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Φ-MEASURE OF AZIMUTHAL FLUCTUATIONS

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The event-by-event azimuthal fluctuations in high-energy heavy-ion collisions are analyzed by means of the so-called Φ-measure. The fluctuations due to the collective transverse flow and those caused by the quantum statistics in the hadron gas are discussed in detail.

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Various phenomena can lead to the nontrivial azimuthal fluctuations in (ultra-)relativistic heavy-ion collisions. One mentions here jets and minijets being the result of (semi-)hard parton-parton scattering [1] and the collective transverse flow due to the anisotropic pressure gradient [2,3]. The latter phenomenon, which is well known at the intermediate collision energies (for a review see *e.g.* [4]), has been recently observed in the nucleus-nucleus collisions at CERN SPS [5, 6]. As argued in the series of our papers [7], the color plasma instabilities also generate a transverse collective flow in heavy-ion collisions at RHIC and LHC. Since the final state azimuthal fluctuations appear as remnants of the inhomogeneous or anisotropic early stage of the collision, when the minijets are copiously produced [8] or the color instabilities occur [7], the effects of interests are expected to be rather small [9]. Thus, it is a real challenge to extract them from the statistical noise.

A method, which has been successfully applied to study the collective flow in heavy-ion collisions at SPS energies [6], is based on the Fourier analysis of the azimuthal distribution [11–13]. Specifically, one expands the distribution of the particle azimuthal angle measured with respect to the reaction plane, which is reconstructed on the event-by-event basis, into the Fourier series. Then, the first harmonic tells us about the so-called directed flow, the second one about the elliptic one, *etc.* Although the method is very powerful there are subtleties in applying it to the real data. It has been shown in a very recent paper [12] that the Bose-Einstein correlations of pions significantly influence the measured Fourier coefficients. The finite statistics and non-flow interparticle correlations — the jets are expected to be an important source of such correlations at RHIC — distort the reconstructed event plane [11]. Due to the finite resolution of the reconstruction procedure, it is difficult to measure the Fourier coefficients higher than the second one [13].

The Fourier expansion method has been designed to study the collective flow which is correlated with the reaction plane. However, the jets or color instabilities, which have been mentioned above, generate the transverse collective motion being, at least approximately, independent of the reaction plane orientation. Then, the Fourier expansion method, as developed in [10,11,13], does not seem to be a right tool to study such phenomena and it is worth to consider other methods of the data analysis, for example that one which focuses on the two-particle large-angle azimuthal correlations [14].

In this letter we propose to study the azimuthal fluctuations by means of the previously introduced fluctuation measure Φ [15] which, in fact, has been invented for other purposes. When the fluctuations of any observable of heavy-ion collisions are studied one faces a problem how to disentangle the 'dynamical' fluctuations from the 'trivial' geometrical ones due to the impact parameter variation. The latter fluctuations are large and dominate the fluctuations of all extensive event characteristics such as multiplicity or transverse energy. Using the fluctuation measure Φ resolves the problem in a specific way. By construction, Φ is exactly the same for nucleon-nucleon (N–N) and nucleus-nucleus (A–A) collisions if the A–A collision is a simple superposition of N–N interactions. Consequently, Φ is independent of the centrality of A–A collision in such a case. In other words, the strength of the correlation is not influenced by the number of uncorrelated particle sources and one can look for the 'dynamical' fluctuations among the events of very different multiplicity. Φ is also defined is such a way that it equals zero when the inter-particle correlations are entirely absent. Let us note here as well that from the dynamical point of view the Φ-measure provides basically the same information as the two-particle correlation function [16]. However, the information is filtered in a very specific way. The measure Φ has been successfully applied to the NA49 experimental data and it has been found [17, 18] that the dynamical transverse momentum correlations, which are present in N–N collisions, are significantly reduced in the central Pb–Pb reactions. The p_{T}-correlations observed in these collisions are fully explained by the effect of Bose statistics of pions [19, 21].

