

STUDY OF ONE-QUASIPROTON BANDS OF ^{129}La USING THE PROJECTED SHELL MODEL*

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The projected shell model is applied to the nucleus ^{129}La . The results of theoretical calculations about the one-quasiproton bands are compared with experimental data, the agreement with the yrast $\pi h_{11/2}$ band and $\pi g_{7/2}$ band is satisfactory. We also assign the $\pi g_{7/2} \otimes [\nu h_{11/2}]^2$ configuration with an oblate shape for one of bands in ^{129}La .

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1. Introduction

High spin states in the odd proton nucleus ^{129}La have been investigated using the techniques of in-beam γ -ray spectroscopy by He *et al.* at Daresbury Laboratory in 1992 [1]. The excited states in ^{129}La were populated using the inverse reaction $^{51}\text{V}(^{82}\text{Se}, 4n)^{129}\text{La}$ at 290 MeV. Nine rotational bands have been observed in ^{129}La , seven of them for the first time. The data are mainly discussed within the framework of the Cranked Shell Model (CSM) [1]. The rotational bands are assigned quasiparticle configurations originating from $\pi h_{11/2}$, $\pi g_{7/2}$, $\pi [h_{11/2}]^2 \otimes \pi g_{7/2}$ and $\pi h_{11/2} \otimes \nu [h_{11/2} g_{7/2}]$ states, respectively.

The CSM calculations [1] for ^{129}La were performed using the following parameters: quadrupole deformation $\varepsilon_2 = 0.22$ and hexadecapole deformation $\varepsilon_4 = 0.00$; triaxiality $\gamma = 0^\circ$, *i.e.* prolate shape; pairing gap parameters $\Delta_p = 1.30$ MeV for protons and $\Delta_n = 1.06$ MeV for neutrons. The deformation parameters were taken from the Total Routhian Surfaces (TRS) calculations [2] which show that the deformation of ^{129}La is dominated by a near prolate shape ($\gamma \cong 0^\circ$) with $\varepsilon_2 \cong 0.20$ – 0.22 and a very small ε_4

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for $0 < I \lesssim 30\hbar$. The pairing gap parameters have been estimated from the odd–even mass difference. In addition, the κ and μ parameters in the Nilsson potential have been taken from [3].

Low-spin states of the nucleus ^{129}La have been investigated by means of in-beam γ -ray spectroscopy by Kühn *et al.* in 1995 [4]. They used the reaction $^{119}\text{Sn}(^{14}\text{N}, 4n\gamma)^{129}\text{La}$ to populate the excited states in ^{129}La . Three new negative-parity bands have been observed for the first time which are identified as the unfavoured yrast, the favoured and unfavoured yrare bands on a $\pi h_{11/2}$ quasiparticle configuration, respectively. These three bands and the favoured yrast band are discussed within the framework of the Rigid Triaxial Rotor plus Particle model (RTRP) considering excitation energies, branching and multipole mixing ratios, a value of $\beta = 0.25$ and $\gamma \approx 16^\circ$ for the triaxial deformation parameter for ^{129}La has been established. There are some discrepancies between the experiment data and the model predictions using these parameters, particularly at higher excitation energies, there are no other theoretical discussions toward other positive parity bands.

Low-lying levels of ^{129}La isotope have also been investigated through the β^+/EC decay of ^{129}Ce by Gizon *et al.* in 1997 [5], the positive parity levels of the level scheme are compared with the calculations made in the frame of the neutron–proton Interacting Boson-Fermion Model (IBFM-2).

In this work the results of investigation about rotational bands in ^{129}La using the projected shell model are presented, especially about one-quasi-proton bands of ^{129}La . In Section 2.1, the projected shell model is briefly described, in Section 2.2, the theoretical predictions are given and a comparison with the experimental results is presented. Finally, the paper is summarized in Section 3.

