

ON THE ORIGIN OF THE VIOLATION OF HARA'S THEOREM FOR CONSERVED CURRENT

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(Received September 9, 2000)

I elaborate on the argument that the violation of Hara's theorem for conserved current requires that the current is not sufficiently well localized. It is also stressed that whatever sign of asymmetry is measured in the $\Xi^0 \rightarrow \Lambda \gamma$ decay, one of the following three statements must be incorrect: (1) Hara's theorem is satisfied, (2) vector meson dominance is applicable to weak radiative hyperon decays, and (3) basic structure of our quark-model description of nuclear parity violation is correct.

PACS numbers: 14.20.Jn, 11.40.Ha, 11.10.Lm

1. Introduction

In 1964 Hara proved a theorem [1], according to which the parity-violating amplitude of the $\Sigma^+ \rightarrow p \gamma$ decay should vanish in the limit of exact SU(3) symmetry. The assumptions used in the proof were fundamental. Over the years, however, there appeared several theoretical, phenomenological and experimental indications that, despite the proof, Hara's theorem may be violated. Quark model calculations of Kamal and Riazuddin [2], VDM-prescription [3] and experiment [4, 5] provide such hints. In particular only these models that violate Hara's theorem provide a reasonably good description of the overall body of experimental data on weak radiative hyperon decays [5], as it stands now.

Obviously, if Hara's theorem is violated in Nature it follows that at least one of its fundamental assumptions is not true. This in turn means that some unorthodox and totally new physics must manifest itself in weak radiative hyperon decays (WRHD). Although in general any non-orthodox physics should be avoided as long as possible, the problem with WRHD is that we are on the verge of being forced to accept it. Namely, there exists a clean experimental way of distinguishing between the orthodox and

nonorthodox physics. The decisive measurable parameter is the asymmetry of the $\Xi^0 \rightarrow \Lambda \gamma$ decay. Its absolute value is expected to be large (of order 0.7) independently of the type of physics involved. One may show that the sign of the asymmetry should be negative (positive) if physics is orthodox (unorthodox). Present experimental number is $+0.43 \pm 0.44$, almost 3σ away from the orthodox prediction. Of course the relevant experiment may have been performed or analysed incorrectly. However, this is just one (though the most crucial) of the hints against Hara's theorem. Other hints, more theoretical in nature, are provided by the calculations in the naive quark model [2] and by the VDM approach [3,5] in which VDM was combined with our present knowledge on parity violating weak coupling of vector mesons to nucleons [6].

There is a growing agreement that the calculation originally presented by Kamal and Riazuddin (KR) is technically completely correct [7,8]. (However, there is no consensus as to the meaning of the KR result [7,8].) The VDM approach is based on two pillars: VDM itself and the DDH paper on nuclear parity violation [6], in which parity violating weak couplings of mesons to nucleons are discussed. The DDH paper forms the foundation of our present understanding of the whole subject of nuclear parity violation, with the basis of the paper hardly to be questioned [8]. Similarly, VDM has an extraordinary success record in low energy physics. If Hara theorem is correct at least one of the above two pillars of the VDM approach to WRHD must be incorrect. This would be an important discovery in itself.

Given this situation, I think it is a timely problem to pinpoint precisely what it is that might lead to the violation of Hara's theorem. Some conjectures in this connection were presented in [5] (and even earlier, see references cited therein). These conjectures pointed at the assumption of locality. In fact, in a recent Comment [9] it was shown that one can obtain violation of Hara's theorem for conserved current provided the current is not sufficiently well localized. As proved in [9], the Hara's-theorem-violating contribution comes from $r = \infty$. However, as the example of the Reply [10] to my Comment shows, the content and implications of the Comment are not always understood. Therefore, in this paper I will try to shed some additional light on the problem.

Before I discuss the question of the implication of current (non)locality on Hara's theorem I will show that the argument raised in [10] against the technical correctness of the KR calculation is logically incorrect.

After disposing of the argument against the technical correctness of the KR calculation I will present a simple example in which current conservation *alone* does not ensure that Hara's theorem holds, unless an *additional physical* assumption is made.

Then, I will proceed to discuss the main relevant point made in Ref. [10].

In fact, Ref. [10] agrees with my standpoint that any violation of Hara's theorem must result from a new phenomenon. However, identification of the origin of this phenomenon therein proposed is mathematically incorrect. This shall be proved below in several ways.

