THE SOLITON SOLUTIONS OF THE SCHWINGER MODEL

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The soliton solutions of the bosonised version of the massive Schwinger model for $N_f = 2$ in the first approximation are found. We discus scattering such solutions on each other in both elastic and non-elastic cases.

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1. Introduction

The Schwinger model [1] is a well-understood theoretical model which is usually used to simulate QCD. It reveals some crucial phenomena known from more realistic four-dimentional models. Indeed, confinement, screening, chiral symmetry breaking and topological vacua appear at once. It is not hard to investigate here a formation of bound states. A technical advantage of the Schwinger model, in comparison with the standard QCD, is the fact that it is a two-dimentional model and many calculations can be carry out explicitly [2–7]. In particular, it would be very intriguing to analyze the possibility of existence of the mesons composed of fundamental quarks. The aim of our work is to find these mesons in the limit of quark masses much bigger than the coupling constant. We will describe their scattering as well.

The paper is organized as follows. In Section 2, we will introduce the Schwinger model in the standard fermionic form as well as in the bosonised version. The topological charges and the equations of motion will be presented. In Section 3, we solve a flavor-charged meson solution while a flavor-neutral meson (in the first approximation) will be found in Section 4. A brief description of a double-mesonic state is also included. Section 5 is devoted to find solutions for elastic scattering of such mesons. In particular, the scattering of flavor-neutral mesons is analyze in Section 6 using an adiabatic approximation. Section 7 contains the discussion and conclusions. In this Section we comment on non-elastic scattering solutions.

2. Lagrangian

We will consider N_f fundamental charged and massive Dirac fermions in two-dimensional spacetime, with the well-known Lagrangian density [1],

$$L = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}^a(i\gamma^\mu\partial_\mu - \mathrm{e}\gamma^\mu A_\mu - m_a)\psi^a \,, \tag{1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,. \tag{2}$$

In general, we allow for different masses m_a of fermion fields ψ_a.

We are going to adopt standard-Abelian bosonisation rules,

$$N_{m_a}[\bar{\psi}^a\gamma^\mu\psi^a] \;=\; \frac{1}{\sqrt{\pi}}\varepsilon^{\mu\nu}\partial_\nu\phi^a \,, \tag{3}$$

$$N_{m_a}[\bar{\psi}^a\psi^a] \;=\; -cm_a N_{m_a}[\cos\sqrt{4\pi}\phi^a] \,, \tag{4}$$

where ϕ^a is the family of canonical pseudoscalar fields and N_{m_a} denotes normal-ordering with respect to fermion masses.

Substituting (3), (4) into the Schwinger's Lagrangian it yields, after integrating out the electric field F_0, the bosonized version of (1),

$$L_{\mathrm{eff}} \;=\; \sum_{a=1}^{N_f}\frac{1}{2}(\partial_\mu\phi^a)^2 - V_{\mathrm{eff}} \,, \tag{5}$$

$$V_{\mathrm{eff}} \;=\; \frac{\mathrm{e}^2}{2\pi}\Big(\sum_{a=1}^{N_f}\phi^a + \frac{\theta}{\sqrt{4\pi}}\Big)^2 - \sum_{a=1}^{N_f} cm_a^2 N_{m_a}[\cos\sqrt{4\pi}\phi^a] + \mathrm{const.} \tag{6}$$

Different vacua are labeled by the angle parameter θ.

In this paper, we limit ourselves to the two-flavor case and set $\theta = 0$ (no CP breaking). We separate the Lagrangian into two parts,

$$L^0_{\mathrm{eff}} \;=\; \sum_{a=1}^{2}\frac{1}{2}(\partial_\mu\phi^a)^2 + \sum_{a=1}^{2} cm_a^2 N_{m_a}[\cos\sqrt{4\pi}\phi^a] \,, \tag{7}$$

$$L^{\mathrm{int}}_{\mathrm{eff}} \;=\; -\frac{e^2}{2\pi}(\phi^1 + \phi^2)^2 \,. \tag{8}$$