2. Theory and calculations

2.1. Theory

The projected shell model [6–9] employed in this paper is a microscopic theory, which solves the problem fully quantum mechanically. The ansatz for the angular-momentum-projected wave function is given by:

$$|IM\rangle = \sum_k f_k \hat{P}_{MK}^I |\varphi_k\rangle, \quad (1)$$

where k labels the basis states. \hat{P}_{MK}^I is the angular momentum projection operator, it is given explicitly by the expression

$$\hat{P}_{MK}^I = \frac{I + \frac{1}{2}}{4\pi^2} \int d\Omega \hat{R}(\Omega) \hat{D}_{MK}^I(\Omega), \quad (2)$$

where $\hat{R}(\Omega)$ is the rotation operator, $\hat{D}_{MK}^I(\Omega)$ the D -function (irreducible representation of the rotation group) and Ω the Euler angle. Acting on an intrinsic state $|\varphi_k\rangle$, the operator \hat{P}_{MK}^I generates states of good angular momentum, thus restoring the necessary rotational symmetry violated in the deformed mean field. In this way the new shell model basis is constructed in which the Hamiltonian is diagonalized, this shell model basis taken in the present work is as follow:

$$\hat{P}_{MK}^I|\varphi_k\rangle. \quad (3)$$

The basis states $|\varphi_k\rangle$ are spanned by the set

$$\{ a_p^+|0\rangle, a_{n_1}^+ a_{n_2}^+ a_p^+|0\rangle \}, \quad (4)$$

for odd proton nuclei. $|0\rangle$ denotes the quasiparticle vacuum state and a_n^+ (a_p^+) is the neutron (proton) quasiparticle creation operator for this vacuum; the index $n_{1(2)}(p)$ runs over selected neutron (proton) quasiparticle states and k in Eq. (1) runs over the configuration of Eq. (2). The vacuum is obtained by diagonalizing a deformed Nilsson Hamiltonian [10] followed by a BCS calculation. In the calculations we use three major shells, *i.e.* ($N = 3, 4$, and 5) for neutrons (protons) as the configuration space.

In this work we have used the Hamiltonian [9]

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi \sum_{\mu} \hat{Q}_{\mu}^+ \hat{Q}_{\mu} - G_M \hat{P}^+ \hat{P} - G_Q \sum_{\mu} \hat{P}_{\mu}^+ \hat{P}_{\mu}, \quad (5)$$

where \hat{H}_0 is the spherical single-particle shell model Hamiltonian, \hat{Q}_{μ} is the quadrupole moment operator, \hat{P} and \hat{P}_{μ} are monopole pairing operator and quadrupole pairing operator, respectively. Though the theory itself is not bound to any particular form of Hamiltonian, the advantage of using such a separable-force Hamiltonian is that the role of each interaction is well known and, therefore, the interpretation of the numerical result becomes easier. The interaction strengths are determined as follows: the strength of the quadrupole–quadrupole interaction χ is adjusted by the self-consistent relation such that the input quadrupole deformation ε_2 and the one resulting from the HFB (Hartree–Fock–Bogoliubov) procedure coincide with each other [9]. The monopole pairing strength constant is adjusted to give the known energy gap

$$G_M = \left[20.12 \mp 13.13 \frac{N-Z}{A} \right] A^{-1}, \quad (6)$$

where “ $-$ ” for neutrons and “ $+$ ” for protons. Finally the quadrupole pairing strength G_Q is simply assumed to be proportional to G_M

$$\left(\frac{G_Q}{G_M} \right)_n = \left(\frac{G_Q}{G_M} \right)_p = \gamma. \quad (7)$$

The proportionality constant γ is chosen as 0.14–0.18 [6] in the rare-earth region where the nuclei are known as well-deformed, the nucleus ^{129}La studied in this paper is in the transitional region, so we choose $\gamma = 0.20$ [11].

