VACUUM ENERGY OF A CAVITY WITH A MOVING BOUNDARY

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The problem of the quantum vacuum in a one-dimensional cavity is discussed. We put forward a new method to solve Moore's equation and search for fundamental classical basis. This new method is applicable to problems with general wall trajectories and enables us to calculate both phase functions and their derivatives by iterations. We calculate energy densities and total energies for oscillating cavities. In particular, off resonant motions are studied.

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1. Introduction

More than fifty years ago Casimir [1] indicated that the presence of metallic plates changes the structure of the ground state of the electromagnetic field existed between those plates. In the 70's [2, 3], many authors began to investigate the non-stationary modifications of the electromagnetic vacuum between moving boundaries [4] (for more extended list of papers see [5]).

Non-stationary boundary conditions can result in creation of real photons. This effect is known in literature as "dynamical" Casimir effect, being in fact some generalization of the Unruh effect. The physical interpretation of production of particles, or in other words the meaning of quantum radiation from the vacuum state due to the perturbation induced by moving boundaries, is however a difficult problem. The time-dependence of boundary conditions brings about the non-existence of the Hamiltonian and the Schrödinger picture. Together with the difficulties with the existence of many unitary inequivalent representations of the quantum field theory, it makes the formal definition of 'particles' hard to introduce. To help this, one usually assumes that the motion of the wall occurs only during some finite time interval.

This paper comes back to the case of one-dimensional cavity. Fulling and Davies [3] proved that photons are generated for a single non-uniformly accelerated mirror, but the number of generated photons is very small unless we change the wall velocity rapidly. On the other hand, the rapid change of the wall velocity obscures physical interpretation. In spite of this, it was found [4, 6–9] that it were possible to enhance the energy and the number of created photons to an observable amount. This is the case of vibrating cavities, especially for resonant cases with the period of oscillations of some eigenfrequencies of the unperturbed cavity modes. Small numbers of photons generated in successive periods can constructively interfere together.

The attention of many authors was to strictly resonant cases, when a frequency of oscillation is equal to a cavity eigenfrequency $\omega_m = m\omega_0$, where $\omega_0 = \frac{\pi c}{L_0}$, $m = 1, 2, 3, \ldots$, and L_0 is the initial distance between the boundaries before cavity oscillations start. Usually, the first point to carry out the quantization is to start with the symplectic space of fundamental solutions, which will be mapped onto the space of linear operators acting in the Hilbert space of physical states. Moore [2] constructed a convenient basis of the space of solutions. The basis is spanned by the analogue of standing waves. Time-dependent boundary conditions cause that the universal (*i.e.* the same for all modes) phase function of generalized standing waves is now subject to some non-local equation. Some earlier papers studied approximated solutions to this equation (called usually Moore's equation) either in case of short times $\varepsilon\omega_0 t \ll 1$ (εL_0 is the amplitude of oscillations) [10] or for a long time domain [4]. The first exact analytical solution to the resonant case was given in [7]. A general method to derive solutions for a broad class of wall trajectories was presented in [11]. In fact, for practical purposes this method works only numerically and does not provide equally helpful recipes for calculating the derivatives of the solution for the phase function. Therefore, some methods based on the renormalization-group idea [12, 13] were invented to improve the perturbation approaches to solve the Moore's equation. Only few papers discussed off resonant cases [5], where a wall oscillates with frequency close to resonant one.

The paper is organized as follows. In Section 2 we will recall Moore's equation and present general methods of evaluating its solutions for any prescribed wall motion. Section 3 gives the calculation of the energy density for an arbitrary motion. Section 4 is devoted to the solution of Moore's equation for off resonant motions. Energy densities and total energies for off resonant cases are discussed in Section 5.

2. Fundamental solutions and Moore's equation

We consider a one-dimensional cavity formed by two walls. For the sake of simplicity, the left wall is fixed at position $x = 0$, while the right one is moving with a given trajectory $q(t)$. In the past, say for times $t \leq 0$, the right wall is assumed to be static, its initial position is L_0. As usual, we have to restrict ourselves to trajectories for which a speed of the wall is not close to the speed of light. Therefore, the allowed trajectories of the moving wall are specified by the following requirements:

$$\begin{aligned} &1. \quad q(t) = L_0 \quad \text{for} \quad t \leq 0\,, \\ &2. \quad |\dot{q}| \leq \text{const} < 1\,, \\ &3. \quad q(t) > 0\,. \end{aligned} \tag{1}$$

The electrodynamics on a line simplifies to the problem of the single scalar field (being the only component of the vector potential in the Coulomb gauge) with the corresponding field equation:

$$(-\partial_t^2 + \partial_x^2)A(x,t) = 0\,. \tag{2}$$

The walls are considered as perfectly reflecting metallic plates. It corresponds to boundary conditions for the electromagnetic field inside the cavity,

$$A(x=0,t) = A(x=q(t),t) = 0 \qquad \text{for all times.} \tag{3}$$

Time-dependent Dirichlet boundary conditions implements the continuity of the electric field in the Lorentz frames in which the walls are instantaneously at rest.

