# SINGLE PARTICLE POTENTIAL OF A $\boldsymbol{\Sigma}$ HYPERON IN NUCLEAR MATTER II. REARRANGEMENT EFFECTS 

Janusz Dąbrowski<br>Theoretical Division, Sołtan Institute for Nuclear Studies<br>Hoża 69, 00-681 Warsaw, Poland

(Received March 24, 2000)

The rearrangement contribution to the real part of the single particle potential of a $\Sigma$ hyperon in nuclear matter, $U_{\Sigma}$, is investigated. The isospin and spin dependent parts of $U_{\Sigma}$ are considered. Results obtained for four models of the Nijmegen baryon-baryon interaction are presented and discussed.

PACS numbers: 21.80.+a

## 1. Introduction

The knowledge of the single particle (s.p.) potential $U_{\Sigma}$ of the $\Sigma$ hyperon in nuclear matter is essential in the description of the structure of $\Sigma$ hypernuclear states. This knowledge should include the isospin and spin dependence of $U_{\Sigma}$. Let us mention that the existence of the only observed $\Sigma$ hypernuclear bound state ${ }_{\Sigma}^{4} \mathrm{He}$ is strictly connected with a strong isospin dependent (Lane) part of $U_{\Sigma}[1,2]$.

A calculation of $U_{\Sigma}$ and its isospin and spin dependence was presented in [3] and [4] (hereafter referred as I). The calculation in [3] and I was performed in the simplest way, and did not include the so called rearrangement effects. Because of the importance of $U_{\Sigma}$ in the theory of $\Sigma$ hypernuclear states, we feel that these rearrangement effects should be included in the calculation of $U_{\Sigma}$. This is done in the present paper.

Let us consider a $\Sigma$ hyperon moving in nuclear matter composed of $Z_{\uparrow}\left(Z_{\downarrow}\right)$ protons with spin up (down), and $N_{\uparrow}\left(N_{\downarrow}\right)$ neutrons with spin up (down). As discussed in I, the s.p. potential of a $\Sigma^{+}$hyperon with spin up/down and momentum $\boldsymbol{k}_{\Sigma}$ has the form (in the linear approximation in the $\alpha$ parameters):

$$
\begin{equation*}
U_{\Sigma}\left(\uparrow / \downarrow, \Sigma^{+}, k_{\Sigma}\right)=U_{0}\left(k_{\Sigma}\right)+\frac{1}{2} \alpha_{\tau} U_{\tau}\left(k_{\Sigma}\right) \pm \frac{1}{4} \alpha_{\sigma} U_{\sigma}\left(k_{\Sigma}\right) \pm \frac{1}{2} \alpha_{\sigma \tau} U_{\sigma \tau}\left(k_{\Sigma}\right) \tag{1}
\end{equation*}
$$

where the proton or isospin excess parameter $\alpha_{\tau}=\left(Z_{\uparrow}+Z_{\downarrow}-N_{\uparrow}-N_{\downarrow}\right) / A$, the spin excess parameter $\alpha_{\sigma}=\left(Z_{\uparrow}+N_{\uparrow}-Z_{\downarrow}-N_{\downarrow}\right) / A$, and the spin-isospin excess parameter $\alpha_{\sigma \tau}=\left(Z_{\uparrow}+N_{\downarrow}-Z_{\downarrow}-N_{\uparrow}\right) / A$. For the assumed charge independence of the baryon-baryon interaction, the expression for

$$
U_{\Sigma}\left(\uparrow / \downarrow, \Sigma^{-}, k_{\Sigma}\right)
$$

differs from (1) only by the sign at the $\tau$ and $\sigma \tau$ terms, whose coefficients become $-\alpha_{\tau}$ and $\mp \alpha_{\sigma \tau}$. The expression for

$$
U_{\Sigma}\left(\uparrow / \downarrow, \Sigma^{0}, k_{\Sigma}\right)
$$

differs from (1) by the absence of the $\tau$ and $\sigma \tau$ terms ${ }^{1}$.
In our derivation of the expression for $U_{\Sigma}$, we assume that the effective $\Sigma N$ interaction $\mathcal{K}$ and $N N$ interaction $K$ in nuclear matter are obtained by applying the Brueckner theory.

We define the s.p. potential $U_{\Sigma}\left(k_{\Sigma}\right)$ of a $\Sigma$ hyperon (whose spin and charge is not indicated here) together with its kinetic energy $\varepsilon_{\Sigma}\left(k_{\Sigma}\right)=$ $\hbar^{2} k_{\Sigma}^{2} / 2 M_{\Sigma}$ as the removal energy:

$$
\begin{equation*}
\varepsilon_{\Sigma}\left(k_{\Sigma}\right)+U_{\Sigma}\left(k_{\Sigma}\right)=E\left(A, 1_{\Sigma}\right)-E(A), \tag{2}
\end{equation*}
$$

where $E\left(A, 1_{\Sigma}\right)$ is the energy of the system of nuclear matter plus the $\Sigma$ hyperon, and $E(A)$ is the energy of pure nuclear matter.