The weights f_k in Eq. (1) are determined by diagonalizing the Hamiltonian \hat{H} in the basis given by Eq. (4). This will lead to the eigenvalue equation (for a given spin I)

$$\sum_{k'} (H_{kk'} - EN_{kk'}) f_{k'} = 0, \quad (8)$$

with the Hamiltonian and norm overlaps given by

$$H_{kk'} = \langle \varphi_k | \hat{H} \hat{P}_{K_k K'_{k'}}^I | \varphi_{k'} \rangle, \quad N_{kk'} = \langle \varphi_k | \hat{P}_{K_k K'_{k'}}^I | \varphi_{k'} \rangle. \quad (9)$$

Projection of good angular momentum onto each intrinsic state generates the rotational band associated with this intrinsic configuration $|\varphi_k\rangle$. For example, $\hat{P}_{MK}^I a_p^+ |0\rangle$ will produce a one-quasiproton band. The energies of each band are given by the diagonal elements of Eq. (9)

$$E_k(I) = \frac{\langle \varphi_k | \hat{H} \hat{P}_{KK}^I | \varphi_k \rangle}{\langle \varphi_k | \hat{P}_{KK}^I | \varphi_k \rangle} = \frac{H_{kk}}{N_{kk}}. \quad (10)$$

A diagram in which $E_k(I)$ for various bands are plotted against spin I is referred to as a band diagram [9]. The solution of Eq. (8) can be compared with the experiment, and the lowest eigenvalue of the Hamiltonian for a given spin is named the yrast energy.

The projected shell model has at least two advantages by this token:

1. The procedure of angular momentum coupling, which must be done troublesomely in the conventional shell model, is done automatically by the projector irrespective of the number of quasiparticles involved.
2. It allows us to choose various multi-quasiparticle bases according to physical importance.

Unfortunately, our present computer code [12] provided by Sun and Hara assumes axial symmetry so that we cannot investigate those γ -deformed nuclei quantitatively [11]. Since the nucleus in question is dominated by a near prolate shape ($\gamma \cong 0^\circ$) as indicated in Section 1, such a constraint will not prevent us from investigating the physics at hand. This model has achieved considerable success when it was applied to rare-earth region where the nucleus is well-deformed. In this paper, we try to apply this model to the $A \sim 130$ and to show the potential of this model via the study of high/low-lying spin states of ^{129}La .

In our calculations, the following formulae are used to calculate the pairing gap parameters Δ_p and Δ_n [13]:

$$\Delta_p = \frac{1}{4}\{B(N, Z-2) - 3B(N, Z-1) + 3B(N, Z) - B(N, Z+1)\}, \quad (11)$$

$$\Delta_n = \frac{1}{4}\{B(N-2, Z) - 3B(N-1, Z) + 3B(N, Z) - B(N+1, Z)\}, \quad (12)$$

the values of the total nuclear binding energy B are taken from the Ref. [14]. The results are $\Delta_p = 1.2975$ MeV and $\Delta_n = 1.065$ MeV, which are almost equal to those used in the CSM calculations [1]. The κ and μ parameters in the Nilsson potential are taken from Ref. [15] where the κ and μ are more reasonable as compared with the Ref. [3]. The deformation parameters are obtained from minima energy calculations using the Nilsson+BCS method. The deformation energy E_{HFB} (equivalent to the deformation energy from the calculations using the Nilsson+BCS method) of ^{129}La as a function of quadrupole deformation ε_2 is shown in Fig. 1. We can find the deformation energy E_{HFB} has two minima in Fig. 1, they correspond to the prolate shape ($\varepsilon_2 = 0.24$) and oblate shape ($\varepsilon_2 = -0.22$), respectively. There is no significant barrier between the two minima when going from one to the another via quadrupole deformation, so the nucleus seems to be soft to quadrupole deformation.

