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THE $g_{K_0^* K\pi}$ COUPLING CONSTANT IN QCD

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The strong coupling constant $g_{K_0^* K\pi}$ of the scalar K_0^* meson decay to $K\pi$ is calculated in light cone QCD sum rule. The predicted value of the coupling constant $g_{K_0^* K\pi}$ is in a good agreement with the experimental result.

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1. Introduction

The number leptons that are expected to be produced yearly at planned B factories and the proposed τ–charm factories [1], is of the order of 10^7, hence a detailed investigation of the decay properties of the τ lepton becomes an important issue. The decays of τ lepton can serve not only as a useful tool in investigation of some aspects of the standard model (SM) but also as a powerful experimental probe of new physics [2]. CP violation plays one of the most promising role in this direction. In light of this aspect, the decay of τ lepton into hadrons has recently been investigated as probes of CP violation in the scalar sector of physics beyond the SM [3]– [7].

In [7] the Cabibbo suppressed $\tau \to K\pi\nu_\tau$ decay to probe the CP violation with polarized τ's was studied. This decay mode is dominated by the contributions of the two lowest vector K^* and scalar K_0^* resonances, and the mode is expected to have larger scalar contribution. The matrix element for $\tau \to (K\pi)^-\nu_\tau$ in the general form

$$\mathcal{M} = \frac{G}{\sqrt{2}} \left[\bar{u}(k) \gamma_\mu (1 - \gamma_5) u(p) J_\mu + \bar{u}(p)(1 + \gamma_5) u(k) J_s \right] , \tag{1}$$

where p and k are the τ lepton and the τ neutrino four momenta, respectively. The vector and scalar hadronic matrix elements can be parametrized as (see [7])

$$\begin{aligned} J_\mu &= \sin\theta_c \left\langle K\pi \left| \bar{s}\gamma_\mu u \right| 0 \right\rangle \\ &= \sqrt{2}\sin\theta_c \left[F_K(q^2) \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) (q_1 - q_2)^\nu + \frac{m_{K_0^*}^2}{q^2} C_K F_s(q^2) q_\mu \right] , \\ J_s &= \sqrt{2}\sin\theta_c \left[\frac{m_{K_0^*}^2}{m_s - m_u} \right] C_K F_s(q^2) , \end{aligned} \tag{2}$$

where θ_c is the Cabibbo angle, $\sin\theta_c = 0.23$, q_1 and q_2 are the four–momenta of π and K respectively, m_s and m_u are the s and u current quark masses and $q = q_1 + q_2$ is the four–momentum of the $K\pi$ system. The coupling strength C_K denoting the scalar contributions is determined as (see [7])

$$C_K = \frac{f_{K_0^*}\, g_{K_0^* K\pi}}{\sqrt{3}\, m_{K_0^*}^2} , \tag{3}$$

where $f_{K_0^*}$ is the leptonic decay constant of scalar K_0^* meson and $g_{K_0^* K\pi}$ is the coupling constant of the $K_0^* \to K\pi$ decay. In deriving Eq. (3) we assumed $\mathcal{B}(K_0^* \to K\pi) = 100\%$. From the measured $K_0^* \to K\pi$ decay width $\Gamma(K_0^* \to K\pi) \simeq 287$ MeV, the value of $g_{K_0^* K\pi}$ is obtained to be 4.87 GeV.

In this work we employ light cone QCD sum rule to calculate $g_{K_0^* K\pi}$ coupling constant in a model independent way and compare our results with the experimental data.

The light cone QCD sum rule is quite different from the "classical" sum rule which is based on the short distance operator product expansion (OPE). This version of QCD sum rule is based on the OPE on the light cone, which is governed by the twist of the operators rather than by their dimension and the vacuum expectation values of local operators are replaced by the light cone hadron wave functions, and it is quite suitable in studies of the hard exclusive processes in QCD. Light cone QCD sum rule has been successfully applied so far in the study of many different problems of hadron physics such as rare, radiative and semileptonic decays of B meson, $\Sigma \to p\gamma$ decay, nucleon magnetic moment, the strong couplings $g_{\pi NN}$, $g_{\rho\omega\pi}$ and $g_{B^* B\pi}$ *etc.* (for an application of this method, see for example, the recent review [8, 9] and references therein).