Following the quantization rules given in [2,3], the field is represented in terms of creation $\hat{a}^\dagger_{k0}$ and annihilation $\hat{a}_{k0}$ operators for photons in the form (we adopt Schrödinger picture):

$$\hat{A}(x,t) = \sum_{k=1}^{\infty} \hat{a}_{k0} A_k(x,t) + \hat{a}^\dagger_{k0} A^*_k(x,t)\,. \tag{4}$$

For the case discussed here, Moore [2] found the complete set of mode functions $A_k(x,t)$, being classical solutions of the field equation, in the following general form with some auxiliary phase function $R(z)$:

$$A_k(x,t) = \mathrm{e}^{ik\pi R(t+x)} - \mathrm{e}^{ikR(t-x)}\,. \tag{5}$$

The boundary condition on the right wall is fulfilled provided that the phase function R satisfies the equation:

$$R(t+q(t)) = R(t-q(t)) + 2\,. \tag{6}$$

To determine phase function $R(z)$ for an arbitrary motion of the wall $q(t)$, Cole and Schieve [11] proposed the following iteration procedure. The crucial point of this method is that Eq. (6) allows us to relate $R(z)$ at later points of time with those at earlier ones.

Let us calculate the value of $R(z)$ at some given z. We can always put $z = t_1 + q(t_1)$, and find t_1 as a time for which a null line drawn backward from z intersects the moving wall. We have then $R(z) = R(t_1 + q(t_1))$. When we use the boundary condition (6) for t_1 we obtain,

$$R(z) = R(t_1 - q(t_1)) + 2\,. \tag{7}$$

Further, we consider the argument value $z' \equiv t_1 - q(t_1)$ and find the time t_2, for which a null line drawn backward from z' intersects the moving wall: $t_2 + q(t_2) = z'$. If we use again the boundary condition (6) and set $z'' \equiv t_2 - q(t_2)$, we obtain,

$$R(z) = R(z') + 2 = R(z'') + 4\,. \tag{8}$$

The procedure can be continued until we reach the initial static region where the function $R(z)$ is known. For times $t \leq 0$ or equivalently for arguments $z \leq L_0$, the solution of equation (6) is just $R(z) = z/L_0$.

Following the construction, the value of $R(z)$ increases by 2 every time the null line intersects the trajectory of the wall. Therefore, we obtain,

$$R(z) = 2n + t_{\mathrm{stat}}/L_0\,, \tag{9}$$

where n is the number of reflections from the moving wall and t_{stat} is the time when the last null line intersects the time axis in the static region $t \leq L_0$. Cole and Schieve [11] evaluated t_{stat} in terms of the wall positions at the reflection points,

$$t_{\mathrm{stat}} = z - 2\sum_{i=1}^{n} q(t_i). \tag{10}$$

Finally, for $R(z)$ we obtain formula,

$$R(z) = 2n + \frac{z - 2\sum_{i=1}^{n} q(t_i)}{L_0}\,. \tag{11}$$

The above described method of solution of Moore's equation works for an arbitrary trajectory of the wall. However, to find points of intersection t_i for a specific wall motion, we should use numerical procedures. Moreover, we cannot handle calculations of derivatives of $R(z)$ here.

Now, we present another method of solution of Moore's equation. This method do not refer to auxiliary reflection points and enables us to find derivatives of R.

At the beginning, let us define an auxiliary function $f(z)$, which obeys the equation:

$$f(t+q(t)) = t - q(t). \tag{12}$$

This function exhibits the following properties:

$$\begin{aligned} &1. \quad f(z) = z - 2L_0 \qquad \text{for} \;\; z \leq 0\,, \\ &2. \quad 0 < \mathrm{const}_1 \leq f'(z) \leq \mathrm{const}_2 < \infty\,, \\ &3. \quad f(z) < z\,. \end{aligned} \tag{13}$$

It can be easily verified that the relation (12) establishes a one-to-one correspondence between the allowed trajectories (1) and the functions defined by properties (13). To specify the motion of the moving wall, we can either describe directly its trajectory $q(t)$ or give some function $f(z)$. The latter way omits the retardation problem and simplifies the calculation of physical observables. If we specify the function $f(z)$, the trajectory $q(t)$ can be given immediately in a parametric form:

$$\begin{cases} t = \frac{z+f(z)}{2}\,, \\ q = \frac{z-f(z)}{2}\,. \end{cases} \tag{14}$$

Let us also note that periodic trajectories, $q(t+T) = q(t)$ for all $t \geq 0$, correspond to functions f satisfying $f(z+T) = f(z) + T$ for all $z \geq L_0$.

Using the function $f(z)$, we can find the corresponding solution $R(z)$ and evaluate its derivatives. Note that Moore's equation (6) can be now rewritten as:

$$R(z) - R(f(z)) = 2\,. \tag{15}$$

Obviously, for all $z \leq L_0$ we have always $R(z) = z/L_0$. We define a sequence of points L_n by:

$$L_n = (f^{-1})^n(L_0)\,, \tag{16}$$

where $(f^{-1})^n \equiv f^{-1} \circ f^{-1} \circ ... \circ f^{-1}$ denotes the action of n times with the inverse function. The solution of Moore's equation is unique and given by:

$$R(z) = 2n + \frac{f^n(z)}{L_0} \qquad \text{for} \;\; z \in [L_{n-1}, L_n]\,, \tag{17}$$

where again we denote the multi-composition as $f^n \equiv f \circ ... \circ f$.

