CONSISTENT INTERACTIONS IN THE HAMILTONIAN BRST FORMALISM

C. Bizdadea

Faculty of Physics, University of Craiova
13 A.I. Cuza Str., Craiova 1100, Romania
e-mail: bizdadea@central.ucv.ro

(Received June 18, 2001)

A Hamiltonian BRST deformation procedure for obtaining consistent interactions among fields with gauge freedom is proposed. The general theory is exemplified on the three-dimensional Chern–Simons models and two-dimensional nonlinear gauge theories.

PACS numbers: 11.10.Ef

1. Introduction

The analysis of consistent interactions that can be introduced among fields with gauge freedom without changing the number of gauge symmetries [1–4] has been transposed lately at the level of the deformation of the master equation [5] from the antifield-BRST formalism [6–10]. This cohomological deformation technique has been applied, among others, to Chern–Simons models [5], Yang–Mills theories [11], p-form gauge theories, and chiral p-forms [12–19]. In this light, the antifield-BRST method was proved to be an elegant tool for investigating the problem of consistent interactions. On the other hand, the Hamiltonian formulation [10,20–24] appears to be the most natural background for investigating various topics in gauge theories, such as the implementation of the BRST symmetry in quantum mechanics [10], the analysis of anomalies [25], the link between the local BRST cohomologies in both Lagrangian and Hamiltonian formalisms [26] (see Theorem 6 from this reference), or for establishing a proper connection between the BRST symmetry and canonical quantization methods [27]. These considerations strongly stimulate a Hamiltonian BRST approach to other interesting problems.

In this paper we analyze the problem of constructing consistent interactions among fields with gauge freedom in the framework of the Hamiltonian

BRST formalism. Our strategy includes two main steps: *(i)* initially, we show that the problem of introducing consistent interactions among fields with gauge freedom can be reformulated as a problem of deforming the BRST charge and the BRST-invariant Hamiltonian of a given "free" theory, and consequently we deduce the general equations that govern these two types of deformations; *(ii)* next, on behalf of the relationship between the Hamiltonian and antifield BRST formalisms for constrained systems, we prove that the general equations possess solutions. In the sequel, we reformulate the general equations in a manner that accounts for locality, and subsequently illustrate our general procedure in the case of three-dimensional Chern-Simons models and two-dimensional nonlinear gauge theories.

2. General equations of the Hamiltonian deformation approach

We begin with a system described by the canonical variables z^A, subject to the first-class constraints

$$G_{a_0}\left(z^A\right) \approx 0\,, \quad a_0 = 1, \ldots, M_0 \tag{1}$$

which are assumed to be L-stage reducible

$$G_{a_0} Z^{a_0}_{\ a_1} = 0\,, \quad a_1 = 1, \ldots, M_1\,, \tag{2}$$

$$Z^{a_{k-2}}_{\ a_{k-1}} Z^{a_{k-1}}_{\ a_k} \approx 0\,, \quad a_k = 1, \ldots, M_k, \quad k = 2, \ldots, L\,, \tag{3}$$

and suppose that there are no second-class constraints in the theory. The Grassmann parities of the canonical variables and first-class constraints are respectively denoted by $\varepsilon\left(z^A\right) = \varepsilon_A$ and $\varepsilon\left(G_{a_0}\right) = \varepsilon_{a_0}$. We denote the first-class Hamiltonian by H_0, such that the gauge algebra is expressed by

$$\left[G_{a_0}, G_{b_0}\right] = G_{c_0} C^{c_0}_{\ a_0 b_0}\,, \quad \left[H_0, G_{a_0}\right] = G_{b_0} V^{b_0}_{\ a_0}\,. \tag{4}$$