2. QCD sum rule for the $g_{K_0^* K\pi}$ coupling constant

The aim of this section is to calculate the coupling constant $g_{K_0^* K\pi}$, which characterizes the $K_0^* \to K\pi$ decay. We start by considering the two point correlation function

$$\Pi(p,q) = i \int d^4x\, \mathrm{e}^{iqx} \left\langle \pi(p) \left| T\bar{d}(x) i\gamma_5 s(x)\, \bar{s}(0) u(0) \right| 0 \right\rangle , \tag{4}$$

which is calculated around the light cone $x^2 = 0$. Here $\bar{d} i\gamma_5 s$ and $\bar{s}u$ are the interpolating currents for pseudoscalar K and the scalar K_0^* mesons, respectively.

According to the basic idea of the QCD sum rule, we must calculate this correlator in terms of the physical particles (hadrons) and in quark–gluon language, and then equate both representations.

First let us calculate the physical part of the correlator Eq. (4). Saturating this correlator by K_0^* and K meson states, we have

$$\Pi^{\mathrm{phys}} = -g_{K_0^* K\pi} \frac{f_{K_0^*} m_{K_0^*}^2}{(m_s - m_u)} \frac{f_K m_K^2}{(m_s + m_d)} \frac{1}{\left((p+q)^2 - m_{K_0^*}^2\right)} \frac{1}{(q^2 - m_K^2)} , \tag{5}$$

where $(p+q)$ and q are the four momenta of the scalar K_0^* and pseudoscalar K mesons, respectively. In deriving the above equation we have used

$$\begin{aligned} \left\langle 0 \left| \bar{d} i\gamma_5 s \right| K \right\rangle &= \frac{f_K m_K^2}{m_s + m_d} , \\ \langle K^* | \bar{s}u | 0 \rangle &= i\, \frac{f_{K_0^*} m_{K_0^*}^2}{m_s - m_u} . \end{aligned} \tag{6}$$

The strong coupling constant for the $K_0^{*-} \to K^0 \pi^-$ decay is defined as follows:

$$\langle \pi K | K_0^* \rangle = -g_{K_0^* K\pi} .$$

Our next task is the calculation of the theoretical part of the correlator function (4). The full light quark propagator with both perturbative term and contributions from vacuum fields can be written as

$$\begin{aligned} i\mathcal{S}(x) &= \langle 0 | T\{s(x)\bar{s}(0)\} | 0 \rangle \\ &= i\, \frac{\not{x}}{2\pi^2 x^4} - \frac{\langle \bar{s}s \rangle}{12} - \frac{x^2}{192}\, m_0^2 \langle \bar{s}s \rangle \\ &\quad - i\, \frac{g_s}{16\pi^2} \int_0^1 du \left\{ \frac{\not{x}}{x^2}\, \sigma_{\alpha\beta}\, G^{\alpha\beta}(ux) - 4iu\, \frac{x_\mu}{x^2}\, G^{\mu\nu}(ux) \gamma_\nu \right\} + \cdots , \end{aligned} \tag{7}$$

where $\not x = x_\mu \gamma^\mu$. Note that here and in the following formulas the strange quark mass is set zero, though in numerical analysis, the mass of the strange quark is taken account.