It is straightforward to evaluate R and its derivatives at points L_n,

$$R(L_n) = 2n+1\,, \qquad R'(L_n) = \frac{f'(L_1)...f'(L_n)}{L_0}\,. \tag{18}$$

3. The energy density of the field inside the vibrating cavity

The energy density inside the oscillating cavity was computed first in the paper [3]. Let us follow these results and begin with:

$$\langle T_{00}(x,t)\rangle = \frac{1}{2}\left\{\left\langle\left(\frac{\partial \hat{A}(x,t)}{\partial x}\right)^2\right\rangle + \left\langle\left(\frac{\partial \hat{A}(x,t)}{\partial t}\right)^2\right\rangle\right\}, \tag{19}$$

where the expectation values are taken with respect to the initial vacuum state. Of course, this expression is divergent. Finite results are usually obtained through time splitting regularization procedure. To avoid evaluating a product of a mode function and its conjugate at the same space-time point (t,x), the latter factor is evaluated at $(t+\varepsilon,x)$, where ε is some infinitesimal quantity $\varepsilon \to 0^+$. After that, we obtain $T_{00}(x,t)$ in terms of phase function R,

$$\begin{aligned}\langle T_{00}(u,v)\rangle \;=\; & \frac{\pi}{4}\sum_{n=1}^{\infty} n\Big\{R'(v)R'(v+\varepsilon)\mathrm{e}^{in\pi[R(v+\varepsilon)-R(v)]} \\ & +R'(u)R'(u+\varepsilon)\mathrm{e}^{in\pi[R(u+\varepsilon)-R(u)]}\Big\}.\end{aligned} \tag{20}$$

introduced light-cone coordinates $u=t+x, v=t-x$. Direct calculations lead to the expression for $\langle T_{00}(x,t)\rangle$ with an explicit quadratic divergence, which is independent of the particular physical state parameters:

$$\langle T_{00}(x,t)\rangle = -\frac{1}{2\pi\varepsilon^2} - [T(u)+T(v)]\,, \tag{21}$$

where

$$T(z) = \frac{1}{24\pi}\left[\frac{R'''(z)}{R'(z)} - \frac{3}{2}\left(\frac{R''(z)}{R'(z)}\right)^2 + \frac{1}{2}\pi^2(R'(z))^2\right]. \tag{22}$$

The final expression for the energy density average in the initial vacuum state is just obtained after the quadratic divergence is removed,

$$\langle T_{00}(x,t)\rangle_{\mathrm{ren}} = -T(u) - T(v)\,. \tag{23}$$

Because of the presence of second and third derivatives in the function $T(z)$, any discontinuities of the wall velocity or its acceleration during the motion will generate δ-function peaks in the energy density. These peaks will propagate through the cavity with the speed of light, they are reflected back and forth between mirrors. Usually, these peaks are ignored and one focuses on the energy density with delta peaks extracted.

4. Behavior of the phase function $R(z)$

The basic period in the problem of the cavity defined in Section 2 is $T = 2L_0$ (not L_0 as in usual resonant problems). In this paper, we will investigate systems in which the prescribed periodic wall motion is given by:

$$L(t) = L_0\left[1 + d\sin\left((m+\varepsilon)\frac{\pi}{L_0}t\right)\right] . \tag{24}$$

We assume m to be a positive integer (the order of the resonance) and ε is a perturbation of the resonant frequency (called here detuning).

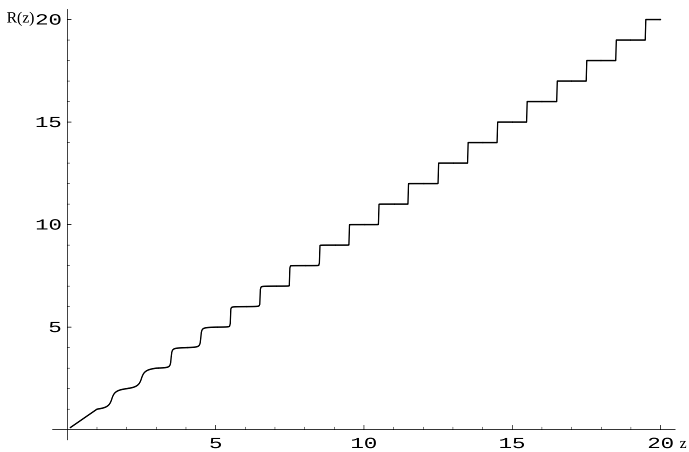

Fig. 1. Function $R(z)$ for parameters: $L_0 = 1$, $d = 0.1$, $m = 1$, $\varepsilon = 0$.

At the beginning, let us start with the case of resonant motion (see Fig. 1). The phase function $R(z)$ for long times $t \to \infty$ looks like staircase with almost perfectly flat steps and very sharp jumps between the steps. A careful analysis of the behavior of the phase function $R(z)$ for many different resonant motion is elaborated in [11]. The authors proved a nice theorem that $R(z)$ becomes staircase not only for strict periodic resonant motions. There must exist only a finite set of points $\{\tau_i\}$ in the interval $[0, 2L_0)$ (one can call them 'return points'), such that $q(\tau_i + 2mL_0) = L_0$ for all $m \geq 0$. Then $R(z)$ will have a perfect staircase-shape with steps at $z = \tau_i + 2mL_0$, respectively for each return point τ_i. The proof of the theorem is given in the appendix of the paper [11]. Let us note, that with the help of our method (17) we can prove this theorem immediately. The assumptions about the periodicity and about the existence of return points lead to the the following function $f(z)$,

$$f(z) = (2n-1)L_0 + g(\xi) \; ; \; z = (2n+1)L_0 + \xi \; , \; g : [0, 2L_0) \to [0, 2L_0) \; . \tag{25}$$

The function $g(z)$ is growing and has fixed points at the return points. It is now obvious that n-fold iteration of the function g develops staircase for $R(z)$, and steps are located at the shifted return points.