The first part describes the free system of two sine-Gordon fields. The second part stands for the Coulomb interaction between them. The boundary values of ϕ^a at the spatial infinity are associated with the flavor quantum numbers of the fundamental fermion fields through the relations:

$$Q^a = \frac{1}{\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\partial_x\phi^a dx = \frac{1}{\sqrt{\pi}}\phi^a\Big|_{-\infty}^{\infty} \,. \tag{9}$$

We expect that all finite-energy solutions should approach the vacuum asymptotically. The Lagrangian gives the following field equations ($a = 1, 2$),

$$\partial_t^2 \phi^a - \partial_x^2 \phi^a + \frac{e^2}{\pi}(\phi^1 + \phi^2) + c\sqrt{4\pi} m_a^2 \sin(\sqrt{4\pi}\phi^a) = 0\,. \tag{10}$$

It is convenient to denote $\beta = \sqrt{4\pi}$, $\mu^2 = N_f \frac{e^2}{\pi}$ and $M_a^2 = c\beta^2 m_a^2$. Finally, after rescaling of fields $\phi^a \to \beta\phi^a$ and the coordinates $x \to Mx$, $t \to Mt$ we obtain the standard form of the coupled sine-Gordon equation (we set $M_1 = M_2 = M$):

$$\partial_t^2 \phi^a - \partial_x^2 \phi^a + \frac{\varepsilon}{2}(\phi^1 + \phi^2) + \sin\phi^a = 0\,. \tag{11}$$

The classical Euler–Lagrange field equations depend only on one parameter $\varepsilon = \frac{e^2}{\mu^2}$. In the present paper, we try to solve these equations in the limit of weak interaction ($\varepsilon \ll 1$). We find localized solutions, which correspond to "mesons" composed of fundamental "quark" fields ϕ^a.

3. Flavor-charged mesons

The simplest case is a solution which describes the meson with nonzero topological charges, with the assumption $\phi^1 = -\phi^2$. In this case, we find two separated sine-Gordon equations. The straightforward solution can be written in the form:

$$\Phi_1 = (\phi_s, \phi_a), \Phi_2 = (\phi_a, \phi_s)\,, \tag{12}$$

where

$$\phi_{s,a} = 4Q \arctan \mathrm{e}^{\gamma(x - x_0 - vt)} \tag{13}$$

and $Q_{s,a} = \pm 1$ for soliton or for antisoliton respectively [8]. The energy and mass for the meson solutions (12) are the same,

$$E = E_a + E_s = 16\gamma\,, \tag{14}$$

$$m = m_a + m_s\,. \tag{15}$$

These mesons are the lowest energy states being the composition of soliton and antisoliton pair localized at some point x_0. However, if we try to separate components for small distance Δx, they start to attract themselves and the energy grows like the confining force [9],

$$\Delta E \sim (\Delta x)^2\,. \tag{16}$$

The asymptotic freedom and the confinement in the Schwinger model are observed already at the classical level provided that we use the dual description (bosonization).

4. Flavor-neutral mesons

Our aim is to find a classical solution of nonlinear equations (11), which describes a meson with vanishing topological charges in the weak interaction limit $\varepsilon << 1$. It corresponds to the following expansion.

$$\phi^1 = \phi^1_0 + \varepsilon\phi^1_1 \,, \tag{17}$$

$$\phi^2 = \varepsilon^2_1 \,. \tag{18}$$

On account of the fact that the topology of the vacuum states or asymptotical behavior of the fields is unchanged this expansion appears to be correct. It shows that both fields are crucial for the topological reasons. However, as we see it at the end of this Section, only one of them gives the contribution to the energy. Using this assumption we derive from Eq. (11) the following equations for the expansion coefficients,

$$(\partial_t^2 - \partial_x^2)\phi^1_0 + \sin\phi^1_0 = 0 \,, \tag{19}$$

$$(\partial_t^2 - \partial_x^2)\phi^1_1 + \frac{1}{2}\phi^1_0 + \phi^1_1\cos\phi^1_0 = 0 \,, \tag{20}$$

$$(\partial_t^2 - \partial_x^2)\phi^2_1 + \frac{1}{2}\phi^1_0 + \phi^2_1 = 0 \,. \tag{21}$$