We are going to show in this note that Φ is also useful in the studies of azimuthal fluctuations. The Φ-measure analysis does not demand the reaction plane reconstruction and can be easily applied to the data. The measure is sensitive to various sources of correlations. We compute here the Φ-measure of fluctuations caused by the collective transverse flow and those caused by the quantum statistics and resonance decays in the hadron gas. The latter correlations are always present in heavy-ion collisions. The fact that the measure is sensitive to the very different correlations is advantage and disadvantage at the same time. It is difficult to disentangle different contributions but, as mentioned above, other methods are not free of the problem as well. The integrated information provided by Φ can be combined with that offered by other methods, in particular the Fourier expansion. Since all harmonics contribute to Φ one can check, for example, whether the first two Fourier coefficients, which are usually measured, saturate the observed value of Φ. The analysis of Φ-measure of ϕ-fluctuations can be also combined with the Φ-measure studies of other kinematical variables. Φ of $p_{\rm T}$-fluctuations has been already shown to be very sensitive to jets [22]. Thus, the simultaneous measurement of ϕ- and $p_{\rm T}$-fluctuations can be helpful in disentangling various contributions.

Let us introduce the correlation (or fluctuation) measure Φ. We define the variable $z \stackrel{\rm def}{=} x-\bar{x}$, where x is a single particle characteristics such as the particle transverse momentum or the azimuthal angle. The overline denotes averaging over a single particle inclusive distribution. In our further considerations, x is identified with the particle azimuthal angle. The event variable Z, which is a multiparticle analog of z, is defined as $Z \stackrel{\rm def}{=} \sum_{i=1}^{N}(x_i - \bar{x})$, where the summation runs over particles from a given event. By construction, $\langle Z \rangle = 0$, where $\langle ... \rangle$ represents averaging over events (collisions). The measure Φ is finally defined in the following way

$$\Phi \stackrel{\rm def}{=} \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2}}\,. \tag{1}$$

We first compute the Φ-measure of the azimthal fluctuations caused be the transverse collective flow. The inclusive $\phi-$distrubtion is, of course, flat *i.e.*

$$P_{\rm inc}(\phi) = \frac{1}{2\pi}\,\Theta(\phi)\,\Theta(2\pi - \phi)\;, \tag{2}$$

which gives $\bar{\phi} = \pi$ and $\overline{\phi^2} = \frac{4}{3}\pi^2$, and consequently, $\overline{z^2} = \frac{1}{3}\pi^2$. Following [10], the azimuthal distribution of a single event is chosen in the form of the

Fourier series *i.e.*

$$P_{\rm ev}(\phi) = \frac{1}{2\pi}\left[1 + \sum_{n=1}^{\infty}\left(X_n \cos(n\phi) + Y_n \sin(n\phi)\right)\right]\Theta(\phi)\,\Theta(2\pi - \phi)\,, \qquad (3)$$

where the parameters X_n and Y_n change from event to event. The distribution (3) is usually rewritten as

$$P_{\rm ev}(\phi) = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos(n(\phi - \psi_n))\right]\Theta(\phi)\,\Theta(2\pi - \phi)\;, \qquad (4)$$

where

$$X_n = 2v_n \cos(n\psi_n)\;, \qquad Y_n = 2v_n \sin(n\psi_n)\,.$$

The first term of the Fourier series from Eq. (3) or (4) corresponds to the so-called transverse directed flow. The amplitude v_1 controls strength of the flow while ψ_1 determines the reaction plane. The meaning of v_n and ψ_n for $n > 1$ is analogous.

The angles ψ_n vary from event to event and their distributions are flat. Therefore, the event distribution (4) averaged over events provides, as it should, the inclusive distribution (2). Further, we consider two extreme cases. In the first one, which seems to be appropriate for the hydrodynamic flow analysis, the ψ_n angles are maximally correlated to each other and uniquely determined by the reaction plane angle ψ_r *i.e.* $\psi_r = n\psi_n + \alpha_n$. Then, the averaging over events corresponds to the integration over the angle ψ_r. In the second case, the angles ψ_n are independent from each other and one integrates over all ψ_n to average over events.

