT and Max Planck Institute for financial support.

REFERENCES

[1] G. Racah, *Phys. Rev.* **63**, 367 (1943).
[2] B.H. Flowers, *Proc. Roy. Soc.* **A212**, 248 (1952).
[3] A.R. Edmonds, B.H. Flowers, *Proc. Roy. Soc.* **A214**, 515 (1952).
[4] J. Bardeen, L.N. Cooper, I.R. Schrieffer, *Phys. Rev.* **108**, 1175 (1957).
[5] B. Mottelson, in: *The Many-Body Problem*, University Grenoble, 1958.
[6] S.T. Bielajev, *Mat. Phys. medd. Kgl. Danske vid. sel.* **31**, 11 (1959).
[7] Y. Wada, F. Takano, N. Fokuda, *Progr. Theor. Phys.* (Japan) **19**, 597 (1958).
[8] A.K. Kerman, *Ann. Phys.* **12**, 300 (1961).
[9] A.K. Kerman, R.D. Lawson, M.H. Macfarlane, *Phys. Rev.* **124**, 162 (1961).
[10] K. Helmers, *Nucl. Phys.* **23**, 594 (1961).
[11] B.H. Flowers, S. Szpikowski, *Proc. Phys. Soc.* **84**, 193 (1964).
[12] B.H. Flowers, S. Szpikowski, *Proc. Phys. Soc.* **84**, 673 (1964).
[13] M. Ichimura, *Prog. Theor. Phys.* (Japan) **32**, 757 (1964).
[14] M. Ichimura, *Prog. Theor. Phys.* (Japan) **33**, 215 (1965).
[15] K.T. Hecht, *Nucl. Phys.* **63**, 177 (1965).
[16] K.T. Hecht, *Phys. Rev.* **139B**, 794 (1965).
[17] J.C. Parkih, *Nucl. Phys.* **63**, 214 (1965).
[18] J.C. Lipkin, *Lie Groups for Pedestrians*, Amsterdam 1965.
[19] J. Engel, K. Langanke, P. Vogel, *Phys. Lett.* **B389**, 211 (1996).
[20] J. Engel, S. Pittel, M. Stoitsov, P. Vogel, J. Dukelsky, *Phys. Rev.* **C55**, 1781 (1997).
[21] O. Civitarese, M. Reboiro, P. Vogel, *Phys. Rev.* **C56**, 1840 (1997).
[22] S. Szpikowski, *Acta Phys. Pol.* **B22**, 641 (1991).
[23] S. Szpikowski, *Z. Phys.* **A339**, 37 (1991).
[24] S. Szpikowski, *Acta Phys. Pol.* **29**, 853 (1966).
[25] W.D. Myers, W.J. Swiatecki, *Nucl. Phys.* **A601**, 141 (1996).
[26] W.D. Myers, W.J. Swiatecki, *Nucl. Phys.* **A612**, 249 (1997).
[27] W. Satula, R. Wyss, *Phys. Lett.* **B393**, 1 (1996).
[28] M. Gerceklioglu, A.A. Kuliev, H.H. Erbil, *Europhys. Lett.* **1**, 25 (1999).