Substituting Eq. (7) into correlator (4) and performing Fourier transformation, for the theoretical part we get

$$\begin{aligned} \Pi^{\text{theor}} &= -f_\pi \int_0^1 du \Bigg\{ \varphi_\pi(u) \frac{pq}{\Delta} - 4\frac{pq}{\Delta^2}\Big(g_1(u) + G_2(u)\Big) + 2g_2(u)\frac{1}{\Delta} \\ &+ \int \frac{\mathcal{D}\alpha_i}{\Delta_1^2} \Big[\big(2\varphi_\perp(\alpha_i) - \varphi_{\|}(\alpha_i) + 2\tilde{\varphi}_\perp(\alpha_i) - \tilde{\varphi}_{\|}(\alpha_i)\big)\, pq \\ &+ 2u\left(\varphi_{\|}(\alpha_i) - 2\varphi_\perp(\alpha_i)\right) \Big] \Bigg\}, \end{aligned} \tag{8}$$

where

$$\begin{aligned} \mu_\pi &= \frac{m_\pi^2}{(m_u + m_d)} \\ \Delta &= -q^2 \bar{u} - (p+q)^2 u \\ \Delta_1 &= -\left[q + p(\alpha_1 + u\alpha_3)\right]^2 , \\ \mathcal{D}\alpha_i &= d\alpha_1\, d\alpha_2\, d\alpha_3\, , \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)\, , \\ \varphi(\alpha_i) &= \varphi(\alpha_1, \alpha_2, \alpha_3)\, . \end{aligned} \tag{9}$$

Here φ_π is the leading twist 2 distribution amplitude, φ_P is the two–particle distribution amplitude of twist 3; $g_1,\ g_2,\ \varphi(\alpha_i)$ and $\tilde{\varphi}(\alpha_i)$ are the distribution amplitude of twist 4 and

$$G_2(u) = -\int_0^u g_2(v) dv\, .$$

All these functions are defined as follows:

$$\begin{aligned} \left\langle \pi(p) \left| \bar{d} i \gamma_\mu \gamma_5 u(0) \right| 0 \right\rangle &= -i f_\pi p_\mu \int_0^1 du\, \mathrm{e}^{iupx} \left(\varphi(u) + x^2 g_1(u)\right) \\ &+ f_\pi \left(x_\mu - \frac{x^2 p_\mu}{px}\right) \int_0^1 du\, \mathrm{e}^{iupx} g_2(u), \end{aligned}$$

$$\left\langle \pi(p) \left| \bar{d} i\gamma_5 u(0) \right| 0 \right\rangle = \frac{f_\pi m_\pi^2}{(m_u + m_d)} \int_0^1 du \, \mathrm{e}^{iupx} \varphi_P(u),$$

$$\left\langle \pi(p) \left| \bar{d}\gamma_\mu \gamma_5 g_s G_{\alpha\beta}(ux) u(0) \right| 0 \right\rangle = f_\pi \left[p_\beta \left(g_{\alpha\mu} - \frac{x_\alpha p_\mu}{px} \right) - p_\alpha \left(g_{\beta\mu} - \frac{x_\beta p_\mu}{px} \right) \right] \times \int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) \, \mathrm{e}^{ipx(\alpha_1 + u\alpha_3)}$$

$$\left\langle \pi(p) \left| \bar{d}\gamma_\mu g_s \tilde{G}_{\alpha\beta}(ux) u(0) \right| 0 \right\rangle = i f_\pi \left[p_\beta \left(g_{\alpha\mu} - \frac{x_\alpha p_\mu}{px} \right) - p_\alpha \left(g_{\beta\mu} - \frac{x_\beta p_\mu}{px} \right) \right] \times \int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) \, \mathrm{e}^{ipx(\alpha_1 + u\alpha_3)} + i f_\pi \frac{p_\mu}{px} (p_\alpha x_\beta - p_\beta x_\alpha) \times \int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) \, \mathrm{e}^{ipx(\alpha_1 + u\alpha_3)} , \tag{10}$$

where the operator $\tilde{G}_{\alpha\beta}$ is the dual of $G_{\alpha\beta}$, *i.e.*,

$$\tilde{G}_{\alpha\beta} = \tfrac{1}{2} \epsilon_{\alpha\beta\delta\rho} G^{\delta\rho} .$$

Due to the choice of the gauge $x_\mu A^\mu(x) = 0$, the path ordered gauge factor

$$\mathcal{P} \exp \left(i g_s \int_0^1 du x^\mu A_\mu(ux) \right) ,$$

has been omitted. Note that the radiative corrections to the leading twist wave functions are neglected, since their contribution is small (about 6–7%, see [10]).