The s.p. potential $U_{\Sigma}$ consists of two parts, the model potential $V_{\Sigma}$ and the rearrangement potential $V_{\Sigma R}$,

$$
\begin{equation*}
U_{\Sigma}=V_{\Sigma}+V_{\Sigma R} \tag{3}
\end{equation*}
$$

The s.p. model potential $V_{\Sigma}$, represented by diagram (a) in Fig. 1, is introduced in the Brueckner theory for the sole purpose of calculating the total energy of the system, and is not equal to the potential part of the removal energy, $U_{\Sigma}$. For illustration, let us consider the case of a $\Sigma$ particle with $k_{\Sigma}=0$, in which $U_{\Sigma}$ is equal to the $\Sigma$ separation energy. To separate the particle from nuclear matter, we first have to perform the work equal to $-V_{\Sigma}$. However, the system of nuclear matter left without the $\Sigma$ particle has the possibility to rearrange itself to an energetically more favorable state, and while doing it releases the rearrangement energy $V_{\Sigma R}$. Hence, the $\Sigma$ separation energy $-U_{\Sigma}=-\left(V_{\Sigma}+V_{\Sigma R}\right)$. The leading contribution to $V_{\Sigma R}$ is represented by the diagram (b) in Fig. 1.

[^0]

Fig. 1. Diagrams representing $V_{\Sigma}$ (a) and $V_{\Sigma R}$ (b). Single lines are nucleons, the double line denotes $\Sigma$.

Our definition of $U_{\Sigma}$ is obviously the proper one for calculating the $\Sigma$ binding (i.e., separation) energy in nuclear matter. Also in calculating $\Sigma$ scattering, i.e., states with positive s.p. energies, one should apply our definition of $U_{\Sigma}$. Namely, the scattering process is determined by the change of the wave length of the particle in the scattering medium, which in turn is determined by the change in energy of the incoming particle when it enters the scatterer, i.e., by the removal energy.

In the present paper, we calculate the rearrangement potential $V_{\Sigma R}$ which has been disregarded in I. Whereas the expression for the hyperon rearrangement potential in isospin and spin saturated nuclear matter is well known since the early work of Dąbrowski and Köhler [5], the rearrangement contributions to the $\tau, \sigma$, and $\tau \sigma$ components of $U_{\Sigma}$ are considered for the first time in the present paper.

This paper is organized as follows. In Section 2, we start from the effective $\Sigma N$ interaction $\mathcal{K}$ and the effective $N N$ interaction $K$ in nuclear matter, and derive the expression for the rearrangement part of $U_{\Sigma}$. In Section 3, we discuss the results for $U_{0}, U_{\tau}, U_{\sigma}$, and $U_{\sigma \tau}$ obtained with $\mathcal{K}$ calculated in [6] for four models of the Nijmegen baryon-baryon interaction. Similarly as in I, we restrict ourselves to the real $\Sigma$ potential.

## 2. The rearrangement potential

Decomposition (3) applies to each of the component of $U_{\Sigma}$ :

$$
\begin{gather*}
U_{0}=V_{0}+V_{0 R}  \tag{4}\\
U_{x}=V_{x}+V_{x R}, \quad x=\tau, \sigma, \sigma \tau . \tag{5}
\end{gather*}
$$

The derivation of the expression for $V_{x R}$ (which automatically leads to the expression for $V_{0 R}$ ) is similar in the three cases of $x=\tau, x=\sigma$, and $x=\sigma \tau$. We shall present the derivation only in the case of $x=\tau$, and in the remaining cases of $x=\sigma$, and $x=\sigma \tau$, we shall only present the results.

While calculating $U_{\tau}$, we put $\alpha_{\sigma}=\alpha_{\sigma \tau}=0$. Nuclear matter becomes then a two-component system with $Z$ protons and $N$ neutrons with the corresponding Fermi momenta $\kappa_{\tau}$ and $\lambda_{\tau}$. Eq. (1) takes now the simpler form:

$$
\begin{equation*}
U_{\Sigma}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right)=U_{0}\left(k_{\Sigma}\right)+\frac{1}{2} \alpha_{\tau} U_{\tau}\left(k_{\Sigma}\right), \tag{6}
\end{equation*}
$$

where $\mu_{\Sigma}= \pm \frac{1}{2}$ for $\Sigma$ hyperon with spin up/down $(\uparrow / \downarrow)$. Notice that in the spin saturated nuclear matter considered here, $U_{\Sigma}$ does not depend on $\mu_{\Sigma}$.