It is known that a constrained Hamiltonian system can be described by the action

$$S_0\left[z^A, u^{a_0}\right] = \int_{t_1}^{t_2} dt\, \left(a_A\left(z\right)\dot{z}^A - H_0 - G_{a_0}u^{a_0}\right), \tag{5}$$

where the Grassmann parities of the Lagrange multipliers are given by $\varepsilon\left(u^{a_0}\right) = \varepsilon_{a_0}$. In (5), $a_A\left(z\right)$ is the one-form potential that gives the symplectic two-form

$$\omega_{AB} = (-)^{\varepsilon_A+1}\frac{\partial^L a_A}{\partial z^B} + (-)^{\varepsilon_B(\varepsilon_A+1)}\frac{\partial^L a_B}{\partial z^A}\,,$$

whose inverse, ω^{AB}, corresponds to the fundamental Dirac brackets $\left[z^A, z^B\right] = \omega^{AB}$. Action (5) is invariant under the gauge transformations

$$\delta_\epsilon z^A = \left[z^A, G_{a_0}\right] \epsilon^{a_0},\ \delta_\epsilon u^{a_0} = \dot{\epsilon}^{a_0} - V^{a_0}_{\ \ b_0}\epsilon^{b_0} - C^{a_0}_{\ \ b_0 c_0}\epsilon^{c_0}u^{b_0} - Z^{a_0}_{\ \ a_1}\epsilon^{a_1}. \quad (6)$$

In order to generate consistent interactions at the Hamiltonian level, we deform the action (5) by adding to it some interaction terms

$$S_0 \to \tilde{S}_0 = S_0 + g \overset{(1)}{S}_0 + g^2 \overset{(2)}{S}_0 + \cdots , \quad (7)$$

and modify the gauge transformations (6) (to be denoted by $\tilde{\delta}_\epsilon z^A$, $\tilde{\delta}_\epsilon u^{a_0}$) in such a way that the deformed gauge transformations leave invariant the new action

$$\frac{\delta^R \tilde{S}_0}{\delta z^A}\tilde{\delta}_\epsilon z^A + \frac{\delta^R \tilde{S}_0}{\delta u^{a_0}}\tilde{\delta}_\epsilon u^{a_0} = 0 \, . \quad (8)$$

Consequently, the deformation of the action (5) and of the gauge transformations (6) produces a deformation of the first-class constraints, first-class Hamiltonian, and accompanying structure functions like

$$G_{a_0} \to \gamma_{a_0} = G_{a_0} + g \overset{(1)}{\gamma}_{a_0} + g^2 \overset{(2)}{\gamma}_{a_0} + \cdots , \quad (9)$$

$$H_0 \to H = H_0 + g \overset{(1)}{H} + g^2 \overset{(2)}{H} + \cdots , \quad (10)$$

$$V^{a_0}_{\ \ b_0} \to \tilde{V}^{a_0}_{\ \ b_0} = V^{a_0}_{\ \ b_0} + g \overset{(1)}{V}{}^{a_0}_{\ \ b_0} + g^2 \overset{(2)}{V}{}^{a_0}_{\ \ b_0} + \cdots , \quad (11)$$

$$C^{a_0}_{\ \ b_0 c_0} \to \tilde{C}^{a_0}_{\ \ b_0 c_0} = C^{a_0}_{\ \ b_0 c_0} + g \overset{(1)}{C}{}^{a_0}_{\ \ b_0 c_0} + g^2 \overset{(2)}{C}{}^{a_0}_{\ \ b_0 c_0} + \cdots , \quad (12)$$

such that the deformed gauge algebra becomes

$$[\gamma_{a_0}, \gamma_{b_0}] = \gamma_{c_0}\tilde{C}^{c_0}_{\ \ a_0 b_0} , \quad [H, \gamma_{a_0}] = \gamma_{b_0}\tilde{V}^{b_0}_{\ \ a_0} \, . \quad (13)$$

In the meantime, we deform the reducibility relations, but we do not explicitly write down these relations.

As the BRST charge and BRST-invariant Hamiltonian contain all the information on the gauge structure of a given theory, we can reformulate the problem of introducing consistent interactions within the Hamiltonian BRST context in terms of these two essential compounds. Indeed, if the interactions can be consistently constructed, then the BRST charge of the undeformed theory, $\overset{(0)}{\Omega}$, can be deformed such as to be the BRST charge of the deformed theory, *i.e.*,

$$\overset{(0)}{\Omega} \to \Omega = \overset{(0)}{\Omega} + g \overset{(1)}{\Omega} + g^2 \overset{(2)}{\Omega} + \cdots , \quad (14)$$