Taking into account the fact that we search for a pure meson state with vanishing topological charges, we should start with the breather solution of Eq. (19),

$$\phi^1_0 = \phi_b = 4\arctan\left[\frac{\gamma\sin\omega t}{\omega\cosh\gamma x}\right] \,, \tag{22}$$

where $\gamma = \sqrt{1-\omega^2}$. This solution is periodic in time and it is the lowest energy composite state which possesses zero topological charge [8]. The next step is the Fourier expansion of the first coefficient fields,

$$\phi^1_1 = \sum_{n=0}^{\infty} \phi^1_{1sn}(x)\sin n\omega t + \phi^1_{1cn}(x)\cos n\omega t \,, \tag{23}$$

$$\phi^2_1 = \sum_{n=0}^{\infty} \phi^2_{1sn}(x)\sin n\omega t + \phi^2_{1cn}(x)\cos n\omega t \,. \tag{24}$$

After some straighforward calculations we compute,

$$\phi^2_{1sn} = \begin{cases} \frac{1}{4\gamma_n}[\mathrm{e}^{-\gamma_n x}\int\limits_{\infty}^{x} \mathrm{e}^{-\gamma_n y}\phi_{bn}dy + \mathrm{e}^{-\gamma_n}\int\limits_{-\infty}^{x}\mathrm{e}^{\gamma_n y}\phi_{bn}dy] & n < [\frac{1}{\omega}] \,, \\ 0 & n > [\frac{1}{\omega}] \,, \end{cases} \tag{25}$$

$$\phi^2_{1cn} = 0\,, \tag{26}$$

where ϕ_{bn} is n-th factor in the Fourier expansion of the breather and $\gamma_n = \sqrt{1-n^2\omega^2}$. With the field ϕ^1_1 the situation is more complicated and for simplicity we restrict ourselves to analyze only the first terms of the Fourier expansion,

$$\phi^1_1 = \phi^1_{1s1}(x)\sin\omega t + \phi^1_{1c1}\cos\omega t\,. \tag{27}$$

This simplification gives, as it is explained in [9] a good approximation to the energy. We put also the approximation for the breather solution in this case,

$$\phi^1_0 \approx \phi\sin\omega t\,, \tag{28}$$

where the function ϕ satisfies the following equation,

$$\partial^2_x\phi + \omega^2\phi - 2J_1(\phi) = 0 \tag{29}$$

and J_1 is the first order Bessel function. After substitution of (28), (29) into (20) and using the approximation (27) we obtain,

$$\phi^1_{1c1} = 0\,, \tag{30}$$

$$-(\omega^2+\partial^2)\phi^1_{1s1} + \frac{1}{2}\phi + \phi^1_{1s1}\frac{2}{\pi}\int\limits_0^\pi \sin^2 t\cos(\phi\sin t)dt = 0\,, \tag{31}$$

where the lates equation can be resolved with the following asymptotical conditions,

$$\phi^1_{1s1} \longrightarrow 0,\ x\to\pm\infty\,. \tag{32}$$

Using a standart procedure we obtain

$$\phi^1_{1s1}(x) = \frac{1}{4}\partial_x\phi\int\limits_0^x(\partial_y\phi)^{-2}\phi^2 dy\,. \tag{33}$$

where the latest equation can be resolved with the following asymptotical (29) together with the asymptotical conditions we derive that,

$$\phi \approx \phi_{b1}\,, \tag{34}$$

where

$$\phi_{b1} = \frac{8}{\sqrt{(\frac{\omega}{\gamma})^2\cosh^2\gamma x + 1} + \frac{\omega}{\gamma}\cosh\gamma x}\,. \tag{35}$$

It is easy to show that the next term of the Fouriar expansion of ϕ_1^1 vanishes. Let us summarize, we have computed the approximated solution of the flavor neutral meson state,

$$\phi^1 = \phi_0 + \frac{\varepsilon}{4}\partial_x\phi_{b1}\sin\omega t\int_0^x (\partial_y\phi_{b1})^{-2}\phi_{b1}^2 dy\,, \tag{36}$$

$$\phi^2 = \varepsilon\sum_{n=1}^{N=[\frac{1}{\omega}]}\frac{1}{4}\left[\mathrm{e}^{\gamma_n x}\int_\infty^x \mathrm{e}^{-\gamma_n y}\phi_{bn}dy + \mathrm{e}^{-\gamma_n x}\int_x^{-\infty}\mathrm{e}^{\gamma_n y}\phi_{bn}dy\right]\sin n\omega t\,. \tag{37}$$

The solution describes time oscillations around the breather state for the field ϕ^1 and the vacuum state for the field ϕ^2. Obviously, the oscillations are localized at the spatial space around the origin.