In the final remarks I will stress once again that the resolution of the whole issue (in favour of Hara's theorem or against it) can be settled once and forever by experiment, that is by a measurement of the asymmetry of the $\Xi^0 \rightarrow \Lambda \gamma$ decay.

2. Conservation of the nonrelativistic current

In Ref. [2] Kamal and Riazuddin obtain gauge-invariant current-conserving covariant amplitude. Ref. [10] accepts correctness of their calculation up to this point. The claim of Ref. [10] is that the authors of [2] incorrectly perform nonrelativistic reduction thereby violating current conservation. According to Ref. [10] this may be seen from Eq. (13) of Ref. [2] which is of the form $H_{PV} \propto \boldsymbol{\epsilon} \cdot (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)$. In this equation the current *seems* to be of the form

$$\mathbf{J} = \boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2 \quad (1)$$

and is not transverse as it should have been for a conserved current.

This claim is logically incorrect. Eq. (13) of Ref. [2] is obtained after *both* performing the nonrelativistic reduction *and* choosing the Coulomb gauge $\boldsymbol{\epsilon} \cdot \hat{\mathbf{q}} = 0$ ($\hat{\mathbf{q}} = \mathbf{q}/|\mathbf{q}|$). The origin of the lack of transversity of the "current" $\mathbf{J}$ in Eq. (1) is *not* the nonrelativistic reduction but the choice of Coulomb gauge $\boldsymbol{\epsilon} \cdot \hat{\mathbf{q}} = 0$, *i.e.* the *restriction* to transverse degrees of freedom only. By choosing the Coulomb gauge we restrict the allowed $\boldsymbol{\epsilon}$ to be transverse only. It is then incorrect to replace $\boldsymbol{\epsilon}$ by (longitudinal) $\hat{\mathbf{q}}$. In other words the correct form of the current-photon interaction insisted upon in Ref. [10], *i.e.*

$$\boldsymbol{\epsilon} \cdot (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2 - \hat{\mathbf{q}}[(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \cdot \hat{\mathbf{q}}]) \quad (2)$$

after choosing the Coulomb gauge $\boldsymbol{\epsilon} \cdot \hat{\mathbf{q}} = 0$ reduces to Eq. (13) of Ref. [2]. Hence, from the form $\boldsymbol{\epsilon} \cdot (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)$ obtained in Ref. [2] *after* choosing the Coulomb gauge one cannot conclude that the current is $\mathbf{J} = \boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2$ and therefore that the nonrelativistic reduction was performed incorrectly.

Having proved that the argument against the KR calculation presented in Ref. [10] is logically incorrect, we proceed to the issue of current (non)locality.

3. A simple example

Let us consider the well-known concept of partially conserved axial current (PCAC). According to this idea the axial current is approximately conserved, with its divergence proportional to the pion mass squared. The weak axial current becomes divergenceless when the pion mass goes to zero, a situation obtained in the quark model with massless quarks. Thus, one may have a nonvanishing coupling of a vector boson to an axial conserved current and a nonvanishing transverse electric dipole moment, *i.e.* violation of Hara's theorem.

The price one has to pay to achieve this in the above example is the introduction of *massless* pions. A massless pion corresponds to an interaction of an infinite range — the pion may propagate to spatial infinity. Thus, vice versa, if one obtains a nonvanishing transverse electric dipole moment in a gauge-invariant calculation (the KR case) this suggests that the relevant current contains a piece that does not vanish at infinity sufficiently fast but *resembles* the pion contribution in the example above. In other words one expects that something happens at spatial infinity.

Of course, for Hara's theorem to be violated, the mechanism of providing the necessary nonlocality must be different from the particular one discussed above. After all, no massless hadrons exist. Consequently, current nonlocality would have to constitute an intrinsic feature of baryons. It might result from baryon compositeness: it is known that composite quantum states may exhibit nonlocal features. In this paper we will not pursue this line of thought any further since here we are primarily interested in proving beyond any doubt that nonlocality is crucial, but not in discussing its deeper justification and implications. Such a discussion will appear timely and desirable if new experiments confirm the positive sign of the $\Xi^0 \rightarrow \Lambda \gamma$ asymmetry.