For cases where the wall oscillates with off-resonant frequencies, respective phase functions $R(z)$ will not develop staircases in general. However, for sufficiently small perturbation to the resonant frequency, there is still a kind of staircase structure in $R(z)$. Large enough perturbations of the frequency lead to approximately linear phase functions $R(z)$.

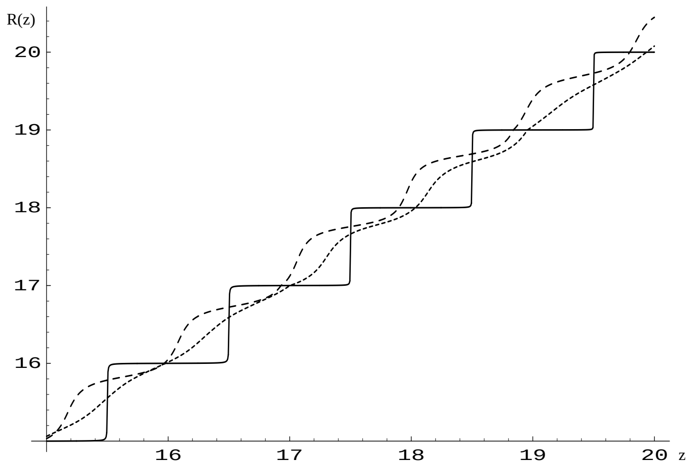

Fig. 2. $R(z)$ dependence for parameters $L_0 = 1$, $d = 0.05$ and for frequencies with detuning $\varepsilon = 0, 0.2, 0.4$.

Among many periodic solutions elaborated in the previous papers, long-time patterns of behavior for non-resonant wall motions were not analyzed. Using our general solution formula (17), we can describe this case as well. For simplicity, let us consider strictly periodic wall motion with the period T. The period T is arbitrary, in particular it may be a non-resonant one. Define the characteristic intervals (16). Suppose now, there exist positive integers M and N such that:

$$L_N = L_0 + MT\,. \tag{26}$$

Then, it follows immediately that: $L_{N+k} = L_k + MT$. Further, the phase function after the 'long period' MT is just reproduced but shifted. All derivatives of $R(z)$ are periodic with respect to the long period MT.

Periodic off-resonant motions develop staircase for the phase function $R(z)$ for small arguments. Later, approximately linear shape will appear (see Fig. 9 in the Appendix). But if the relation (26) holds either exactly or approximately, after the long time MT the phase function will reproduce its exact or approximate initial shape respectively. In the Appendix, we present also figures with long-time oscillations of the total energy and with the dependence of long-time period MT on the detuning ε for off resonant wall motions.

It is clear that the vacuum energy density will develop sharp peaks at points (x,t), where function $R(z)$ suddenly changes. Traveling peaks in the energy density were described in [7]. The total energy for resonant cases grows exponentially. The energy enhancement will be present also for motions close to resonant ones. However, there exists then a long-time oscillation of the total energy (see figures included in the Appendix).

5. Vacuum energy inside the cavity

In this Section, we discuss behavior of the energy density and the total energy inside the vibrating cavity. Let us call the following part of expression (23) as a 'Casimir part',

$$\langle T_{00}\rangle_{\rm Casimir} = -\frac{\pi}{48}\left((R'(u))^2 + (R'(v))^2\right) . \tag{27}$$

This term for static boundaries is responsible for well-known static Casimir force [1], associated with the energy density value (note again that for the static case $R(z) = z/L_0$):

$$\langle T_{00}(x,t)\rangle_{\rm static} = -\frac{\pi}{24L_0^2} . \tag{28}$$

The other part of the expression (23) will be called an 'Unruh part':

$$\langle T_{00}\rangle_{\rm Unruh} = -\frac{1}{24\pi}\left[\frac{R'''(u)}{R'(u)} - \frac{3}{2}\left(\frac{R''(u)}{R'(u)}\right)^2 + (u \longleftrightarrow v)\right] . \tag{29}$$

This term is responsible for quantum radiation from a vacuum state induced by a single moving conducting wall (called usually Unruh–Davies effect).

For resonant or close to resonant cases (small ε), the Unruh part of the energy is significant. It may be hundreds of times greater than the static part, so it may change an overall sign of the total energy. For strong off resonant cases (large ε), the Unruh part is typically only a small correction to the Casimir part.

Now we will study the shape of the energy density inside the cavity. It is very simple to deduce from that, what kind of motion we are dealing with. When the wall oscillates with a strong off-resonant frequency, the energy density fluctuates around the flat static Casimir value. There are well developed peaks in the energy density, if the wall vibrates with a resonant or almost resonant frequency.

The plot above presents behavior of the energy density for times between $80L_0$ and $81L_0$. From now on, energy density values are scaled by the static