To determine $U_{\Sigma}$, we apply definition (2) of the $\Sigma$ s.p. energy, which we write in the form:

$$
\begin{equation*}
U_{\Sigma}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right)=E_{\mathrm{POT}}\left(Z, N, 1_{\Sigma^{+}}\right)-E_{\mathrm{POT}}(Z, N), \tag{7}
\end{equation*}
$$

where $E_{\mathrm{POT}}$ is the potential part of $E$. In the more precize notation in Eq. (7) we indicate that $E$ depends on $Z$ and $N$, and not just on $A$ as the simplified notation in Eq. (2) might suggest. Instead of $E(Z, N)$, we shall use also an equivalent notation $E\left(\kappa_{\tau}, \lambda_{\tau}\right)$.

We split $E_{\text {POT }}(N, Z)$ into three parts:

$$
\begin{equation*}
E_{\mathrm{POT}}(Z, N)=E_{p p}+E_{n n}+E_{p n}, \tag{8}
\end{equation*}
$$

where the three consecutive parts are produced respectively by the effective $p p, n n$, and $p n$ interaction:

$$
\begin{align*}
& E_{p p}=\frac{1}{2} \sum_{\mu_{1} \mu_{2}} \sum_{\boldsymbol{m}_{1}}^{m_{1}<\kappa_{\tau}} \sum_{\boldsymbol{m}_{2}}^{m_{2}<\kappa_{\tau}}\left(\boldsymbol{m}_{1} \mu_{1} p \boldsymbol{m}_{2} \mu_{2} p\left|K\left(\kappa_{\tau} \lambda_{\tau}\right)\right| \boldsymbol{m}_{1} \mu_{1} p \boldsymbol{m}_{2} \mu_{2} p\right)-\text { exchange }  \tag{9}\\
& E_{n n}=\frac{1}{2} \sum_{\mu_{1} \mu_{2}} \sum_{\boldsymbol{m}_{1}}^{m_{1}<\kappa_{\tau}} \sum_{m_{2}}^{m_{2}<\kappa_{\tau}}\left(\boldsymbol{m}_{1} \mu_{1} n \boldsymbol{m}_{2} \mu_{2} n\left|K\left(\kappa_{\tau} \lambda_{\tau}\right)\right| \boldsymbol{m}_{1} \mu_{1} n \boldsymbol{m}_{2} \mu_{2} n\right)-\text { exchange }  \tag{10}\\
& E_{p n}=\sum_{\mu_{1} \mu_{2}} \sum_{\boldsymbol{m}_{1}}^{m_{1}<\kappa_{\tau}} \sum_{m_{2}}^{m_{2}<\kappa_{\tau}}\left(\boldsymbol{m}_{1} \mu_{1} p \boldsymbol{m}_{2} \mu_{2} n\left|K\left(\kappa_{\tau} \lambda_{\tau}\right)\right| \boldsymbol{m}_{1} \mu_{1} p \boldsymbol{m}_{2} \mu_{2} n\right) \tag{11}
\end{align*}
$$

where $\mu_{1(2)}$ is the spin magnetic number of nucleon 1(2).
For $E_{\mathrm{POT}}\left(Z, N, 1_{\Sigma^{+}}\right)$we have:

$$
\begin{equation*}
E_{\mathrm{POT}}\left(Z, N, 1_{\Sigma^{+}}\right)=\tilde{E}_{\mathrm{POT}}(Z, N)+V_{\Sigma}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right), \tag{12}
\end{equation*}
$$

where $V_{\Sigma}$ is the $\Sigma$ s.p. model potential:

$$
\begin{align*}
& V_{\Sigma}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right) \\
& =\sum_{\mu_{1}}\left[\sum_{\boldsymbol{m}_{1}}^{m_{1}<\kappa_{\tau}}\left(\boldsymbol{m}_{1} \mu_{1} p \boldsymbol{k}_{\Sigma} \mu_{\Sigma} \Sigma^{+}\left|\mathcal{K}\left(\kappa_{\tau} \lambda_{\tau}\right)\right| \boldsymbol{m}_{1} \mu_{1} p \boldsymbol{k}_{\Sigma} \mu_{\Sigma} \Sigma^{+}\right)\right. \\
& \left.+\sum_{\boldsymbol{m}_{1}}^{m_{1}<\lambda_{\tau}}\left(\boldsymbol{m}_{1} \mu_{1} n \boldsymbol{k}_{\Sigma} \mu_{\Sigma} \Sigma^{+}\left|\mathcal{K}\left(\kappa_{\tau} \lambda_{\tau}\right)\right| \boldsymbol{m}_{1} \mu_{1} n \boldsymbol{k}_{\Sigma} \mu_{\Sigma} \Sigma^{+}\right)\right] \tag{13}
\end{align*}
$$