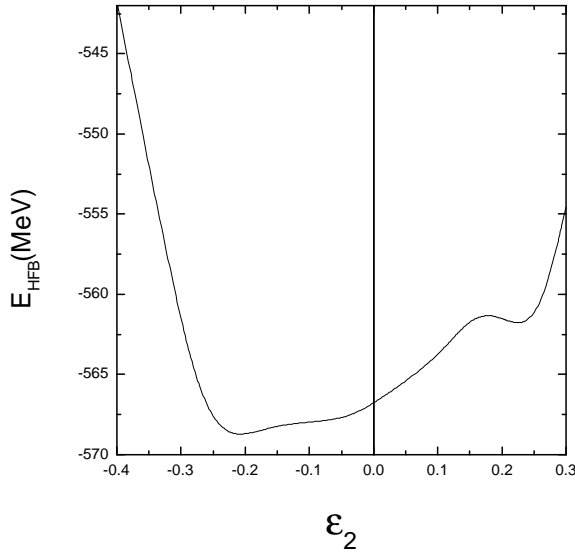


Fig. 1. Deformation energy E_{HFB} of ^{129}La as a function of quadrupole deformation parameter ε_2 .

2.2. Comparison of the calculations with experimental results

The experimental and calculated levels about one-quasiproton bands of ^{129}La are shown in Fig. 2. Among them, the experimental data of the negative parity $\pi h_{11/2}$ unfavoured yrast band, the favoured yrare band and unfavoured yrare band are taken from Ref. [4], we mark them band 2, 3 and 4, respectively. The rest bands are taken from Ref. [1], we mark them band 1, 5 and 6 (whereas $17/2^+$ level of band 6 is taken from Ref. [4]), respectively, they are the negative parity $\pi h_{11/2}$ favoured yrast band, the positive parity $\pi g_{7/2}$ favoured and unfavoured band, respectively. We adopt the Nilsson levels of the $h_{11/2}$ subshell in the $N = 5$ ($N = 5$) major shell as the single particle basis of proton (neutron) for band 1 and 2 in order to correspond to their configuration in the calculations. For the same reason, we adopt the Nilsson levels of the $g_{7/2}$ ($h_{11/2}$) subshell in the $N = 4$ ($N = 5$) major shell as the single particle basis of proton(neutron) for band 5 and 6. The quadrupole deformation parameter used in the calculations is $\varepsilon_2 = 0.24$ (corresponding to the right minimum in Fig. 1) and hexadecapole deformation $\varepsilon_4 = 0.00$, *i.e.* prolate shape. It is deduced by comparing theoretical results with experimental data that the agreement with negative parity $\pi h_{11/2}$ band 1 and 2 is very good, but the agreement with band 3 and 4 is not satisfactory, this is in accord with Ref. [4]. The agreement with positive parity $\pi g_{7/2}$ band 5 and 6 is also very good, this indicates that these two bands can be interpreted by the prolate deformation, this is not given by Ref. [4]. It should be pointed out as follows:

1. The proton Fermi surface lies at the bottom of the intruder subshell $h_{11/2}$ and the neutron Fermi surface lies in the upper middle part of the intruder subshell $h_{11/2}$, respectively, in the transitional region such as the nucleus ^{129}La . In the case of protons, they tend to drive the nuclei toward prolate shapes with $\gamma = 0^\circ$, while for the neutrons they tend to drive the nuclei toward oblateness with $\gamma = -60^\circ$, so the nucleus which is excited to a different band has the possible different shape and deformation due to different configuration.
2. Our results about band 5 and 6 are in principle consistent with that have been discussed by He *et al.* [1] using the prolate shape to explain the high spin states of ^{129}La , except that we take a slightly larger quadrupole deformation parameter $\varepsilon_2 = 0.24$.
3. The every band number is obtained by solving the eigenvalue of the Hamiltonian (namely formula (5)), then it is taken to compare with the experimental result directly, it can not be done by using the Cranked Shell Model by He *et al.* [1].