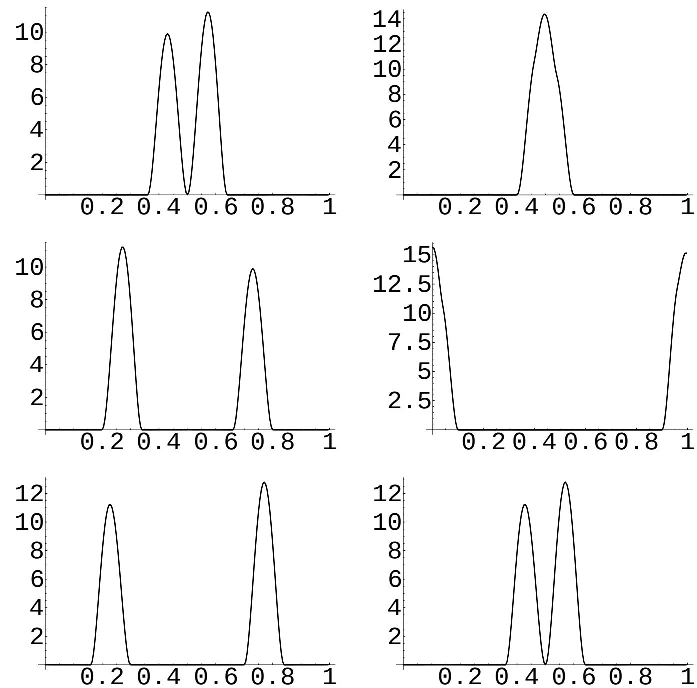

Fig. 3. Behavior of the energy density during one period. Assigned parameters of the motion are $d = 0.01$, $m = 2$, $L_0 = \frac{\pi}{10}$, $\varepsilon = 0$.

energy density $W_0 = \frac{\pi}{24L_0^2}$. We start the analysis of the energy density from the top left plot, *i.e.* at $t = 80L_0$. The right peak is traveling to the to the static wall on the left, and the left one to the right in the direction of moving wall. Then, they meet each other and there is an local enhancement in the density. The peaks pass each other and move on. After bouncing from the static wall, a peak preserves its height. It is not surprising as there is no energy change for reflections from the static wall, peaks only change their directions of movement. While reflected from the oscillating wall, they can gain or lose energy. For strictly resonant cases, the energy is always pumped into the system between the walls. When the boundary oscillates with off-resonant frequency the energy of the cavity oscillates around the static value $-\frac{\pi}{24L_0^2}$, represented by the solid line in Fig. 4. After integrating over the whole cavity, there will be no change of the total energy due to the motion of the wall.

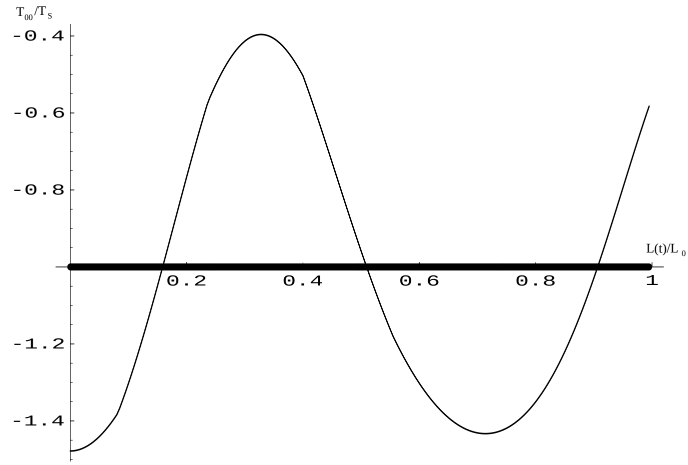

Fig. 4. Energy density for off-resonant wall motion. $\varepsilon = 0.5$. Other parameters as in Fig 3.

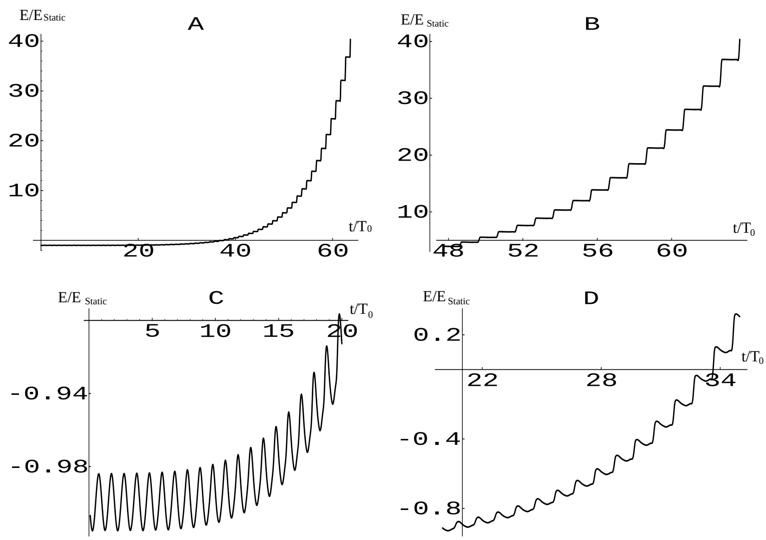

Fig. 5. Time dependence of the total energy accumulated inside the cavity. The parameters are $d = 0.01$, $m = 2$, $L_0 = \pi/10$, $\varepsilon = 0$.

Now, let us trace time dependence of the total energy accumulated inside the cavity. For resonant cases, we can observe generally an exponential growth of energy with time.

Figure 5A shows the overall shape of the energy-time dependence. Figures B,C,D are the magnified parts of A. The total energy in the plots is scaled by the static value of the Casimir energy $E_{\mathrm{C}} = \frac{\pi}{24L_0}$. As it is seen, general behavior of the energy for resonant motions is exponential, but there exist flat regions between following increases. Their explanation come trivially from the picture of traveling peaks in the energy density. The question is why each following increase is greater than previous one. The explanation is quite simple: expression for $\langle T_{00}\rangle$ and $\langle T_{11}\rangle$ parts of the energy-momentum tensor are the same. $\langle T_{11}\rangle$ describes the force from the field acting on the oscillating wall. The force of interaction with the boundary is increasing with each period. To hold our moving wall to oscillate in a prescribed way, we have to apply to it an enhanced force. The greater force we are acting, the greater energy transfer to the slab is pumped.