and $\tilde{E}_{\mathrm{POT}}(Z, N)$ is the potential energy of nuclear matter in the presence of the $\Sigma^{+}$hyperon (with momentum $\boldsymbol{k}_{\Sigma}$ and spin projection $\mu_{\Sigma}$ ). The expression for $\tilde{E}(Z, N)$ is the same as that for $E(Z, N)$, Eqs (8)-(11), except that $K$ must be replaced by $\tilde{K}$, the effective $N N$ interaction in the system of nuclear matter plus the $\Sigma^{+}$hyperon (with momentum $\boldsymbol{k}_{\Sigma}$ and spin projection $\mu_{\Sigma}$ ).

Combining Eqs (12), (7), (3), we may write $V_{\Sigma R}$ in the form:

$$
\begin{equation*}
V_{\Sigma R}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right)=V_{\Sigma R}^{p p}+V_{\Sigma R}^{n n}+V_{\Sigma R}^{p n} . \tag{14}
\end{equation*}
$$

Expressions for $V_{\Sigma R}^{N N^{\prime}}$ are the same as those for $E_{N N^{\prime}}$ except that $K\left(\kappa_{\tau} \lambda_{\tau}\right)$ must be replaced by $\tilde{K}\left(\kappa_{\tau} \lambda_{\tau}\right)-K\left(\kappa_{\tau} \lambda_{\tau}\right)$, e.g.,

$$
\begin{align*}
V_{\Sigma R}^{p p}= & \frac{1}{2} \sum_{\mu_{1} \mu_{2}} \sum_{\boldsymbol{m}_{1}}^{m_{1}<\kappa_{\tau}} \sum_{\boldsymbol{m}_{2}}^{m_{2}<\kappa_{\tau}}\left(\boldsymbol{m}_{1} \mu_{1} p \boldsymbol{m}_{2} \mu_{2} p \mid \tilde{K}\left(\kappa_{\tau} \lambda_{\tau}\right)\right. \\
& \left.-K\left(\kappa_{\tau} \lambda_{\tau}\right) \mid \boldsymbol{m}_{1} \mu_{1} p \boldsymbol{m}_{2} \mu_{2} p\right)- \text { exchange } . \tag{15}
\end{align*}
$$

The procedure of calculating each of the three parts of $V_{\Sigma R}$ is similar, and we shall outline it in the case of $V_{\Sigma R}^{p p}$. From now on, we assume that the effective $N N$ interactions $K$ and $\tilde{K}$ are determined within the Brueckner theory of nuclear matter. We assume the simplest version of this theory, in which pure kinetic energies $\varepsilon_{N}$ are used in the intermediate states of the $K$-matrix equation. This is the so called low order Brueckner (LOB) theory. Thus the equation for $\tilde{K}$ differs from the equation for $K$ only by the appearance of the s.p. nucleon energies of the occupied states in the system of nuclear matter plus the $\Sigma$ hyperon, $\tilde{e}_{N}\left(m_{1,2} \mu_{1,2}\right)$, in place of the energies $e_{N}\left(m_{1,2}\right)$ in pure nuclear matter, which appear in the equation for $K$. From
the equations for $\tilde{K}$ and $K$, one obtains (see, e.g., [7])

$$
\begin{align*}
& \left.\left[\tilde{K}\left(\kappa_{\tau}, \lambda_{\tau}\right)-K\left(\kappa_{\tau}, \lambda_{\tau}\right)\right] \mid \boldsymbol{m}_{1} \mu_{1} p \boldsymbol{m}_{2} \mu_{2} p\right)=\sum_{\mu_{1}^{\prime} \mu_{2}^{\prime}} \sum_{\boldsymbol{k}_{1}}^{k_{1}>\kappa_{\tau}} \sum_{\boldsymbol{k}_{2}}^{k_{2}>\kappa_{\tau}} \\
& \left.\times K\left(\kappa_{\tau} \lambda_{\tau}\right) \mid \boldsymbol{k}_{1} \mu_{1}^{\prime} p \boldsymbol{k}_{2} \mu_{2}^{\prime} p\right)\left[\frac{1}{\tilde{a}}-\frac{1}{a}\right]\left(\boldsymbol{k}_{1} \mu_{1} p \boldsymbol{k}_{2} \mu_{2} p\left|\tilde{K}\left(\kappa_{\tau} \lambda_{\tau}\right)\right| \boldsymbol{m}_{1} \mu_{1} p \boldsymbol{m}_{2} \mu_{2} p\right), \tag{16}
\end{align*}
$$

where

$$
\begin{align*}
& a=e_{p}\left(m_{1}\right)+e_{p}\left(m_{2}\right)-\varepsilon_{p}\left(k_{1}\right)-\varepsilon_{p}\left(k_{2}\right) \\
& \tilde{a}=\tilde{e}_{p}\left(m_{1} \mu_{1}\right)+\tilde{e}_{p}\left(m_{2} \mu_{2}\right)-\varepsilon_{p}\left(k_{1}\right)-\varepsilon_{p}\left(k_{2}\right) . \tag{17}
\end{align*}
$$