Since $Z = \sum_{i=1}^{N}(\phi_i - \bar{\phi})$ one finds in the first case that

$$\begin{aligned}\langle Z^2\rangle \;=\; &\int_0^{2\pi}\frac{d\psi_r}{2\pi}\sum_N \mathcal{P}_N \int_0^{2\pi} d\phi_1 \ldots \int_0^{2\pi} d\phi_N\; P_{\rm ev}(\phi_1)\ldots P_{\rm ev}(\phi_N) \\ &\times(\phi_1 + \ldots + \phi_N - N\bar{\phi})^2\,,\end{aligned} \qquad (5)$$

where $\mathcal{P}_N$ is the multiplicity distribution. The formula analogous to (5), which corresponds to the second case, includes the averaging over all angles ψ_n. It is understood that there is one more averaging in Eq. (5) which is not explicitly shown. Namely, the averaging over the amplitudes v_n which also change from event to event. To simplify the notation we also neglect here a correlation between the event multiplicity and the flow strength. At first glance, the multi-particle distribution from Eq. (5) might look as a simple product of the one-particle distributions. One should note however that every $P_{\rm ev}(\phi)$ depends on the reaction plane angle ψ_r. Therefore, the integration over ψ_r leads to the *correlated* multi-particle distribution.

After elementary calculation, one finds from Eq. (5) that

$$\langle Z^2\rangle = \frac{\pi^2}{3}\langle N\rangle + (\langle N^2\rangle - \langle N\rangle)\, S \ ,$$

with

$$\langle N^k\rangle \stackrel{\rm def}{=} \sum_N N^k \mathcal{P}_N \ ,$$

and

$$S = 2 \left\{ \begin{array}{lll} \left\langle \sum_{n=1}^{\infty} (\frac{v_n}{n})^2 \right\rangle & \text{for} & 1-\text{st case,} \\ \left\langle \left(\sum_{n=1}^{\infty} \frac{v_n}{n}\right)^2 \right\rangle & \text{for} & 2-\text{nd case.} \end{array} \right.$$

Finally, we get

$$\Phi = \sqrt{\frac{\pi^2}{3} + \left(\frac{\langle N^2\rangle - \langle N\rangle}{\langle N\rangle}\right) S} - \frac{\pi}{\sqrt{3}} \ . \tag{6}$$

As expected, $\Phi = 0$ for $S = 0$. When $S \to 0$, $\langle N^2\rangle \cong \langle N\rangle^2$ and $\langle N\rangle \gg 1$, we get an approximate expression

$$\Phi \cong \frac{3}{2\pi^2}\, \langle N\rangle\, S \ . \tag{7}$$

Equations (6), (7) establish the relation between the Φ-measure and the Fourier coefficients v_n. One sees that, in principle, all v_n contribute to Φ. The measure also depends on how the angles ψ_n are actually correlated among each other. We have considered the two extreme cases of the dependence.

Let us now estimate the expected effect. The amplitudes v_1 and v_2 observed in Pb–Pb collision at 158 GeV per nucleon do not exceed for pions the value of 0.03 [6]. We take $v_1 = v_2 = 0.03$ and $v_n = 0$ for $n > 2$. We also neglect here the variation of v_1 and v_2. Then, one finds from Eq. (7) that for $\langle N\rangle = 170$ [6] Φ equals 0.058 in the first case and 0.105 in the second one.