The energy density following from the Lagrangian (5) has the form for this solution,

$$E = \sum_{a=1,2}\frac{1}{2}(\partial_\mu\phi^a)^2 + (1-\cos\phi^a) + \frac{\varepsilon}{4}(\phi^1+\phi^2)^2 \tag{38}$$

$$= E_b + \varepsilon[\partial_x\phi_0\partial_x\phi_1^1 + \partial_t\phi_0\partial_t\phi_1^1 + \phi_1^1\sin\phi_0] + o(\varepsilon)\,, \tag{39}$$

where E_b is the energy density of the breather. The residual field ϕ^2 gives a contribution at the higher orders in ε.

The lowest two-meson state can be obtained in the same assumption like in the one-meson state *i.e.* if we put $\phi^1 = -\phi^2$ in (11). Then the solution is,

$$(Q\bar{Q}q\bar{q}) = (\phi_b, -\phi_b)\,. \tag{40}$$

On the contrary to one-flavor-neutral case this state does not contain the residual fields.

5. Scattering solution

It is easy to find the elastic scattering solution when we scatter two flavor-charged mesons (Q stands here for the field ϕ^1 and q refers to the field ϕ^2),

$$Q\bar{q} + \bar{Q}q \to Q\bar{q} + \bar{Q}q\,. \tag{41}$$

The solution is,

$$\varPhi = (\phi_{sa}, -\phi_{sa})\,, \tag{42}$$

where ϕ_{sa} is well-known solution of the sine-Gordon equation, which describes scattering of a soliton and an antisoliton [8],

$$\phi_{sa} = 4\arctan\left[\frac{\sinh\gamma vt}{v\cosh\gamma x}\right]\,. \tag{43}$$

Asymptotically we find two single meson states which propagate in opposite directions.

The similar case refers to the elastic scattering of a meson on a meson of the same type (both are flavor-charged),

$$Q\bar{q} + Q\bar{q} \to Q\bar{q} + Q\bar{q}\,. \tag{44}$$

The solution can be found in the analogous way,

$$\Phi = (\phi_{ss}, -\phi_{ss})\,, \tag{45}$$

where ϕ_{ss} is the double soliton scattering solution of the sine-Gordon equation [8],

$$\phi_{ss} = \arctan\left[\frac{\cosh\gamma vt}{v\sinh\gamma x}\right]\,. \tag{46}$$

A less straightforward task is to find the solution of the equation (11) which describes the elastic scattering of the two flavor-neutral mesons (at least in the limit of small ε,

$$Q\bar{Q} + q\bar{q} \to Q\bar{Q} + q\bar{q}\,. \tag{47}$$

We compute the above solution in the first order approximation,

$$\Psi = \Psi_0 + \varepsilon\psi_1\,, \tag{48}$$

where

$$\Psi_0 = (\psi_0^1, \psi_0^2) = (\psi_b(x - vt), \psi_b(x + vt)) \tag{49}$$

is solution of two independent sine-Gordon equations which are obtained taking the limit $\varepsilon = 0$. This solution contains two sine-Gordon breathers which move in opposite directions with the same velocity. The first correction Ψ_1 can be interpreted as the interaction of the breathers. The interaction is small when the breathers are separated. It becomes to play a significant way only during the short time when the centers of the breathers come close together. Let us investigate that in a more detailed way. Comparing leading order terms in the parameter ε in the equation of motion (11) we obtain,

$$(\partial_t^2 - \partial_x^2)\psi_1^1 + \psi_1^1\cos\psi_0^1 = -\tfrac{1}{2}(\psi_0^1 + \psi_0^2)\,, \tag{50}$$

where ψ_1^1 is one of two field components of Ψ_1. In order to solve this linear differential equation we divide it into two parts.