Ref. [10] accepts that the current specified in Ref. [9] is conserved and that nonetheless it yields a nonzero value of the electric dipole moment in question. However, it is alleged that this nonzero result originates from $r = 0$ (and not from spatial infinity). In view of the example given above this claim should be suspected as incorrect. In fact its mathematical incorrectness can be proved. Let us therefore see where the arguments of Ref. [10] break down.

4. The origin of the nonzero contribution to the transverse electric dipole moment

In Ref. [9] it is shown that for the current of the form

$$\begin{aligned} \mathbf{J}_5^\varepsilon(\mathbf{r}) = & [\boldsymbol{\sigma} - (\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}] \delta_\varepsilon^3(\mathbf{r}) + \frac{1}{2\pi r^2} [\boldsymbol{\sigma} - 3(\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}] \delta_\varepsilon(r) \\ & - \frac{1}{4\pi r^3} [\boldsymbol{\sigma} - 3(\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}] \operatorname{erf}\left(\frac{r}{2\sqrt{\varepsilon}}\right), \end{aligned} \quad (3)$$

where $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ is the error function, $\hat{\mathbf{r}} = \mathbf{r}/r$, $r = |\mathbf{r}|$ and $\varepsilon \rightarrow 0$, the transverse electric dipole moment is given by

$$T_{1M}^{el} = \lim_{\varepsilon \rightarrow 0} \frac{iq}{2\pi\sqrt{2}} \int_0^\infty dr \text{erf}\left(\frac{r}{2\sqrt{\varepsilon}}\right) j_1(qr) \int d\Omega_{\hat{\mathbf{r}}} \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} Y_{1M}(\hat{\mathbf{r}}) \quad (4)$$

and is nonzero. The question is where does this nonzero result comes from. Ref. [9] (Ref. [10]) claim that the whole contribution is from $r = \infty$ (respectively $r = 0$). We shall show that the claim of Ref. [10] is mathematically incorrect.

The Reply [10] is based on the (true) equality (Eqs. (3), (4) therein)

$$\alpha = \lim_{\epsilon \rightarrow 0} q \int_0^\infty dr j_1(qr) \text{erf}\left(\frac{r}{2\sqrt{\epsilon}}\right) = \left(\frac{2}{q}\right) \int_0^\infty dz j_0(z) \delta\left(\frac{z}{q}\right) \quad (5)$$

in which the left-hand side (l.h.s.) is the original integral appearing in the expression for the electric dipole moment, from which it was concluded in Ref. [9] that violation of Hara's theorem originates from $r = \infty$.

The Reply [10] further claims that as one has to perform the integral first, and only then take the limit $\epsilon \rightarrow 0$, it can be seen from the right-hand side of Eq. (5) that in the limit $\epsilon \rightarrow 0$ the integral on the left-hand side receives all its contribution from the point $r = 0$.

That this claim is mathematically incorrect can be seen in many ways. We shall deal with the integral on the left-hand side *directly* since equality of definite integrals does not mean that the integrands are identical. In particular integration by parts used to arrive at the r.h.s. of Eq. (5) may change the region from which the value of the integral comes as it should be obvious from the following example:

$$\begin{aligned} & \int_0^\infty dx \exp(-x) \theta(x - \epsilon) \\ &= -\exp(-x) \theta(x - \epsilon)|_0^\infty + \int_0^\infty dx \exp(-x) \delta(x - \epsilon) \\ &= \int_0^\infty dx \exp(-x) \delta(x - \epsilon). \end{aligned} \quad (6)$$

Clearly, the integral on the l.h.s. of Eq. (6) does not receive all its contribution from the point $x = \epsilon$ while the r.h.s. does. Let us therefore concentrate on the l.h.s of Eq. (5) since it is the integrand on the l.h.s. which has a physical meaning.

(a) *Mathematical proof*

For *any finite* ϵ the integrand on the l.h.s. of Eq. (5) vanishes for $r = 0$ since $j_1(0) = \text{erf}(0/(2\sqrt{\epsilon})) = 0$. Consequently, already the most naive argument seems to show that the point $r = 0$ does not contribute in the limit $\epsilon \rightarrow 0$ at all. Should one be concerned with the neighbourhood of the point $r = 0$, we notice that both functions $j_1(qr)$ and $\text{erf}(r/(2\sqrt{\epsilon}))$ are *bounded* for any q, r, ϵ of interest. Consequently, the integrand on the left-hand side of Eq. (5) is bounded by $\max_{0 \leq z \leq \infty} j_1(z) \equiv M < \infty$. Hence, the contribution from any interval $[0, \Delta]$, ($0 \leq \Delta \ll 1$) is bounded by $q \int_0^\Delta dr M \approx q\Delta M$ and vanishes when $q\Delta \rightarrow 0$. From the mathematical point of view the incorrectness of Ref. [10] is thus proved.