As already mentioned, the transverse flow is far not the only source of the azimuthal fluctuations. In particular, the quantum correlations contribute here. We compute Φ in the ideal quantum gas to estimate the effect of quantum statistics. Modifying our previous calculations [19–21], one immediately finds

$$\frac{\langle Z^2\rangle}{\langle N\rangle} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3}\, (\phi - \bar\phi)^2\, \frac{\lambda^{-1}e^{\beta E}}{(\lambda^{-1}e^{\beta E} \pm 1)^2} \ , \tag{8}$$

where

$$\rho = \int \frac{d^3p}{(2\pi)^3} \, \frac{1}{\lambda^{-1} e^{\beta E} \pm 1} \; ; \tag{9}$$

$\beta \equiv T^{-1}$ is the inverse temperature; $\lambda \equiv e^{\beta\mu}$ denotes the fugacity and μ the chemical potential; E is the particle energy equal to $\sqrt{m^2 + \mathbf{p}^2}$ with m being the particle mass and $\mathbf{p}$ its momentum; the upper sign is for fermions while the lower one for bosons.

Since the inclusive azimuthal distribution is again given by (2), we get

$$\Phi = \frac{\pi}{\sqrt{3}} \left(\sqrt{\frac{\tilde{\rho}}{\rho}} - 1 \right) , \tag{10}$$

where

$$\tilde{\rho} = \int \frac{d^3p}{(2\pi)^3} \, \frac{\lambda^{-1} e^{\beta E}}{(\lambda^{-1} e^{\beta E} \pm 1)^2} \; . \tag{11}$$

As seen, Φ is an intensive thermodynamic quantity, *i.e.* it is independent of the system volume. It is also independent of the number of the particle internal degrees of freedom. One easily observes that $\Phi < 0$ for fermions, $\Phi > 0$ for bosons and $\Phi = 0$ in the classical limit ($\lambda^{-1} \gg 1$).

When the particles are massless and their chemical potential vanishes ($\lambda = 1$), Φ can be calculated analytically and the result reads

$$\Phi = \frac{\pi}{\sqrt{3}} \left(\sqrt{\frac{\pi^2}{6\zeta(3)} \begin{pmatrix} 2/3 \\ 1 \end{pmatrix}} - 1 \right) \cong \begin{pmatrix} -0.082 \\ 0.309 \end{pmatrix} ,$$

where $\zeta(x)$ is the Riemann zeta function with $\zeta(3) \cong 1.202$.

In Fig. 1 we present with dashed lines the Φ-measure of $\phi-$fluctuations in the ideal pion gas as a function of temperature. The pions are, of course, massive ($m_\pi = 140$ MeV). The calculations are performed for several values of the pion chemical potential. The chemical equilibrium corresponds to $\mu = 0$. As seen, Φ grows with the temperature. In the classical limit, when $\mu \to -\infty$, the Φ-measure vanishes.

It is a far going idealization to treat a fireball at freeze-out as an ideal gas of pions. A substantial fraction of the final state pions come from the hadron resonances. The role of resonances is twofold. On one hand, the pions, which originate form the resonance decays, do not 'feel' the Bose–Einstein statistics at freeze-out and consequently the finite value of Φ due to the quantum effects is significantly reduced. On the other hand, there are extra correlations among pions from the resonances caused by the decay kinematics. In particular, the two-body decays produce the strong correlation. Below, we estimate the role of resonances as in our earlier papers [20, 21], *i.e.* we

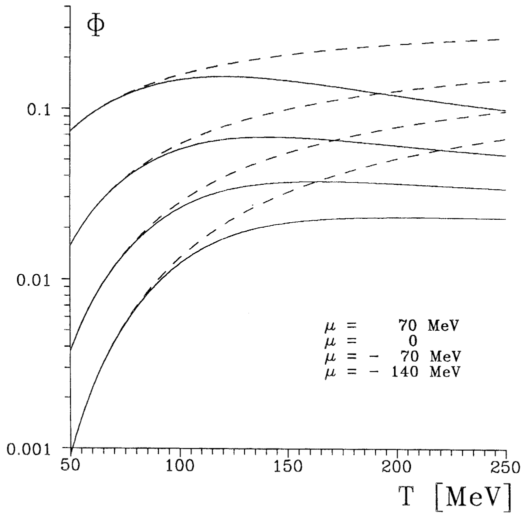

Fig. 1. Φ−measure of ϕ−fluctuations in the hadron gas as a function of temperature for four values of the chemical potential. The resonances are either neglected (dashed lines) or taken into account (solid lines). The most upper dashed and solid lines correspond to $\mu = 70$ MeV, the lower ones to $\mu = 0$, *etc.*

take into account only the first effect. The correlations due to the decay kinematics will be discussed elsewhere.