$$[\Omega, \Omega] = 0\,. \tag{15}$$

Equation (15) can be analyzed order by order in the deformation parameter g, leading to

$$\left[\overset{(0)}{\Omega}, \overset{(0)}{\Omega}\right] = 0\,, \tag{16}$$

$$2\left[\overset{(0)}{\Omega}, \overset{(1)}{\Omega}\right] = 0\,, \tag{17}$$

$$2\left[\overset{(0)}{\Omega}, \overset{(2)}{\Omega}\right] + \left[\overset{(1)}{\Omega}, \overset{(1)}{\Omega}\right] = 0\,, \tag{18}$$

$$\vdots$$

At the same time, the deformation of the BRST charge induces the deformation of the BRST-invariant Hamiltonian of the undeformed theory, $\overset{(0)}{H}_B$,

$$\overset{(0)}{H}_B \rightarrow H_B = \overset{(0)}{H}_B + g\overset{(1)}{H}_B + g^2\overset{(2)}{H}_B + \cdots\,, \tag{19}$$

in such a way that H_B is the BRST-invariant Hamiltonian of the interacting theory, *i.e.*,

$$[H_B, \Omega] = 0\,. \tag{20}$$

The equation (20) splits, according to the powers of the deformation parameter, as

$$\left[\overset{(0)}{H}_B, \overset{(0)}{\Omega}\right] = 0\,, \tag{21}$$

$$\left[\overset{(0)}{H}_B, \overset{(1)}{\Omega}\right] + \left[\overset{(1)}{H}_B, \overset{(0)}{\Omega}\right] = 0\,, \tag{22}$$

$$\left[\overset{(0)}{H}_B, \overset{(2)}{\Omega}\right] + \left[\overset{(1)}{H}_B, \overset{(1)}{\Omega}\right] + \left[\overset{(2)}{H}_B, \overset{(0)}{\Omega}\right] = 0\,, \tag{23}$$

$$\vdots$$

Equations (16)–(18), *etc.* and (21)–(23), *etc.* stand for the general equations of our deformation procedure. With the help of their solutions we can reach the Hamiltonian version of the interacting theory. More precisely, from the deformed BRST charge one identifies the deformed first-class constraints, their corresponding algebra, the new reducibility relations, *etc.* In the meantime, from the deformed BRST-invariant Hamiltonian one draws the new first-class Hamiltonian, the Dirac brackets among the deformed first-class constraints and this first-class Hamiltonian, *etc.* The equations (16) and (21) are checked by hypothesis. Then, it appears naturally the question whether the remaining equations possess solutions. This will be investigated in the next section.

3. Solution to the general equations

In order to prove that the equations (17)–(18), *etc.* and (22)–(23), *etc.* possess solutions, we use the link between the antifield and Hamiltonian BRST formalisms for constrained Hamiltonian systems [28]. First-class constrained Hamiltonian systems can be approached from the point of view of the BRST formalism in two different manners. One is based on the antibracket–antifield formulation [6–10], while the other relies on the standard Hamiltonian BRST treatment [10, 20–24]. The starting point of the antibracket–antifield formalism is represented by the invariance of the action (5) under the gauge transformations (6). In agreement with the general prescriptions of the antibracket–antifield procedure, we introduce the ghosts $\left(\eta^{a_{k-1}}\right)_{k=1,\ldots,L+1}$ and $\left(u^{a_k}\right)_{k=1,\ldots,L}$, with

$$\varepsilon\left(\eta^{a_k}\right)=\left(\varepsilon_{a_k}+k+1\right) \bmod 2,\ \mathrm{gh}\left(\eta^{a_k}\right)=k+1,\ k=0,\ldots,L, \tag{24}$$

$$\varepsilon\left(u^{a_k}\right)=\left(\varepsilon_{a_k}+k\right) \bmod 2,\ \mathrm{gh}\left(u^{a_k}\right)=k,\ k=1,\ldots,L\,, \tag{25}$$