For further clarification, however, the following two points may be consulted. Point (b) below provides simple and intuitive visual demonstration of what happens on the l.h.s. of Eq. (5) in the limit $\epsilon \rightarrow 0$. In point (c) the integral is actually performed *before* taking the limit $\epsilon \rightarrow 0$, the procedure considered in Ref. [10] to be correct.

(b) *Intuitive “proof”*

The integral on the left of Eq. (5) can be evaluated for any ϵ (formula 2.12.49.6 in Ref. [11]) and one obtains

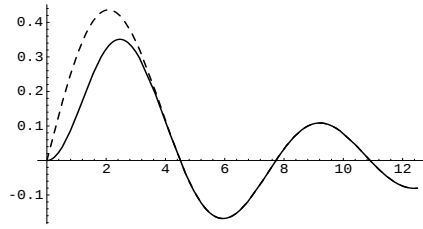
$$q \int_0^\infty dr j_1(qr) \text{erf}\left(\frac{r}{2\sqrt{\epsilon}}\right) = \frac{\sqrt{\pi}}{2q} \frac{1}{\sqrt{\epsilon}} \text{erf}(q\sqrt{\epsilon}) \quad (7)$$

which for small $q\sqrt{\epsilon}$ is equal to

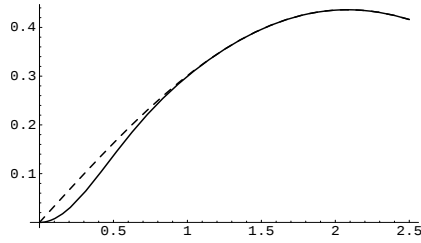
$$1 - \frac{q^2\epsilon}{3} + O((q^2\epsilon)^2). \quad (8)$$

This approach to 1 from below (when $q^2\epsilon \rightarrow 0$) can be seen from a series of plots shown in Fig. 1.

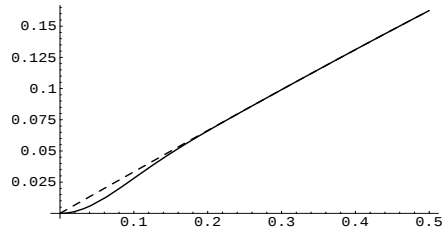
In Fig. 1 one can see that for small $q\sqrt{\epsilon}$ the integrand in Eq. (5) differs significantly from $j_1(qr)$ only for very small $qr < q\Delta$, where the integrand is *smaller* than $j_1(qr)$. It is also seen that in the limit $q\sqrt{\epsilon} \rightarrow 0$ the contribution from the region of small qr *grows* (thus the whole



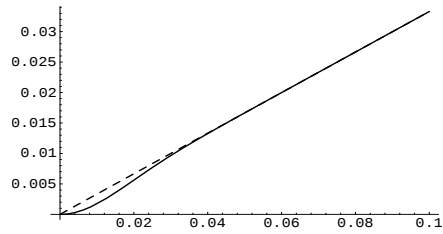
(a)



(b)



(c)



(d)

Fig. 1. The integrand $j_1(z)\text{erf}(z/(2q\sqrt{\epsilon}))$ (solid line), close to $z \equiv qr = 0$, for $q\sqrt{\epsilon} = 1.25, 0.25, 0.05, 0.01$ in plots (a), (b), (c), (d) respectively. Dashed line: $j_1(z)$.

integral grows in agreement with Eq. (8)) but *never* exceeds the integral $\int_0^\Delta q \, dr \, j_1(qr)$. It is intuitively obvious that the latter integral is smaller than $j_1(q\Delta) \cdot q\Delta$ and cannot yield the value 1 in Eq. (8) for $\Delta \rightarrow 0$! For more details consult point (c2) below.

(c) *Doing integrals first*

Should one be not satisfied for any reasons with the above two arguments, and insist that one has to perform the integral first, an appropriate rigorous proof of mathematical incorrectness of Ref. [10] follows. In this proof the integral is performed *before* taking the limit $\epsilon \rightarrow 0$, as argued in Ref. [10] to be the only correct procedure.