The first one reads,

$$(\partial_t^2 - \partial_x^2)\psi_1^1 + \psi_1^1\cos\phi_b(x - vt) = -\tfrac{1}{2}\phi_b(x - vt) \tag{51}$$

and it is identical with the equation considered previously for $(Q\bar{Q})$ meson state. Therefore the solution can be presented in the following form,

$$\psi_1^1 = \phi_1^1(x - vt)\,. \tag{52}$$

Now let us turn to the second equation,

$$(\partial_t^2 - \partial_x^2)\psi_1^1 + \psi_1^1 \cos\phi_b(x - vt) = -\tfrac{1}{2}\phi_b(x + vt)\,. \tag{53}$$

The solution of (53) can be written as,

$$\psi_1^1 = f(x,t)\phi_1^2(x + vt)\,. \tag{54}$$

The shape function $f(x,t)$ should posses the above mentioned asymptotical properties namely $f(x,t) \to 1$ if the breathers are separated. It is easy to verify that this limit is approached exponentially.

Thus the approximated solution for the contact interactions during the scattering of two flavor-neutral mesons can be written as follows,

$$\Psi = (\Psi^1, \Psi^2)\,, \tag{55}$$

where

$$\Psi^1 = \phi_b(x - vt) + \varepsilon[\phi_1^1(x - vt) + f(x,t)\phi_1^2(x + vt)]\,. \tag{56}$$

In the formula for Ψ^2 we have to interchange $x - vt$ with $x + vt$.

This solution has the required asymptotical behavior and a finite energy.

In a similar way the solution which describes elastic scattering of the mesons $Q\bar{Q}$ and $Q\bar{q}$ can be found,

$$Q\bar{Q} + Q\bar{q} \to Q\bar{Q} + Q\bar{q}\,. \tag{57}$$

If we put again $\varepsilon = 0$, then the equation (11) separates into two independent sine-Gorgon equations and the respective solution is,

$$\Phi = (\phi_{b-s}, \phi_a)\,. \tag{58}$$

The function ϕ_{b-s} is the scattering solution of the sine-Gordon equation where a breather scatters on a soliton [10]. This solution represents an interaction between the meson $Q\bar{Q}$ and the quark Q while the quark $\bar{q}$ is non-interacting in the zero order approximation. When we consider first order corrections we find new effects. The quarks, which were primary free, are confined around the same point. The point, usually called center, moves during the time evolution. This is the mechanism of the meson $Q\bar{q}$ formation.

6. Adiabatic evolution

The most general form of the breather which solves the standard sine-Gordon equation can be written down,

$$\phi_b = 4\arctan\left[\tan\mu\frac{\sin(\eta\cos\mu)}{\cosh(z\sin\mu)}\right], \tag{59}$$

$$z = \frac{x-\xi}{\sqrt{1-V^2}}, \quad \xi = Vt+\xi_0, \tag{60}$$

$$\eta = \frac{t-Vx}{\sqrt{1-V^2}} + \eta_0 . \tag{61}$$

The parameter μ is the amplitude of the breather. If we add a small term $P[\phi]$ to the sine-Gordon equation, then the solution of the extended equation can be found as the perturbation of the breather (59). In the adiabatic approximation we can compute a general perturbation-induced evolution equation for the parameters μ, V and η [11]. We are going to analyze two parameters (the amplitude and the velocity) in details.

$$\frac{d\mu}{dt} = \varepsilon(1-V^2)^{1/2}(4\cos\mu)^{-1}I_1 , \tag{62}$$

$$\frac{dV}{dt} = -\varepsilon(1-V^2)^{3/2}(4\cos\mu)^{-1}I_2 , \tag{63}$$

where

$$I_1 = \int\limits_{-\infty}^{\infty} \frac{\cosh x\cos(\cos\mu\eta)}{\cosh^2 x+\tan^2\mu\sin^2(\cos\mu\eta)}P[\phi_b]dx , \tag{64}$$

$$I_2 = -\int\limits_{-\infty}^{\infty} \frac{\sinh x\sin(\cos\mu\eta)}{\cosh^2 x+\tan^2\mu\sin^2(\cos\mu\eta)}P[\phi_b]dx , \tag{65}$$