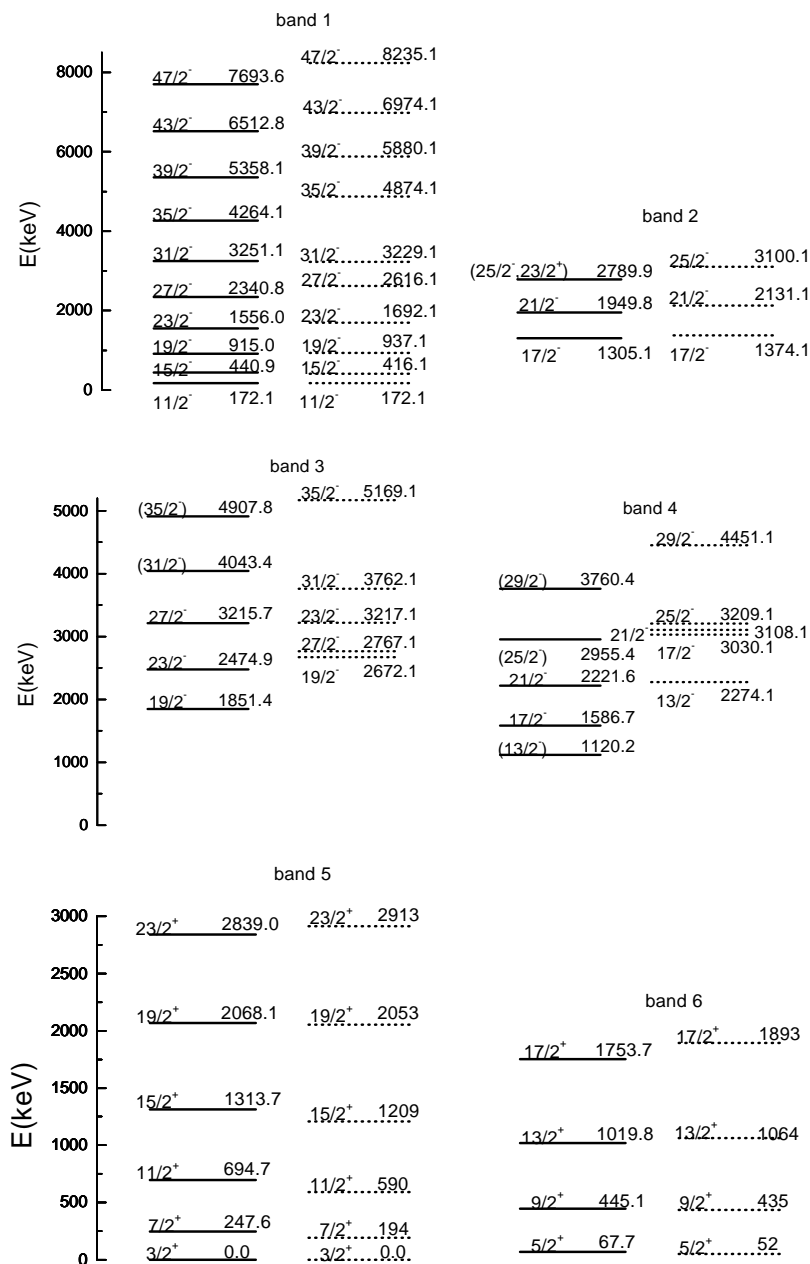


Fig. 2. Levels of one-quasiproton bands of ^{129}La (solid lines) and the corresponding calculated levels (dotted lines).

Though there is no corresponding basis in the actual projected shell model for the other bands showed in Ref. [1] (see also formulae (3) and (4)), namely the $\pi h_{11/2} \otimes \nu[h_{11/2}g_{7/2}]$ band and $\pi g_{7/2} \otimes [\pi h_{11/2}]^2$ band, we try to carry on some calculations, unfortunately, the agreement with the experimental data is not very good. It looks as if the basis states $|\varphi_k\rangle$, which are spanned by the set $\{a_p^+|0\rangle, a_{n_1}^+ a_{n_2}^+ a_p^+|0\rangle\}$, should be extended in order to describe the other bands more nicely [9].