where gh denotes the ghost number. The antifields associated with the fields $\left(z^A,u^{a_0},\eta^{a_{k-1}},u^{a_k}\right)$ are denoted by $\left(z^*_A,u^*_{a_0},\eta^*_{a_{k-1}},u^*_{a_k}\right)$ and display the properties $\varepsilon\,(\text{antifield})=\varepsilon\,(\text{field})+1$, $\mathrm{gh}\,(\text{antifield})=-\mathrm{gh}\,(\text{field})-1$. Up to terms that are quadratic in the antifields, the solution to the master equation reads as

$$\begin{aligned}\overset{(0)}{S} = \int_{t_1}^{t_2} dt \Bigg(& a_A(z)\,\dot{z}^A+\sum_{k=0}^{L} u^*_{a_k}\dot{\eta}^{a_k}-H_0-G_{a_0}u^{a_0}+z^*_A\left[z^A,G_{a_0}\right]\eta^{a_0} \\ & -u^*_{a_0}V^{a_0}_{\ \ b_0}\eta^{b_0}+(-)^{\varepsilon_{b_0}+1}u^*_{a_0}C^{a_0}_{\ \ b_0c_0}\eta^{c_0}u^{b_0}+\frac{1}{2}(-)^{\varepsilon_{b_0}}\eta^*_{a_0}C^{a_0}_{\ \ b_0c_0}\eta^{c_0}\eta^{b_0} \\ & +\sum_{k=0}^{L-1}\eta^*_{a_k}Z^{a_k}_{\ \ a_{k+1}}\eta^{a_{k+1}}-\sum_{k=1}^{L}u^*_{a_{k-1}}Z^{a_{k-1}}_{\ \ a_k}u^{a_k}+\ldots\Bigg)\,.\end{aligned} \tag{26}$$

The Hamiltonian point of view is based on extending the phase-space by introducing the canonical pairs ghost–antighost $\left(\eta^{a_k},\mathcal{P}_{a_k}\right)$, with $\left[\eta^{a_k},\mathcal{P}_{a_k}\right]=\delta^{a_k}_{\ \ b_k}$ and $\varepsilon\left(\mathcal{P}_{a_k}\right)=\left(\varepsilon_{a_k}+k+1\right) \bmod 2$, $\mathrm{gh}\left(\mathcal{P}_{a_k}\right)=k+1$. The BRST charge starts like

$$\overset{(0)}{\Omega}=G_{a_0}\eta^{a_0}+\tfrac{1}{2}(-)^{\varepsilon_{b_0}}\mathcal{P}_{a_0}C^{a_0}_{\ \ b_0c_0}\eta^{c_0}\eta^{b_0}+\sum_{k=0}^{L-1}\mathcal{P}_{a_k}Z^{a_k}_{\ \ a_{k+1}}\eta^{a_{k+1}}+\cdots, \tag{27}$$

such that $\left[\overset{(0)}{\Omega},\overset{(0)}{\Omega}\right]=0$. The BRST-invariant extension of H_0

$$\overset{(0)}{H}_B = H_0 + \mathcal{P}_{a_0} V^{a_0}_{\ \ b_0} \eta^{b_0} + \cdots, \tag{28}$$

satisfies the equation $\left[\overset{(0)}{H}_B,\overset{(0)}{\Omega}\right]=0$. By employing the identifications

$$u^*_{a_k} = \mathcal{P}_{a_k},\ k=0,\ldots,L, \tag{29}$$

and extending the Dirac bracket such that $\left[\eta^{a_k},u^*_{a_k}\right]=\delta^{a_k}_{\ \ b_k}$, we get that

$$\begin{aligned}\frac{1}{2}\left(\overset{(0)}{S},\overset{(0)}{S}\right) &= \int_{t_1}^{t_2} dt\left(-\frac{d}{dt}\overset{(0)}{\Omega}-\left[\overset{(0)}{H}_B,\overset{(0)}{\Omega}\right]+\frac{1}{2}z^*_A\left[z^A,\left[\overset{(0)}{\Omega},\overset{(0)}{\Omega}\right]\right]\right.\\ &\left.+\frac{1}{2}\sum_{k=0}^{L}\eta^*_{a_k}\left[\eta^{a_k},\left[\overset{(0)}{\Omega},\overset{(0)}{\Omega}\right]\right]+\frac{1}{2}\sum_{k=0}^{L}\left[\left[\overset{(0)}{\Omega},\overset{(0)}{\Omega}\right],u^*_{a_k}\right]u^{a_k}\right). \end{aligned}\tag{30}$$