The second plot of Fig. 5 shows energy versus time dependence for a long-time domain between $47.5L_0$ and $63L_0$. The structure of the graph consists of well developed steps. The lower plot shows energy behavior for short times ($t < 20T_0$). At the beginning of motion, the energy oscillates around the static energy, quite similar as for off-resonant cases (compare to Fig 6). After several periods the field excitations interfere constructively and the falls in the graph are flattened.

We would like to describe further what we called 'Casimir' and 'Unruh' parts in the energy. The above plots show that resonant and off-resonant cases are dramatically different. For the non-resonant case, the Unruh part (top left graph)is negligible while the 'Casimir' part oscillates around the static value. Thus, there is no energy enhancement and the number of radiated photons will be very small. Quite different picture is for resonant and close to resonant wall motions. We have already learned that the Unruh part dominates and its plot is staircase, but we have also a significant change in the Casimir part.

Let us now look into the dependence of the total energy on the frequency of wall oscillations. The energy increases not only for strictly resonant cases, but for also for cases close to resonant ones.

As we can see in the picture for time $20L_0$ for both frequencies allowed detuning is about 0.1 whereas for time $60L_0$ it is only about 0.03. It is not very surprising, if we think about photons created in the cavity in each cycle. Every period there are created photons with frequency of creation $\omega_c = m\frac{2\pi}{L(t)}$. For resonant motion, photons from one cycle can constructively interfere with photons from the following cycle, because they have the same phase, and there will be some energy increase. But for slightly off-resonant oscillations photons originating from the previous period are a little bit too early or too late in the next cycle to interfere exactly. They have a little bit different phase. After sufficient long time phase shift will be big enough, and destructive interference will destroy any energy increase.

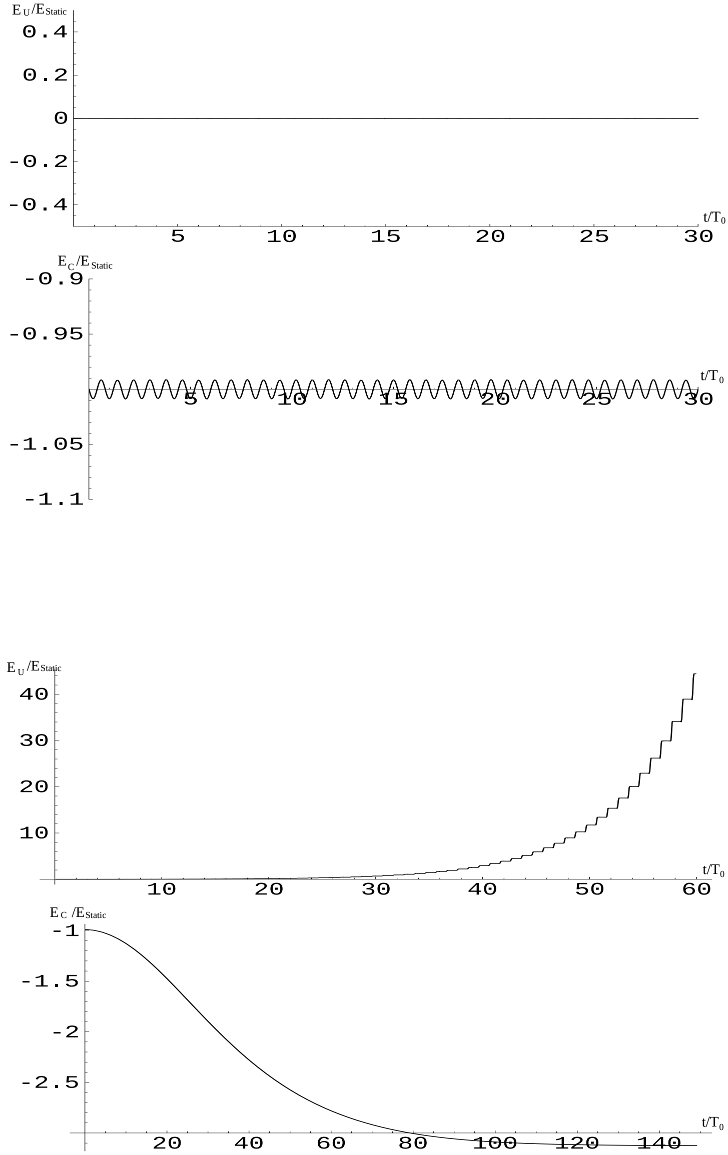

Fig. 6. Time dependence of 'Casimir' and 'Unruh' parts of the energy. The parameters of motion are: for the left off-resonant case $\varepsilon = 0.5$, for the right resonant one $\varepsilon = 0$, other parameters as those for Fig. 3.