Now we follow [5] and make the following approximations: First, we introduce the expansion

$$
\begin{equation*}
\frac{1}{\tilde{a}}-\frac{1}{a} \simeq-\frac{1}{a^{2}}\left(\left[\tilde{e}_{p}\left(m_{1} \mu_{1}\right)-e_{p}\left(m_{1}\right)\right]+\left[\tilde{e}_{p}\left(m_{2} \mu_{2}\right)-e_{p}\left(m_{2}\right)\right]\right) \tag{18}
\end{equation*}
$$

second, we approximate the differences in the s.p. energies by

$$
\begin{equation*}
\tilde{e}_{p}\left(m_{i} \mu_{i}\right)-e_{p}\left(m_{i}\right) \simeq\left(\boldsymbol{m}_{i} \mu_{i} p \boldsymbol{k}_{\Sigma} \mu_{\Sigma} \Sigma^{+}\left|\mathcal{K}\left(\kappa_{\tau} \lambda_{\tau}\right)\right| \boldsymbol{m}_{i} \mu_{i} p \boldsymbol{k}_{\Sigma} \mu_{\Sigma} \Sigma^{+}\right) \tag{19}
\end{equation*}
$$

where $i=1,2$, and third, we apply the first iteration to Eq. (16), i.e., we replace $\tilde{K}$ by $K$ on the right hand side of this equation.

Now, we apply the above approximations in (16) and obtain an approximate expression for $\tilde{K}-K$ which we insert into expression (15) for $V_{\Sigma R}^{p p}$. In this expression for $V_{\Sigma R}^{p p}$, we introduce the total spin of the two protons and its $z$ projection, $s m_{s}$, and also the total isospin and its third component $T T_{3}{ }^{2}$. In this way, we get after some algebra,

$$
\begin{align*}
V_{\Sigma R}^{p p}= & -\sum_{\boldsymbol{m}_{1}}^{m_{1}<\kappa_{\tau}} \sum_{\boldsymbol{m}_{2}}^{m_{2}<\kappa_{\tau}} \sum_{s m_{s}} \frac{1}{\Omega} \int d \boldsymbol{r} d \xi\left|\chi_{\boldsymbol{m}}^{\boldsymbol{M} s m_{s} T=1 T_{3}=1}\left(\boldsymbol{r} \xi ; \kappa_{\tau} \lambda_{\tau}\right)\right|^{2} \\
& \times \sum_{\mu_{1}}\left(\boldsymbol{m}_{1} \mu_{1} p \boldsymbol{k}_{\Sigma} \mu_{\Sigma} \Sigma^{+}\left|\mathcal{K}\left(\kappa_{\tau} \lambda \tau\right)\right| \boldsymbol{m}_{1} \mu_{1} p \boldsymbol{k}_{\Sigma} \mu_{\Sigma} \Sigma^{+}\right) \tag{20}
\end{align*}
$$

where $\chi_{\boldsymbol{m}}^{\boldsymbol{M} s m_{s} T T_{3}}\left(\boldsymbol{r} \xi ; \kappa_{\tau} \lambda_{\tau}\right)$ is the defect function of two nucleons with total momentum $\boldsymbol{M}=\boldsymbol{m}_{1}+\boldsymbol{m}_{2}$ and relative momentum $\boldsymbol{m}=\left(\boldsymbol{m}_{1}-\boldsymbol{m}_{2}\right) / 2$ in the spin and isospin state $s m_{s} T T_{3}$. The relative position vector of the two

[^1]nucleons is denoted by $\boldsymbol{r}$, and their spin and isospin coordinates are denoted by $\xi$. The defect function is the difference between the two nucleon wave function in nuclear matter $\psi$ and plane wave $\phi, \chi=\psi-\phi$, and is determined by $K$ through the relation $\frac{1}{a} K \phi=\chi$, which leads to the appearance of $\chi$ in expression (20). Notice that plane wave states in our expressions are normalized in the periodicity box of volume $\Omega^{3}$.