Performing double Borel transformation with the variables $(p+q)^2$ and q^2 in Eqs. (5) and (8), we get the following sum rule for the $g_{K_0^* K\pi}$ coupling constant.

$$g_{K_0^* K\pi} f_{K_0^*} f_K = \frac{1}{\mu_{K_0^*}} \frac{1}{\mu_K} \, \mathrm{e}^{\frac{m_{K_0^*}^2}{M_1^2} + \frac{m_K^2}{M_2^2}} f_\pi M^4$$
$$\times \left\{ - \frac{1}{2} \varphi'_\pi(u_0) f_1 \frac{s_0}{M^2} + 2 \frac{g'_1(u_0)}{M^2} f_0 \frac{s_0}{M^2} \right.$$
$$+ \frac{1}{M^2} \left(\int_0^{u_0} d\alpha_1 \frac{F(\alpha_1, 1 - u_0, u_0 - \alpha_1)}{2(u_0 - \alpha_1)} \right.$$

$$
\begin{aligned}
& - \int_0^1 d\alpha_3 \frac{F(u_0, 1-u_0-\alpha_3, \alpha_3)}{2\alpha_3} \\
& + \int_0^{u_0} d\alpha_1 \frac{\varphi_{\|}(\alpha_1, 1-u_0, u_0-\alpha_1) - 2\varphi_{\perp}(\alpha_1, 1-u_0, u_0-\alpha_1)}{(u_0-\alpha_1)} \\
& - \int_0^{u_0} d\alpha_1 \int_{u_0-\alpha_1}^{1-\alpha_1} \frac{1}{\alpha_3} \Big[\varphi_{\|}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3) - 2\varphi_{\perp}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)\Big]\Bigg)\Bigg\}.
\end{aligned}
\tag{11}
$$

where

$$
\begin{aligned}
\mu_{K_0^*} &= \frac{m_{K_0^*}^2}{m_s - m_u}\,, \\
\mu_K &= \frac{m_K^2}{m_s + m_d}\,, \\
u_0 &= \frac{M_2^2}{(M_1^2 + M_2^2)}\,, \\
M^2 &= \frac{M_1^2 M_2^2}{(M_1^2 + M_2^2)}\,,
\end{aligned}
$$

with M_1^2 and M_2^2 are being the Borel parameters,

$$
\begin{aligned}
\varphi'_\pi(u_0) &= \left.\frac{d\varphi(u)}{du}\right|_{u=u_0} \qquad \text{and,} \\
F(\alpha_i) &= 2\varphi_\perp(\alpha_i) - \varphi_{\|}(\alpha_i) + 2\tilde{\varphi}_\perp(\alpha_i) - \tilde{\varphi}_{\|}(\alpha_i)\,,
\end{aligned}
$$

and s_0 is the continuum threshold. Here the function

$$
f_n(x) = 1 - \mathrm{e}^{-x} \sum_{k=0}^{n} \frac{x^k}{k!}\,,
$$

is used to subtract the continuum and higher resonance contributions. This contribution is modeled by the dispersion integral, by invoking duality in the region $s_1,\ s_2 \geq s_0$. In deriving Eq. (11), we have used the double Borel transformation formula:

$$
B_{1\,(p+q)^2}^{M_1^2}\, B_{2\,q^2}^{M_2^2} \frac{\Gamma(n)}{[m^2 - q^2\bar{u} - u(p+q)^2]^n} = \left(M^2\right)^{2-n} \mathrm{e}^{-\frac{m^2}{M^2}} \delta(u-u_0)\,.
$$

Note that in presenting Eq. (11) we omit the terms that are proportional to the strange quark mass in order to simplify the expressions, but in numerical analysis we take into account these terms. The sum rule (11) is asymmetric with respect to the Borel parameters M_1^2 and M_2^2 due to the significant mass difference of K_0^* and K. We choose M_1^2 and M_2^2 to be $M_1^2 = 2m_{K_0^*}^2\beta$ and $M_2^2 = 2m_K^2\beta$ respectively, where β is a scale factor and hence in regard to this assignment we have $M^2 = 0.44\beta$ GeV^2, $u_0 = 0.107$.