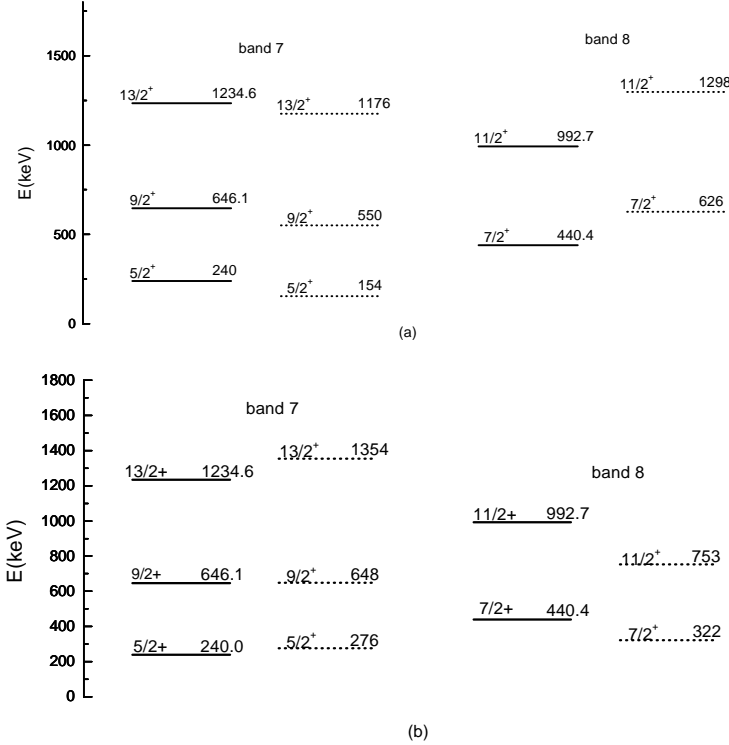


Fig. 3. Experimental levels of band 7 and 8 of ^{129}La (solid lines) and the corresponding calculated levels (dotted lines) using the quadrupole deformation parameter (a) $\varepsilon_2 = 0.24$ (b) $\varepsilon_2 = -0.22$.

Whereas the $\pi h_{11/2} \otimes [\nu h_{11/2}]^2$ oblate band and the possible $\pi g_{7/2} \otimes [\nu h_{11/2}]^2$ oblate band have been detected in the odd mass isotope ^{131}La beside ^{129}La [16, 17], we attempt to do the theoretical calculations about the band O and P in ^{129}La showed in Ref. [4]. In our calculations, we adopt the Nilsson levels of $g_{7/2}[h_{11/2}]$ subshell in the $N = 4$ ($N = 5$) major shell as the single particle basis of proton (neutron), and use the quadrupole deformation $\varepsilon_2 = 0.24$ and $\varepsilon_2 = -0.22$, respectively. Considering the band

TABLE I

The energy E_{Qp} of quasiproton, its projection K of the total angular momentum \vec{I} on the intrinsic (body) coordinate axis and the subshell it belongs to.

ε_2	E_{Qp}	K	subshell
-0.22	-6.2931	9/2	1g _{9/2}
	-4.8756	-7/2	1g _{9/2}
	-4.0610	5/2	1g _{9/2}
	-3.4985	-3/2	1g _{9/2}
	-3.2455	1/2	1g _{9/2}
	-1.8165	-7/2	1g _{7/2}
	-1.5203	5/2	1g _{7/2}
	1.4426	-3/2	1g _{7/2}
	1.5226	1/2	1g _{7/2}
	1.5753	5/2	2d _{5/2}
	1.8920	-3/2	2d _{5/2}
	2.2507	1/2	2d _{5/2}
	4.2353	-3/2	2d _{3/2}
	4.4773	1/2	2d _{3/2}
	7.3427	1/2	3s _{1/2}
0.24	-6.1900	1/2	1g _{9/2}
	-5.5398	-3/2	1g _{9/2}
	-4.4530	5/2	1g _{9/2}
	-3.0756	-7/2	1g _{9/2}
	-2.3491	1/2	1g _{7/2}
	-1.6627	9/2	1g _{9/2}
	-1.3989	1/2	2d _{5/2}
	-1.3970	-3/2	1g _{7/2}
	1.8463	5/2	1g _{7/2}
	1.9033	-3/2	2d _{5/2}
	3.1596	1/2	2d _{3/2}
	3.6545	-7/2	1g _{7/2}
	3.8505	5/2	2d _{5/2}
	6.0339	3/2	2d _{3/2}
	6.0988	1/2	3s _{1/2}