Since the integral $\int d \boldsymbol{r} d \xi|\chi|^{2}$ depends weakly on the nucleon momenta, we approximate it by its average value in the Fermi sea:

$$
\begin{align*}
& \int d \boldsymbol{r} d \xi\left|\chi_{\boldsymbol{m}}^{\boldsymbol{M} s m_{s} 11}\left(\boldsymbol{r} \xi ; \kappa_{\tau} \lambda_{\tau}\right)\right|^{2} \simeq \int d \boldsymbol{r} d \xi\left|\chi^{s m_{s} 11}\left(\boldsymbol{r} \xi ; \kappa_{\tau} \lambda_{\tau}\right)\right|^{2} \\
\equiv & \sum_{\boldsymbol{m}_{1}}^{m_{1}<\kappa_{\tau}} \sum_{\boldsymbol{m}_{2}}^{m_{2}<\kappa_{\tau}} \int d \boldsymbol{r} d \xi\left|\chi_{\boldsymbol{m}}^{\boldsymbol{M} s m_{s} 11}\left(\boldsymbol{r} \xi ; \kappa_{\tau} \lambda_{\tau}\right)\right|^{2} / \sum_{\boldsymbol{m}_{1}}^{m_{1}<\kappa_{\tau}} \sum_{\boldsymbol{m}_{2}}^{m_{2}<\kappa_{\tau}} \tag{21}
\end{align*}
$$

With this approximation, we may write expression (20) in the form:

$$
\begin{align*}
V_{\Sigma R}^{p p}= & -\frac{1}{4} \rho_{\kappa_{\tau}} \sum_{s m_{s}} \int d \boldsymbol{r} d \xi\left|\chi^{s m_{s} 11}\left(\boldsymbol{r} \xi ; \kappa_{\tau} \lambda_{\tau}\right)\right|^{2} \\
& \times \sum_{\boldsymbol{m}_{1}}^{m_{1}<\kappa_{\tau}} \sum_{\mu_{1}}\left(\boldsymbol{m}_{1} \mu_{1} p \boldsymbol{k}_{\Sigma} \mu_{\Sigma} \Sigma^{+}\left|\mathcal{K}\left(\kappa_{\tau} \lambda \tau\right)\right| \boldsymbol{m}_{1} \mu_{1} p \boldsymbol{k}_{\Sigma} \mu_{\Sigma} \Sigma^{+}\right) \tag{22}
\end{align*}
$$

where

$$
\begin{equation*}
\rho_{\kappa_{\tau}}=4 \frac{1}{\Omega} \sum_{m_{1}}^{m_{1}<\kappa_{\tau}}=\frac{2 \kappa_{\tau}^{3}}{3 \pi^{2}}=\frac{2 Z}{\Omega}=\rho\left(1+\alpha_{\tau}\right) \tag{23}
\end{equation*}
$$

where $\rho$ is the density of nuclear matter, $\rho=A / \Omega=2 k_{F}^{3} / 3 \pi^{2}\left(k_{F}\right.$ is the Fermi momentum of spin and isospin saturated nuclear matter).

From now on, we neglect the dependence of $\rho_{\kappa_{\tau}} \int d \boldsymbol{r} \xi|\chi|^{2}$ on the proton excess parameter $\alpha_{\tau}$ :

$$
\begin{equation*}
\rho_{\kappa_{\tau}} \int d \boldsymbol{r} d \xi\left|\chi^{s m_{s} 11}\left(\boldsymbol{r} \xi ; \kappa_{\tau} \lambda_{\tau}\right)\right|^{2} \simeq \bar{\kappa}^{s m_{s} T=1} \tag{24}
\end{equation*}
$$

where the "wound integral" in the state $s m_{s} T, \bar{\kappa}^{s m_{s} T}$, is

$$
\begin{equation*}
\bar{\kappa}^{s m_{s} T}=\bar{\kappa}^{s m_{s} T}\left(k_{F}\right) \equiv \rho \int d \boldsymbol{r} d \xi\left|\chi^{s m_{s} T}\left(\boldsymbol{r} \xi ; k_{F}\right)\right|^{2} \tag{25}
\end{equation*}
$$

where $\chi^{s m_{s} T}\left(\boldsymbol{r}, \xi ; k_{F}\right)=\chi^{s m_{s} T T_{3}}\left(\boldsymbol{r}, \xi ; \kappa_{\tau}, \lambda_{\tau}\right)_{\kappa_{\tau}=\lambda_{\tau}=k_{F}}$ (notice that it does not depend on $T_{3}$ ). The accuracy of approximation (24) is discussed in the next section.