3. Numerical analysis

For numerical analysis we need the explicit forms of the wave functions. Following [11] we define the relevant wave functions as:

$$\varphi_\pi(u,\mu) = 6u(1-u)\Big[1+a_2(\mu)C_2^{3/2}(2u-1)+a_4(\mu)C_4^{3/2}(2u-1)+\cdots\Big], \quad (12)$$

where

$$C_{2n}^{3/2}(2u-1)$$

are the Gegenbauer polynomials and the coefficients $a_2 = \frac{2}{3}$, $a_4 = 0.43$ corresponding to the normalization point $\mu = 0.5$ GeV,

$$\begin{aligned}
\varphi_\perp(\alpha_i) &= 30\delta^2(\alpha_1-\alpha_2)\alpha_3^2\left[\tfrac{1}{3}+2\varepsilon(1-2\alpha_3)\right], \\
\varphi_{||}(\alpha_i) &= 120\delta^2\varepsilon(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3, \\
\tilde{\varphi}_\perp(\alpha_i) &= 30\delta^2\alpha_3^2(1-\alpha_3)\left[\tfrac{1}{3}+2\varepsilon(1-2\alpha_3)\right], \\
\tilde{\varphi}_{||}(\alpha_i) &= -120\delta^2\alpha_1\alpha_2\alpha_3\left[\tfrac{1}{3}+\varepsilon(1-3\alpha_3)\right]. \qquad (13)
\end{aligned}$$

$$\begin{aligned}
g_1(u) &= \tfrac{5}{2}\delta^2\bar{u}^2u^2+\tfrac{1}{2}\varepsilon\delta^2\left[\bar{u}u(2+13\bar{u}u)+10u^3\ln u(2-3u+\tfrac{6}{5}u^2)\right. \\
&\quad \left.+10\bar{u}^3\ln\bar{u}(2-3\bar{u}+\tfrac{6}{5}\bar{u}^2)\right],
\end{aligned}$$

where $\delta = 0.2$ GeV^2 at $\mu = 1$ GeV.

Here we would like to make the following remark. At the point $\mu = 0.107$ the expansion of the wave function in terms of the Gegenbauer polynomials converges very slowly. For this reason it is necessary to take into account many terms in this expansion (at least for the leading wave function $\varphi(u,\mu)$. For more details, see second ref. in [11]). The terms that have not been presented in Eq. (12), bring uncertainty to the value of the leading wave function. Following second ref. in [11], we estimate this uncertainty to the leading wave function $\varphi'(u,\mu)$ in the sum rules (11) as follows:

$$\varphi'(u,\mu=0.107) \simeq -10.4\pm1.5\,.$$

The values of the main input parameters, which appear in further numerical analysis are as follows: $\mu_\pi(1\ \text{GeV}) \simeq 1.65$, $\mu_K(1\ \text{GeV}) \simeq 1$, $f_\pi = 132$ MeV, $f_K = 156$ MeV and $m_s = 155$ MeV [12]. If the strange quark mass is chosen to be $m_s = 175$ MeV (see for example [13]), the results change marginally, about 2–3%.

The other free parameter of the QCD sum rules, namely, the continuum threshold s_0 is chosen as the mass square of the next scalar state, which approximately corresponds to $s_0 = (m_{K_0^*} + 0.5)^2\ \text{GeV}^2 \approx 3.6\ \text{GeV}^2$. To check the sensitivity of our results to the continuum threshold s_0 , different choices of s_0, such as $s_0 = 3.8\ \text{GeV}^2$; $4.0\ \text{GeV}^2$ and $4.2\ \text{GeV}^2$ are considered as well.