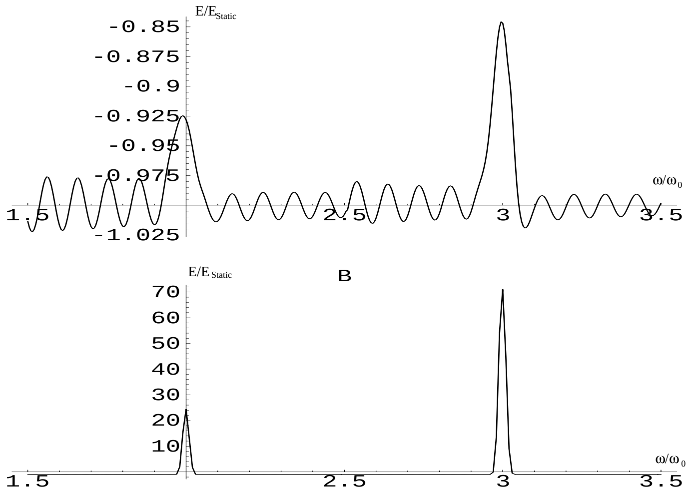

Fig. 7. Frequency dependence of the energy for times $t = 20L_0$ and $60L_0$, the parameters are $d = 0.01$, $L_0 = \pi/10$, the energy is scaled by $\pi/24L_0$.

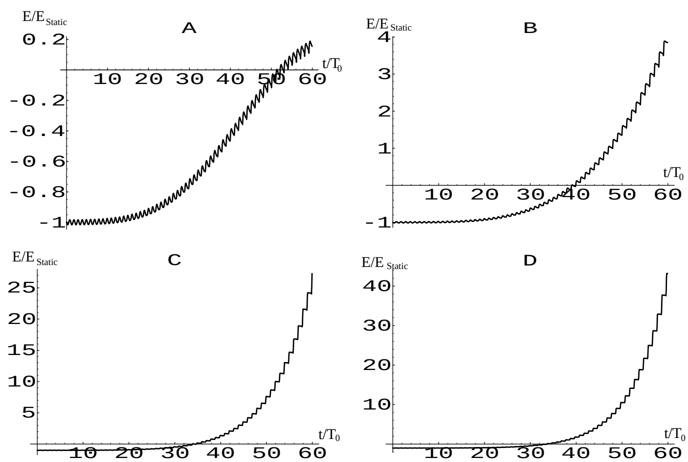

Fig. 8. Energy versus time dependence for different frequencies. Parameters are $L_0 = \frac{\pi}{10}$, $\delta = 0.01$, and frequencies are: A — $\varepsilon = 0.025$, B — $\varepsilon = 0.02$, C — $\varepsilon = 0.01$, D — $\varepsilon = 0$.

For short times, Fig. 8 compares the energy initial growth for different frequencies, less or more detuned with respect to a resonant value. The long-time behavior is presented in the Appendix.

Appendix

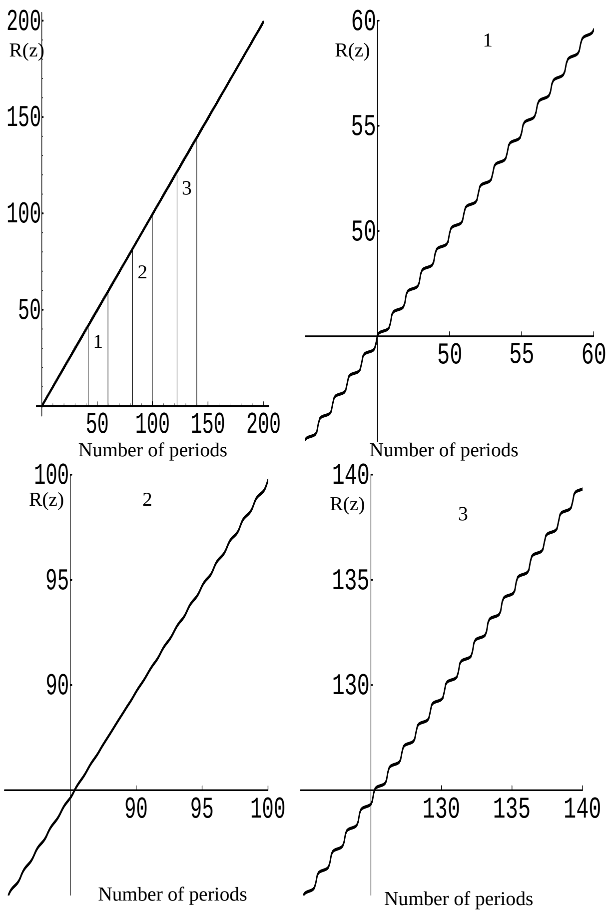

Fig. 9. The phase function for one off-resonant case. The plots 1, 2 and 3 are the magnified part of the top-left plot. We observe staircase and almost linear structures interchangeably.