[^2]With approximation (24), Eq. (22) takes the form:

$$
\begin{equation*}
V_{\Sigma R}^{p p}=-\frac{1}{4} \sum_{s m_{s}} \bar{\kappa}^{s m_{s} 1} \sum_{\boldsymbol{m}_{1}}^{m_{1}<\kappa_{\tau}} \sum_{\mu_{1}}\left(\boldsymbol{m}_{1} \mu_{1} p \boldsymbol{k}_{\Sigma} \mu_{\Sigma} \Sigma^{+}\left|\mathcal{K}\left(\kappa_{\tau} \lambda \tau\right)\right| \boldsymbol{m}_{1} \mu_{1} p \boldsymbol{k}_{\Sigma} \mu_{\Sigma} \Sigma^{+}\right) \tag{26}
\end{equation*}
$$

Proceeding in the same way with $V_{\Sigma R}^{n n}$ and $V_{\Sigma R}^{p n}$, and adding the expressions so obtained to expression (26), we get

$$
\begin{equation*}
V_{\Sigma R}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right)=-\bar{\kappa} V_{\Sigma}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right) \tag{27}
\end{equation*}
$$

where the wound integral $\bar{\kappa}$ is:

$$
\begin{equation*}
\bar{\kappa}=\frac{1}{8} \sum_{s m_{s} T}(2 T+1) \bar{\kappa}^{s m_{s} T} \tag{28}
\end{equation*}
$$

Eq. (27) leads to

$$
\begin{align*}
V_{0 R}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right) & =-\bar{\kappa} V_{0}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right)  \tag{29}\\
U_{0}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right) & =(1-\bar{\kappa}) V_{0}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right) \tag{30}
\end{align*}
$$

and

$$
\begin{align*}
V_{x R}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right) & =-\bar{\kappa} V_{x}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right)  \tag{31}\\
U_{x}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right) & =(1-\bar{\kappa}) V_{x}\left(\mu_{\Sigma}, \Sigma^{+}, k_{\Sigma}\right) \tag{32}
\end{align*}
$$

where $x$ stands for $\tau$. Expressions for $V_{0}$ and $V_{x}$, which may be obtained from Eq. (13), are given in I.

The procedure with $V_{\sigma R}$ and $V_{\sigma \tau R}$ is similar, and the resulting expressions turn out to be identical with (31), (32). This means that expressions (31), (32) are valid not only for $x=\tau$, but also for $x=\sigma$ and $x=\sigma \tau$.

## 3. Discussion of the results

In deriving the simple relation between $V_{x R}$ and $V_{x}$, Eq. (31), we made an additional approximation which for $x=\tau$ has the form of Eq. (24). It is relatively simple to estimate the effect of the $\alpha_{\tau}$ dependence of $\rho_{\kappa_{\tau}}$ neglected in approximation (24), and of $\rho_{\lambda_{\tau}}$ neglected in similar approximations used in calculating $V_{\Sigma R}^{n n}$ and $V_{\Sigma R}^{p n}$. This leads to the conclusion that the contribution of this neglected $\alpha_{\tau}$ dependence to $V_{\tau R}$ is a few times smaller than our contribution (31). We believe that this conclusion would not be changed by including the $\alpha_{\tau}$ dependence of $\int d \boldsymbol{r} d \xi|\chi|^{2}$ also neglected in approximation (24). Namely if we apply the single density approximation applied in [8], we may express this neglected $\alpha_{\tau}$ dependence through the dependence of
$\int d \boldsymbol{r} d \xi\left|\chi\left(\boldsymbol{r} \xi ; k_{F}\right)\right|^{2}$ on $k_{F}$, which appears to be much weaker than this dependence of $\rho$ (see e.g. [9]). For $x=\sigma, \sigma \tau$, the situation is similar.

Our final result for the effect of the rearrangement potential on the isospin and spin dependent component of the $\Sigma$ s.p. potential in nuclear matter, expressions (31), (32), is very simple. It turns out that the simple form of this effect in isospin and spin saturated nuclear matter, Eqs (29), (30), also applies in the case of nuclear matter with isospin and spin excess.

The magnitude of the whole effect depends on one parameter: the wound integral $\bar{\kappa}$. As was discussed in [5], $\bar{\kappa}$ is determined predominantly by acting of the hard core in the $S$ state of the interacting nucleons, which leads to the result $\bar{\kappa}=0.15$ for the had core radius $r_{c}=0.4 \mathrm{fm}$. Results obtained with soft core $N N$ interactions are smaller. In general, values of the wound integral considered in the literature are in the range: $0.15 \gtrsim \bar{\kappa} \gtrsim 0.10$. Here, we shall use the value $\bar{\kappa}=0.15$ calculated in [5].