The dependence of $g_{K_0^* K\pi} f_{K_0^*} f_K$ on the Borel parameter β, for different values of the threshold s_0 is given in Fig. 1. The lower bound of the Borel parameter β is determined by the requirement that, the terms of higher twists in the operator expansion must be smaller than the leading twist term (say 3 times). This leads to $\beta \geq 1$ for the sum rule (11). The upper limit of β is restricted from the condition that the continuum contribution must be less than 30% of the main one. Under this condition the upper bound is determined to be $\beta = 1.6$. It follows from this figure that our result is quite stable in the working region of β and practically insensitive to the choice of the continuum threshold.

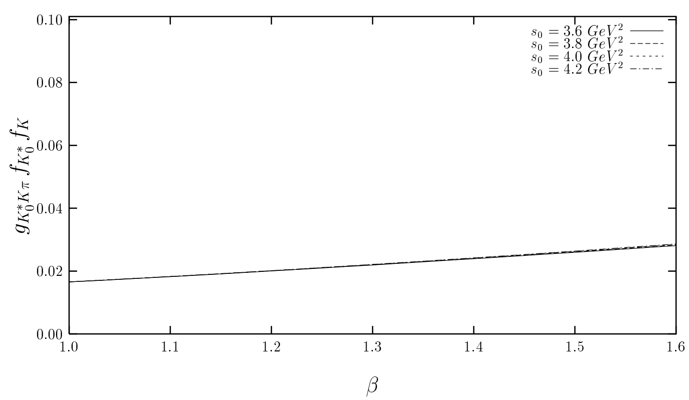

Fig. 1. The dependence of $g_{K_0^* K\pi} f_{K_0^*} f_K$ on the Borel parameter β, at four fixed values of the continuum threshold, $s_0 = 3.6\ \text{GeV}^2$, $3.8\ \text{GeV}^2$, $4.0\ \text{GeV}^2$ and $s_0 = 4.2\ \text{GeV}^2$.

From the analysis of Fig. 1, we finally get

$$g_{K_0^* K\pi} f_{K_0^*} f_K = 0.022 \pm 0.005\,, \tag{14}$$

in which we have included the uncertainties due to the continuum threshold, Borel parameter, radiative corrections to the leading twist wave function, neglection of the four particle components of the wave functions as well as uncertainty due the terms which are not taken into account in the leading wave function expansion in terms of Gegenbauer polynomials. In determination of $g_{K_0^* K\pi}$ the value of the leptonic decay constant $f_{K_0^*}$ is needed. However, it should be noted that, this decay constant has not been measured yet, but it has been estimated in framework of different approaches, such as QCD sum rule which predicts $f_{K_0^*} \simeq 31$ MeV [14]; effective Lagrangian method whose estimation is ~ 45 MeV [15] and pole dominance model's result ~ 50 MeV [16]. Using these values of the leptonic decay constant $f_{K_0^*}$, we get from Eq. (15)

$$g_{K_0^* K\pi} = \begin{cases} 4.6 \pm 1.0 \text{ GeV} & (\text{for } f_{K_0^*} = 31 \text{ MeV})\,, \\ 3.1 \pm 0.7 \text{ GeV} & (\text{for } f_{K_0^*} = 45 \text{ MeV})\,, \\ 2.8 \pm 0.6 \text{ GeV} & (\text{for } f_{K_0^*} = 50 \text{ MeV})\,. \end{cases}$$

Comparing these predictions with the existing experimental result $g_{K_0^* K\pi} \simeq 4.87 \pm 1.0$ GeV, it is observed that if we use $f_{K_0^*} = 31$ MeV, which was estimated from QCD sum rule, the predicted value of $g_{K_0^* K\pi}$ is close to the experimental result.

In summary, we have used light cone QCD sum rule to calculate the strong coupling constant $g_{K_0^* K\pi}$. The prediction we have for $g_{K_0^* K\pi}$ is in good agreement with the experimental result.

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