The deformations (14) and (19) induce a deformation of the solution to the master equation

$$\overset{(0)}{S}\rightarrow S=\overset{(0)}{S}+g\overset{(1)}{S}+g^2\overset{(2)}{S}+\cdots, \tag{31}$$

such that the equation (30) for the deformed theory becomes

$$\begin{aligned}\frac{1}{2}\left(S,S\right) &= \int_{t_1}^{t_2} dt\left(-\frac{d}{dt}\Omega-\left[H_B,\Omega\right]+\frac{1}{2}z^*_A\left[z^A,\left[\Omega,\Omega\right]\right]\right.\\ &\left.+\frac{1}{2}\sum_{k=0}^{L}\eta^*_{a_k}\left[\eta^{a_k},\left[\Omega,\Omega\right]\right]+\frac{1}{2}\sum_{k=0}^{L}\left[\left[\Omega,\Omega\right],u^*_{a_k}\right]u^{a_k}\right). \end{aligned}\tag{32}$$

The equation (32) splits according to the deformation parameter as (30) and

$$\begin{aligned}\left(\overset{(0)}{S},\overset{(1)}{S}\right) &= \int_{t_1}^{t_2} dt\left(-\frac{d}{dt}\overset{(1)}{\Omega}-\left[\overset{(0)}{H}_B,\overset{(1)}{\Omega}\right]-\left[\overset{(1)}{H}_B,\overset{(0)}{\Omega}\right]+z^*_A\left[z^A,\left[\overset{(0)}{\Omega},\overset{(1)}{\Omega}\right]\right]\right.\\ &\left.+\sum_{k=0}^{L}\eta^*_{a_k}\left[\eta^{a_k},\left[\overset{(0)}{\Omega},\overset{(1)}{\Omega}\right]\right]+\sum_{k=0}^{L}\left[\left[\overset{(0)}{\Omega},\overset{(1)}{\Omega}\right],u^*_{a_k}\right]u^{a_k}\right), \end{aligned}\tag{33}$$

$$\begin{aligned}\left(\overset{(0)}{S},\overset{(2)}{S}\right)+\frac{1}{2}\left(\overset{(1)}{S},\overset{(1)}{S}\right) &= \int_{t_1}^{t_2} dt \left(-\frac{d}{dt}\overset{(2)}{\Omega}-\left[\overset{(0)}{H}_B,\overset{(2)}{\Omega}\right]-\left[\overset{(1)}{H}_B,\overset{(1)}{\Omega}\right]\right.\\ &\quad -\left[\overset{(2)}{H}_B,\overset{(0)}{\Omega}\right]+z_A^*\left[z^A,\left[\overset{(0)}{\Omega},\overset{(2)}{\Omega}\right]+\frac{1}{2}\left[\overset{(1)}{\Omega},\overset{(1)}{\Omega}\right]\right]\\ &\quad +\sum_{k=0}^{L}\eta^*_{a_k}\left[\eta^{a_k},\left[\overset{(0)}{\Omega},\overset{(2)}{\Omega}\right]+\frac{1}{2}\left[\overset{(1)}{\Omega},\overset{(1)}{\Omega}\right]\right]\\ &\quad \left.+\sum_{k=0}^{L}\left[\left[\overset{(0)}{\Omega},\overset{(2)}{\Omega}\right]+\frac{1}{2}\left[\overset{(1)}{\Omega},\overset{(1)}{\Omega}\right],u^*_{a_k}\right]u^{a_k}\right),\\ &\quad\vdots \end{aligned} \tag{34}$$