Let us divide the integral on the left-hand side of Eq. (5) into two contributions:

$$\lim_{\epsilon \rightarrow 0} \left[\int_0^\Delta dr \, q \, j_1(qr) \operatorname{erf} \left(\frac{r}{2\sqrt{\epsilon}} \right) + \int_\Delta^\infty dr \, q \, j_1(qr) \operatorname{erf} \left(\frac{r}{2\sqrt{\epsilon}} \right) \right], \quad (9)$$

where Δ is finite, but otherwise arbitrary: $0 < \Delta < \infty$.

According to Ref. [10], the whole contribution to the integral on the left-hand side of Eq. (5) comes from the point $r = 0$ when the limit $\epsilon \rightarrow 0$ is taken *after* evaluating the integral. Hence, the whole contribution to the left-hand side of Eq. (5) should come from the first term in Eq. (9), *i.e.* from

$$f_{[0,\Delta]}(q, \epsilon) \equiv \int_0^\Delta dr \, q \, j_1(qr) \operatorname{erf} \left(\frac{r}{2\sqrt{\epsilon}} \right) \quad (10)$$

when the limit $\epsilon \rightarrow 0$ is taken *after* evaluating the integral.

- (c1) Let us therefore estimate the integral $f_{[0,\Delta]}(q, \epsilon)$. Integrating by parts we obtain

$$\begin{aligned} f_{[0,\Delta]}(q, \epsilon) = & -\frac{1}{q} j_0(q\Delta) \frac{2}{\sqrt{\pi}} \int_0^{\Delta/(2\sqrt{\epsilon})} \exp(-t^2) \, dt \\ & + \frac{1}{q} j_0(q \cdot 0) \frac{2}{\sqrt{\pi}} \int_0^{0/(2\sqrt{\epsilon})} \exp(-t^2) \, dt \end{aligned}$$

$$+ \frac{2}{\sqrt{\pi}} \frac{1}{2\sqrt{\epsilon}} \frac{1}{q} \int_0^{\Delta} dr \, j_0(qr) \exp\left(\frac{-r^2}{4\epsilon}\right). \quad (11)$$

Since we take the limit $\epsilon \rightarrow 0$ only *after* evaluating the integral, the second term above vanishes. Thus

$$f_{[0,\Delta]}(q, \epsilon) = \frac{1}{q} \frac{2}{\sqrt{\pi}} \int_0^{\Delta/(2\sqrt{\epsilon})} dt \exp(-t^2) (j_0(q \cdot 2\sqrt{\epsilon}t) - j_0(q\Delta)). \quad (12)$$

Consequently

$$|f_{[0,\Delta]}(q, \epsilon)| \leq \frac{1}{q} \frac{2}{\sqrt{\pi}} \int_0^{\Delta/(2\sqrt{\epsilon})} dt \exp(-t^2) |j_0(q \cdot 2\sqrt{\epsilon}t) - j_0(q\Delta)|. \quad (13)$$

We are ultimately interested in the limit $q \rightarrow 0$. Hence, let us take $q\Delta \ll 1$. This may be assumed for *any* finite Δ . Since $0 \leq 2\sqrt{\epsilon}t \leq \Delta$, and the function $j_0(z)$ is monotonically decreasing for $z \ll 1$ it follows that

$$|j_0(q2\sqrt{\epsilon}t) - j_0(q\Delta)| \leq |j_0(0) - j_0(q\Delta)|. \quad (14)$$

Hence, for $q \ll 1/\Delta$ we have

$$\begin{aligned} |f_{[0,\Delta]}(q, \epsilon)| &\leq \frac{1}{q} \frac{2}{\sqrt{\pi}} \int_0^{\Delta/(2\sqrt{\epsilon})} dt \exp(-t^2) |j_0(0) - j_0(q\Delta)| \\ &\leq \frac{1}{q} \frac{2}{\sqrt{\pi}} \int_0^{\infty} dt \exp(-t^2) |j_0(0) - j_0(q\Delta)| \\ &= \Delta \left| \frac{j_0(q\Delta) - j_0(0)}{q\Delta} \right|. \end{aligned} \quad (15)$$

For finite Δ , in the limit $q \rightarrow 0$, the factor under the sign of modulus is the definition of the derivative of j_0 at 0, *i.e.*

$$\lim_{q \rightarrow 0} |f_{[0,\Delta]}(q, \epsilon)| \leq \Delta |j_1(0)|. \quad (16)$$

Since $j_1(0) = 0$ we conclude that for *any finite* Δ one has $\lim_{q \rightarrow 0} |f_{[0,\Delta]}(q, \epsilon)| = 0$, and that this occurs for any finite ϵ . We now take the limit $\epsilon \rightarrow 0$ and obviously obtain $\lim_{\epsilon \rightarrow 0} (\lim_{q \rightarrow 0} |f_{[0,\Delta]}(q, \epsilon)|) = 0$.