The spectrum of pions, which originate from the resonance decays, is not dramatically different than that given by the equilibrium distribution [23]. Therefore, we treat the fireball at freeze-out as a mixture of 'quantum' pions — those called 'direct' — and the 'classical' ones which come from the resonance decays. The Φ−measure is again given by Eq. (10) but the formulas (9), (11) are modified as

$$\rho = \int \frac{d^3p}{(2\pi)^3} \left[\frac{1}{\lambda^{-1} e^{\beta E} - 1} + \lambda_r e^{-\beta E} \right] ,$$

$$\widetilde{\rho} = \int \frac{d^3p}{(2\pi)^3} \left[\frac{\lambda^{-1} e^{\beta E}}{(\lambda^{-1} e^{\beta E} - 1)^2} + \lambda_r e^{-\beta E} \right] .$$

The parameter λ_r is chosen is such a way that the number of 'classical' pions equals the number of pions from the resonance decays. Thus, λ_r is temperature dependent. In the actual calculations we have taken into account the lightest resonances: $\rho(770)$ and $\omega(782)$ which give the dominant contribution. The life time of ρ, which is 1.3 fm$/c$, is not much longer than the time of the fireball decoupling and some pions from the ρ decays can still 'feel' the effect of Bose statitistics. Therefore, the contribution of ρ to the 'classical' pions is presumably overestimated in our calculations. Since we neglect the heavier resonances and weakly decaying particles, which also contribute to the final state pions, the two effects partially compensate each other. In any case, our calculations show that the resonances do not change the value of Φ dramatically in the domain of temperatures of interest.

In Figs. 1 the solid lines represent $\Phi-$measure which includes the resonances. The chemical potentials of ρ and ω are assumed to be equal to that of pions. As seen, the role of the resonances is negligible at the temperatures below 100 MeV but above this temperature the resonances reduce the fluctuations noticeably. The freeze-out temperature in Pb–Pb collisions at 158 GeV per nucleon, which is obtained by means of the simultaneous analysis of the single particle spectra and the two-particle correlations, is about 120 MeV [24]. For $T = 120$ MeV and $\mu = 0$ the $\Phi-$measure equals 0.078, when the resonances are neglected, and is reduced to 0.066 when the resonances are taken into account. One observes that the effects of the quantum statistics and transverse flow are of comparable size.

The $\Phi-$measure given by Eq. (1) corresponds to the second moment of the fluctuating quantity. It has been suggested [25] to use the higher moments in an analogous way. However, we have shown [21] that only the third moment measure preserves the advantageous properties of Φ while the higher moment measures do not. We have also argued [21] that the simultaneous usage of Φ_2 and Φ_3 may help in identifying the origin of correlations observed in the final state of heavy-ion collisions. Unfortunately, the third moment measure is useless in the studies of $\phi-$fluctuations. One easily shows that due to the symmetry $\overline{z^3} = 0$ and $\langle Z^3 \rangle = 0$ when the variable x is identified with the azimuthal angle.

We conclude our considerations as follows. The $\Phi-$measure, which can be easily applied to the experimental data, seems to be a useful tool to analyze the azimuthal fluctuations in heavy-ion collisions. It is sensitive to the different nontrivial fluctuations, in particular those caused by the transverse flow and quantum statistics which have been quantitatively discussed here. The $\Phi-$measure analysis combined with other techniques, such as the Fourier expansion method [10–13], will help to identify various sources of the fluctuations.

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