O and P as the $\pi g_{7/2}$ yrare band, and using $\varepsilon_2 = 0.24$, we obtain the results which are shown in Fig. 3(a) (we denote the band O and P by band 7 and 8, respectively). It can be found that the theoretical values of levels $9/2^+$ and $13/2^+$ are smaller than the theoretical values of levels $7/2^+$ and $11/2^+$, respectively, these are contrary to the experimental ones, so the band 7 and 8 cannot be interpreted as the $\pi g_{7/2}$ yrare band. When we use the $\varepsilon_2 = -0.22$, the results are shown in Fig. 3(b). It can be found that the ordering of all calculated levels agrees with that of the experimental ones. There are some discrepancies between the theoretical values and the experimental data for the levels $7/2^+$ and $11/2^+$, but their intervals are near equal. Considering the basis which we take in the calculations, the structure assigned to this band is the $\pi g_{7/2} \otimes [\nu h_{11/2}]^2$ configuration, and this configuration is oblate. In fact, when we choose the quadrupole deformation $\varepsilon_2 = 0.24$ and $\varepsilon_2 = -0.22$, respectively, according to the calculations using the Nilsson+BCS method, the energy E_{Qp} of quasiproton, its projection K of the total angular momentum \vec{I} on the nuclear symmetry axis and the subshell it belongs to are listed in the Table I. The values of E_{Qp} take the Fermi surface as a reference, λ_p is 43.668419 MeV and 43.481304 MeV corresponding to $\varepsilon_2 = -0.22$ and $\varepsilon_2 = 0.24$, respectively. It has been concluded by Gizon *et al.* [5] that the wave functions of the $5/2^+$ and $7/2^+$ states in band 7 and 8, respectively, are strongly mixed; their structures are $14\%s_{1/2} + 10\%d_{3/2} + 28\%d_{5/2} + 48\%g_{7/2}$ and $15\%s_{1/2} + 25\%d_{3/2} + 45\%d_{5/2} + 15\%g_{7/2}$, respectively. That paper did not mention the $g_{9/2}$ subshell, but we can find from Table I, when we choose $\varepsilon_2 = 0.24$, the $K = 9/2$ orbital of $g_{9/2}$ subshell is very near the Fermi surface, by this token it is even more reasonable to take $\varepsilon_2 = -0.22$. It is also expected that the $\pi h_{11/2} \otimes [\nu h_{11/2}]^2$ band in the nucleus ^{129}La will be observed by some experiments in future and that this band will be interpreted using the quadrupole deformation $\varepsilon_2 = -0.22$ (corresponding to the left minimum in Fig. 1).

3. Summary

This paper presents the calculations of the six one-quasiproton bands of ^{129}La using the projected shell model. First of all, we study the deformation energy E_{HFB} of ^{129}La as a function of quadrupole deformation ε_2 using the Nilsson+BCS method, one may find that the deformation energy E_{HFB} has two minima, which correspond to the prolate shape and oblate shape, respectively. We take the quadrupole deformation parameter $\varepsilon_2 = 0.24$ to study the six one-quasiproton bands, the agreement of the theoretical calculations with the $\pi h_{11/2}$ negative parity yrast band and $\pi g_{7/2}$ positive parity band is quite satisfactory, but the agreement with the $\pi h_{11/2}$ yrare band is not satisfactory. In addition, this paper also assigns the $\pi g_{7/2} \otimes$

$[\nu h_{11/2}]^2$ oblate band. The projected shell model can interpret the six of total twelve level bands in ^{129}La , it is the best of all mentioned models in answering for the experimental level scheme of ^{129}La until now.

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