In Table I, we present the results for $V_{0}, U_{0}$, and $V_{x}, U_{x}$ (for $x=\tau, \sigma$, and $\sigma \tau$ ). Results for $V_{0}, V_{x}$ are taken from I where they were calculated with the help of the YNG effective $\Sigma N$ interaction of Yamamoto et al. [6], which represents the reaction matrix $\mathcal{K}$ calculated in the LOB theory from the model D [10], model F [11], and the soft-core (SC) model [12] of the Nijmegen baryon-baryon interaction. Also included are results calculated in I with the help of the YNG interaction obtained from the new soft-core (NSC) model of Rijken et al. [13].

TABLE I
Different components (in MeV ) of $U_{\Sigma}\left(k_{\Sigma}=0\right)$ calculated at $k_{F}=1.35 \mathrm{fm}^{-1}$ with the YNG interaction obtained from the indicated models of the $\Sigma N$ interaction, and with $\bar{\kappa}=0.15$.

| x | Model | $V_{0}$ | $U_{0}$ | $V_{x}$ | $U_{x}$ |
| :---: | :---: | ---: | ---: | ---: | ---: |
|  | D | -13.1 | -11.1 | 55.1 | 46.8 |
| $\tau$ | F | 23.5 | 20.0 | 80.4 | 68.3 |
|  | SC | -9.6 | -8.2 | 31.0 | 26.4 |
|  | NSC | -16.6 | -14.1 | -36.7 | -31.2 |
|  | D | -13.1 | -11.1 | 66.8 | 56.8 |
| $\sigma$ | F | 23.5 | 20.0 | 72.3 | 61.5 |
|  | SC | -9.6 | -8.2 | 3.3 | 2.8 |
|  | NSC | -16.6 | -14.1 | -40.4 | -34.3 |
| $\sigma \tau$ | D | -13.1 | -11.1 | 63.9 | 54.3 |
|  | F | 23.5 | 20.0 | 95.6 | 81.3 |
|  | SC | -9.6 | -8.2 | 56.3 | 47.9 |
|  | NSC | -16.6 | -14.1 | 70.7 | 60.1 |

Because of the rearrangement effect all components of the $\Sigma$ s.p. potential in nuclear matter are reduced by the factor $(1-\bar{\kappa}) \simeq 0.85$. With this reduction taken into account, the discussion in I remains essentially unchanged. In particular, the conclusion remains valid, that the new strangeness exchange experiments at Brookhaven [14,15] favor model F of the Nijmegen interaction.

This research was partly supported by Polish State Committee for Scientific Research (KBN) under Grant No. 2-P03B-048-12.

## REFERENCES

[1] T. Harada, S. Shinmura, Y. Akaishi, H. Tanaka, Nucl. Phys. A507, 715 (1990).
[2] T. Harada, Y. Akaishi, Progr. Theor. Phys. 96, 145 (1996).
[3] J. Dąbrowski, Phys. Rev. C60, 025205 (1999).
[4] J. Dąbrowski, Acta Phys. Pol. B30, 2783 (1999).
[5] J. Dąbrowski, H.S. Köhler, Phys. Rev. 136, B162 (1964).
[6] Y. Yamamoto, T. Motoba, H. Himeno, K. Ikeda, S. Nagata, Progr. Theor. Phys. Suppl. 117, 361 (1994).
[7] J. Dąbrowski, M.Y.M. Hassan, Acta Phys. Pol. B1, 339 (1970).
[8] K.A. Brueckner, J. Dąbrowski, Phys. Rev. 134, B722 (1964).
[9] N. Yamaguchi et al., Progr. Theor. Phys. 70, 459 (1983).
[10] N.M. Nagels, T.A. Rijken, J.J. de Swart, Phys. Rev. D12, 744 (1975; D15, 2547 (1977).
[11] N.M. Nagels, T.A. Rijken, J.J. de Swart, Phys. Rev. D20, 1663 (1979).
[12] P.M.M. Maessen, T.A. Rijken, J.J. de Swart, Phys. Rev. C40, 226 (1989); Nucl. Phys. A547, 245c (1992).
[13] T.A. Rijken, V.G.J. Stoks, Y. Yamamoto, Phys. Rev. C59, 21 (1999).
[14] R. Sawafta, Nucl. Phys. A639, 103c (1998).
[15] T. Nagae et al., Phys. Rev. Lett. 80, 1605 (1998).


[^0]:    ${ }^{1}$ Similarly as in I we consider the case of pure central $\Sigma N$ effective interaction for which the possible dependence of $U_{\sigma}$ and $U_{\sigma \tau}$ on the direction of $\boldsymbol{k}_{\Sigma}$ does not appear.

[^1]:    ${ }^{2}$ We use the convention in which the third component of the isospin of proton (neutron) is $\frac{1}{2}\left(-\frac{1}{2}\right)$.

[^2]:    ${ }^{3}$ The essential steps in obtaining expression (20) are similar to those presented in detail in [7] (see also [5]).