where the integration is over the variable $x = \sin\mu z$. The above equation can be expanded in the parameter ε. Let us introduce the expansions $\mu = \mu_0 + \varepsilon\mu_1$ and $V = V_0 + V_1$. The breather parameters μ_0 and V_0 refer to the amplitude and the velocity of the standard sine-Gordon breather respectively. We obtain the equations for μ_1 and V_1,

$$\frac{d\mu_1}{dt} = (1-V_0^2)^{1/2}(4\cos\mu_0)^{-1}\int\limits_{-\infty}^{\infty} \frac{\cosh x\cos(\cos\mu_0\eta)}{\cosh^2 x+\tan^2\mu_0\sin^2(\cos\mu_0\eta)}P[\phi_b]dx, \tag{66}$$

$$\frac{dV_1}{dt} = (1-V_0^2)^{3/2}(4\cos\mu_0)^{-1}\int\limits_{-\infty}^{\infty} \frac{\sinh x\sin(\cos\mu_0\eta)}{\cosh^2 x+\tan^2\mu_0\sin^2(\cos\mu_0\eta)}P[\phi_b]dx. \tag{67}$$

In the case of the meson $(Q\bar{Q})$ the perturbation is represented by,

$$P[\phi_b] = \phi_b \,. \tag{68}$$

It is easy to see that V_1 vanishes so that the free meson $(Q\bar{Q})$ moves with constant velocity V_0. But behaviour of μ_1 is more sophisticated. The amplitude (thickness) is periodic in time with the period,

$$T = \pi \frac{\sqrt{1 - V_0^2}}{\cos \mu_0} \,. \tag{69}$$

The period is twice smaller then the period of the standard breather oscillations. Then, in our solution the perturbed breather oscillates in the thickness twice during the period of the fundamental oscillations of the sine-Gordon breather. The amplitude of the oscillations is much smaller than the instant width of the breather.

Let us consider the above mentioned elastic scattering mesons $(Q\bar{Q})$ and $(q\bar{q})$. Now the perturbation term has the form,

$$P[\phi] = \phi_b(x + vt) + \phi_b(x - vt) \,. \tag{70}$$

First, we can observe that the solution of such problem tends asymptotically to the free meson solution. The difference is that the velocity is no longer fixed. There is a small correction V_1 being periodic in time. The period of the oscillations is the same as for μ_1.

During the scattering the mesons $(Q\bar{Q})$ and $(q\bar{q})$ attract and push away each other periodically. This interaction is observed when the centers of the meson $(Q\bar{Q})$ and $(q\bar{q})$ are close together.

7. Summary and conclusion

In this article we studied the existence and the properties of soliton solutions in the Schwinger model (a bosonized version) when the quarks masses are equal (SU(2) flavor symmetry). We found the lowest states for the flavor-charged and the flavor-neutral mesons in the approximation of weak interactions. These mesons possess topological charges and finite energies. Their stability is guaranteed by topological charges *i.e.* by the form of the ground state. In both mesons we have the confinement of the fundamental quarks at the classical level. However, the way how the confinement appears in the above mesons is different. In the flavor-charged case it is implied directly by the Coulomb interactions between fields $\phi^{1,2}$ (8), while in the flavor-neutral case the confinement follows from the mechanism which stabilizes the breather and it is crucial based on the interactions with the vacuum.

These mesons can form bound states (*i.e.* the two-flavor-neutral state) as well as scattering states. In particular, we found the elastic scattering solution for various configurations of the mesons. The flavor-neutral case was obtained in the approximation of weak interactions. We observed small and fast vanishing oscillations of the amplitude and the velocity. The period of the oscillations is twice smaller than the basic period of the breather.

The above SU(2)-flavor solutions can be easy embedded in the SU(3)-flavor case. However, non-trivial SU(3) solutions, which contain all fundamental fields should also exist. These mesons and their behaviour will be investigated in the next paper.

It is still too difficult to analyze the non-elastic scattering solution in the above framework. The ground state is given by a soliton of sine-Gordon equation and it cannot be transformed into another solution. Time evolution does not destroy the ground state. We can assume that our approximation is failed when the correction to the amplitude of the breather satisfy,

$$\mu_0 \sim \varepsilon\mu_1\,. \tag{71}$$

In this case fundamental quarks of the scattering mesons are mixed and interchanged.

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