This directly contradicts the claim of Ref. [10]. It is also seen that only for $\Delta = \infty$ the above proof does not go through because then $q\Delta$ is ∞ for any finite q , and $|j_0(0) - j_0(q\Delta)| = |j_0(0) - j_0(\infty)| = |j_0(0)| = 1$. Thus, since for any finite Δ the contribution to the first term in Eq. (9) is 0 in the limit of $q \rightarrow 0$, the whole contribution must come from the second term in Eq. (9). Since Δ is arbitrary, the contribution comes from $r = \infty$. This can be checked by a direct evaluation of the second term in Eq. (9) for *any finite* Δ .

- (c2) Should someone be not convinced by the procedure of bounding the integrand in Eq. (13), one can perform the integral in Eq. (10) directly. Denoting $\delta = q\Delta$, $\epsilon' = q\sqrt{\epsilon}$ we have

$$\begin{aligned} f_{[0,\Delta]}(q, \epsilon) &= \int_0^\delta dz \, j_1(z) \operatorname{erf}\left(\frac{z}{2\epsilon'}\right) \\ &= -j_0(\delta)\operatorname{erf}\left(\frac{\delta}{2\epsilon'}\right) + \int_0^\delta dz \, j_0(z) \cdot \frac{2}{\sqrt{\pi}} \exp\left(-\frac{z^2}{4\epsilon'^2}\right) \frac{1}{2\epsilon'}. \end{aligned} \quad (17)$$

For small ϵ' the second term on the r.h.s. above receives contributions from small z only. Therefore we may expand $j_0(z)$ around $z = 0$:

$$j_0(z) \approx 1 - \frac{1}{6}z^2 + \dots \quad (18)$$

and perform the integrations. We obtain

$$\begin{aligned} &\frac{2}{\sqrt{\pi}} \cdot \frac{1}{2\epsilon'} \int_0^\delta dz \, 1 - \frac{z^2}{6} \exp\left(\frac{-z^2}{4\epsilon'^2}\right) \\ &= \frac{2}{\sqrt{\pi}} \int_0^{\delta/(2\epsilon')} dt \, \exp(-t^2) - \frac{1}{6} \cdot \frac{2}{\sqrt{\pi}} (2\epsilon')^2 \int_0^{\delta/(2\epsilon')} dt \, t^2 \exp(-t^2). \end{aligned} \quad (19)$$

The integral in the second term in Eq. (19) may be evaluated as

$$\frac{2}{\sqrt{\pi}} \left[-\frac{d}{d\lambda} \int_0^{\delta/(2\epsilon')} dt \, \exp(-\lambda t^2) \right]_{\lambda=1}$$

$$\begin{aligned}
 &= -\frac{d}{d\lambda} \left[\lambda^{-1/2} \operatorname{erf} \left(\frac{\delta \lambda^{1/2}}{2\epsilon'} \right) \right]_{\lambda=1} \\
 &= -\frac{1}{2} \operatorname{erf} \left(\frac{\delta}{2\epsilon'} \right) + \frac{2}{\sqrt{\pi}} \frac{\delta}{4\epsilon'} \exp \left(\frac{-\delta^2}{4\epsilon'^2} \right). \tag{20}
 \end{aligned}$$

Putting together Eqs. (13)–(20) one obtains

$$f_{[0,\Delta]}(q, \epsilon) = \left(1 - j_0(\delta) - \frac{\epsilon'^2}{3} \right) \operatorname{erf} \left(\frac{\delta}{2\epsilon'} \right) + \frac{\epsilon'^2}{3} \frac{2}{\sqrt{\pi}} \frac{\delta}{2\epsilon'} \exp \left(\frac{-\delta^2}{4\epsilon'^2} \right). \tag{21}$$