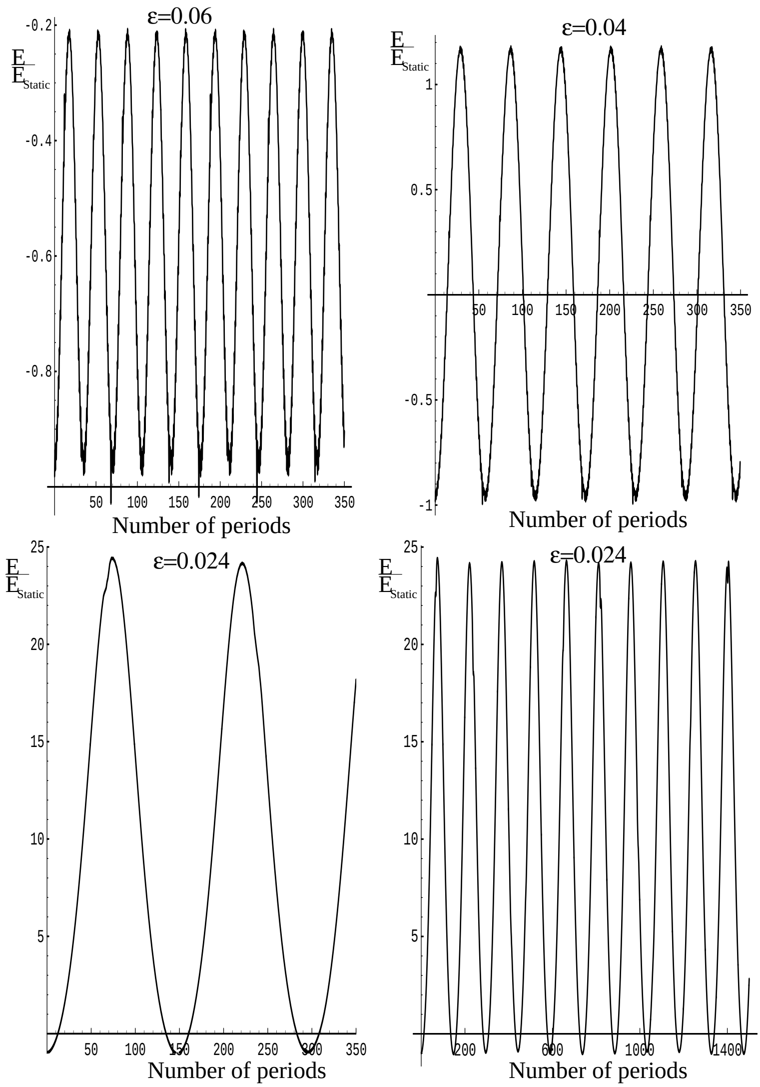

Fig. 10. The total energy for off-resonant cases. The long-time periods and staircase in each period can be easily observed.

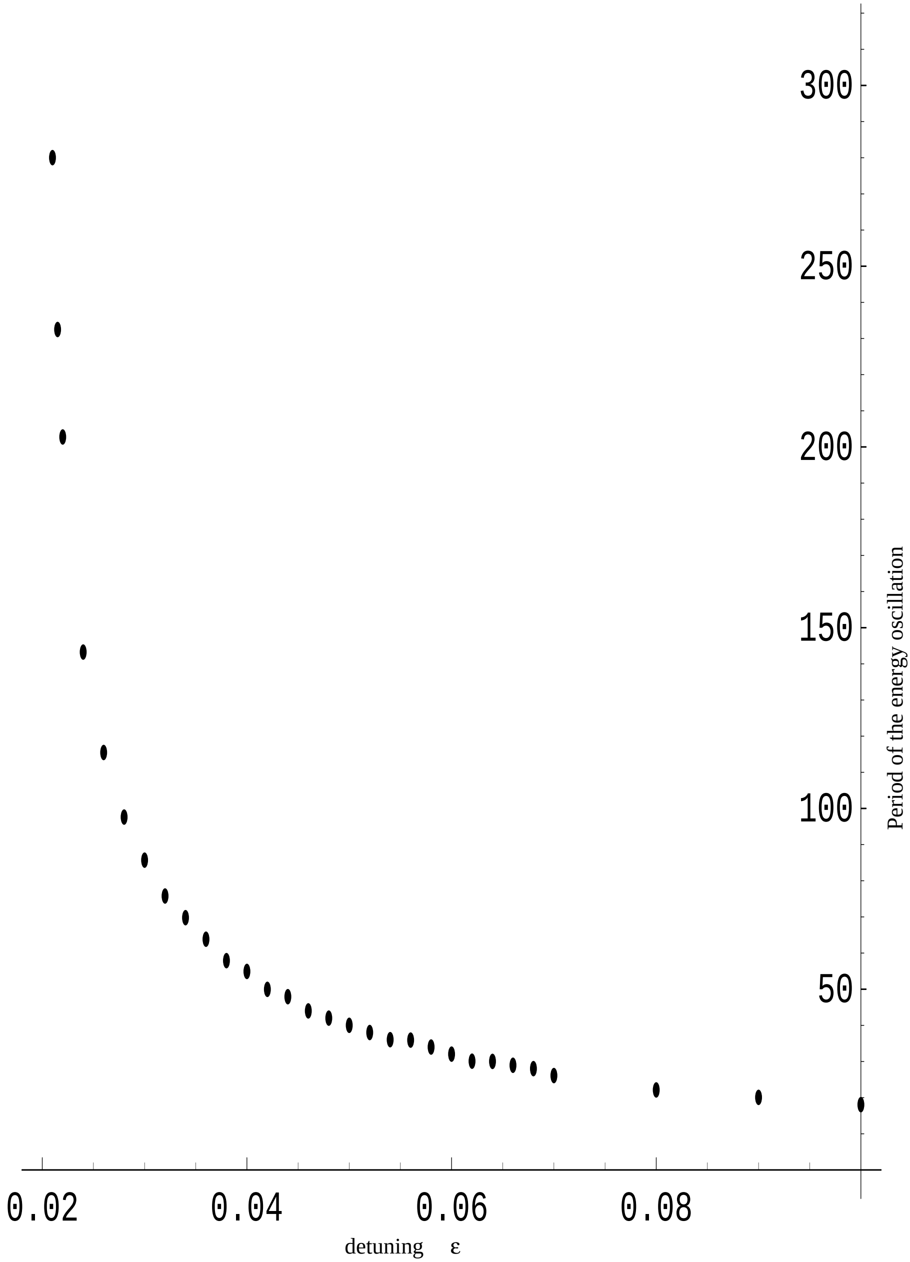

Fig. 11. The dependence of the long-time period MT on the detuning parameter ε.

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