We now recall that $\delta/\epsilon' = \Delta/\sqrt{\epsilon}$ and that we are interested in the limit $\epsilon \rightarrow 0$ for any finite Δ . For very large (but finite) δ and small ϵ' we have $j_0(\delta) \approx 0$, $\operatorname{erf}(\delta/(2\epsilon')) \approx 1$, and

$$\frac{\delta}{2\epsilon'} \exp \left(\frac{-\delta^2}{4\epsilon'^2} \right) \approx 0. \tag{22}$$

Eq. (21) reduces then to

$$f_{[0,\Delta]}(q, \epsilon) \approx 1 - \frac{\epsilon'^2}{3} \tag{23}$$

approaching 1 from below in agreement with Eq. (8) and Fig. 1.

For $\epsilon \rightarrow 0$ and fixed Δ one obtains from Eq. (21)

$$\lim_{\epsilon \rightarrow 0} f_{[0,\Delta]}(q, \epsilon) = 1 - j_0(q\Delta). \tag{24}$$

Clearly, the contribution to the integral in Eq. (5) coming from the interval $[0, \Delta]$ is small and goes to zero when $q\Delta \rightarrow 0$. Thus, for *any finite* Δ , in the limit $q \rightarrow 0$ the contribution to the integral in Eq. (5) comes entirely from the second term in Eq. (5). Since Δ is arbitrary, the contribution comes from $r = \infty$.

5. Final remarks

In summary, violation of Hara's theorem may occur for conserved current as shown in Ref. [9]. One has to pay a price, though: the price is the lack of sufficient localizability of the current. This connection to the *physical* issue of locality has been already suggested in [5]. Thus, violation of Hara's theorem would require a highly non-orthodox resolution. Whether this is a physically reasonable option constitutes a completely separate question.

However, one should remember that what is “physically reasonable” is determined by *experiment* and not by our preconceived ideas about what the world looks like. After all, *all* our fundamental ideas are abstracted from experiment. They do not live their own independent life and must be modified if experiment proves their deficiencies.

In general, we should try to avoid non-orthodox physics as long as we can. The problem is, however, that there are various theoretical, phenomenological, experimental and even philosophical hints that, despite expectations based on standard views, Hara’s theorem may be violated. It is therefore important to ask and answer the question whether one can provide a single and clearcut test, the results of which would unambiguously resolve the issue.

In fact, as already mentioned in the introduction, such a test has been pointed out in [5] (see also [12, 13]). It was shown there that the issue can be experimentally settled by measuring the asymmetry of the $\Xi^0 \rightarrow \Lambda\gamma$ decay. The sign of this asymmetry is strongly correlated with the answer to the question of the violation of Hara’s theorem in $\Sigma^+ \rightarrow p\gamma$. In Hara’s-theorem-satisfying models this asymmetry is negative and around -0.7 . On the contrary, in Hara’s-theorem-violating models this asymmetry is positive and of the same absolute size, (*i.e.* it is around $+0.7$). Present data is $+0.43 \pm 0.44$. The KTeV experiment at Fermilab has 1000 events of $\Xi^0 \rightarrow \Lambda\gamma$ [14]. These data are being analysed. Thus, the question of the violation of Hara’s theorem should be experimentally settled soon.

If the results of the KTeV experiment (and those of an even higher statistics experiment being performed by the NA48 collaboration at CERN [15]) confirm large positive asymmetry for the $\Xi^0 \rightarrow \Lambda\gamma$ decay, one should start to discuss the possible deeper physical meaning of the violation of Hara’s theorem. I tried to refrain from such a discussion so far.

On the other hand, if the asymmetry in the $\Xi^0 \rightarrow \Lambda\gamma$ decay is negative, one must conclude that Hara’s theorem holds in Nature. In this case, however, it follows that either vector meson dominance is inapplicable to weak radiative hyperon decays or our present understanding of nuclear parity violation (Ref. [6]) is incorrect.

In conclusion, whatever sign of asymmetry is measured in the $\Xi^0 \rightarrow \Lambda\gamma$ decay, something well accepted will have to be discarded.

I would like to thank A. Horzela for providing reference [11] and J. Lach and A. Horzela for discussions regarding the presentation of the argument. Comments on the presentation of the material of this paper, received from V. Dmitrasinovic prior to paper’s dissemination, are also gratefully acknowledged.

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