The last equations emphasize that the existence of $\overset{(1)}{S}$ guarantees the existence of $\overset{(1)}{\Omega}$ and $\overset{(1)}{H}_B$, the existence of $\overset{(2)}{S}$ guarantees the existence of $\overset{(2)}{\Omega}$ and $\overset{(2)}{H}_B$, and so on. Moreover, the equations (17)–(18), *etc.* and (22)–(23), *etc.* are equivalent to the equations $\left(\overset{(0)}{S},\overset{(1)}{S}\right)=0$, $\left(\overset{(0)}{S},\overset{(2)}{S}\right)+\frac{1}{2}\left(\overset{(1)}{S},\overset{(1)}{S}\right)=0$, *etc.* modulo imposing some appropriate boundary conditions on Ω [24]. On the other hand, the last equations possess solution. The existence of such solutions was proved in [5] on behalf of the triviality of the antibracket in the cohomology. Thus, the existence of the solutions in the antibracket proves the existence of the solutions to (17)–(18), *etc.* and (22)–(23), *etc.* In conclusion, the equations that describe the Hamiltonian deformation procedure possess solutions, so we can construct consistent Hamiltonian interactions by means of the equations (17)–(18), *etc.* and (22)–(23), *etc.*

At this point, we consider the interactions that can be obtained via a redefinition of the variables

$$z^A \to \bar{z}^A = z^A + g\lambda^A + \cdots . \tag{35}$$

Such a redefinition implies that the first-class constraint functions and the first-class Hamiltonian are transformed like

$$G_{a_0} \to \bar{G}_{a_0} = G_{a_0}\left(z^A+g\lambda^A+\cdots\right) = G_{a_0}+g\lambda^A\frac{\partial^L G_{a_0}}{\partial z^A}+\cdots, \tag{36}$$

$$H_0 \to \bar{H}_0 = H_0\left[z^A+g\lambda^A+\cdots\right] = H_0+g\lambda^A\frac{\delta^L H_0}{\delta z^A}+\cdots . \tag{37}$$

Obviously, the redefinition (35) modifies as well the other structure functions. The transformations (36)–(37) induce the changes

$$\overset{(0)}{\Omega} \rightarrow \Omega = \overset{(0)}{\Omega} + g\lambda^{A}\frac{\partial^{L}G_{a_0}}{\partial z^{A}}\eta^{a_0} + \cdots , \tag{38}$$

$$\overset{(0)}{H}_{B} \rightarrow H_{B} = \overset{(0)}{H}_{B} + g\lambda^{A}\frac{\delta^{L}H_{0}}{\delta z^{A}} + \cdots , \tag{39}$$

at the level of the BRST charge, respectively, of the BRST-invariant Hamiltonian. The interactions that can be eliminated by means of variable redefinitions are usually considered as no interactions and are called trivial interactions. Trivial interactions appear at the level of the solutions to the equations (17)–(18), *etc.* and (22)–(23), *etc.* as follows. The equation (17) implies that $\overset{(1)}{\Omega}$ is an $\overset{(0)}{s}$-co-cycle, where $\overset{(0)}{s}$ denotes the undeformed BRST differential, which decomposes like $\overset{(0)}{s} = \delta + \gamma + \cdots$, with δ the Koszul–Tate differential (graded by the antighost number, antigh) and γ the exterior derivative along the gauge orbits (graded by the pure ghost number, pgh). The overall degree of $\overset{(0)}{s}$, namely, the ghost number, is defined like the difference between the pure ghost number and the antighost number. We suppose that $\overset{(1)}{\Omega}$ is an $\overset{(0)}{s}$-coboundary

$$\overset{(1)}{\Omega} = \left[\overset{(1)}{\sigma}, \overset{(0)}{\Omega}\right] . \tag{40}$$

By expanding the right hand-side of the last relation according to the antighost number, we find

$$\overset{(1)}{\Omega} = u^{A}\frac{\partial^{L}G_{a_0}}{\partial z^{A}}\eta^{a_0} + \cdots , \tag{41}$$

where $u^{A} = \left.\frac{\delta^{R}\overset{(1)}{\sigma}}{\delta z^{B}}\omega^{BA}\right|_{\eta=\mathcal{P}=0}$, such that the solution (40) deforms in a trivial way the BRST charge (as (40) leads to a deformation of the same type with (38)). Using (40), we find

$$\left[\overset{(0)}{H}_{B}, \overset{(1)}{\Omega}\right] = -\left[\left[\overset{(1)}{\sigma}, \overset{(0)}{H}_{B}\right], \overset{(0)}{\Omega}\right] , \tag{42}$$

such that from (22) it results (up to an $\overset{(0)}{s}$-exact term) that

$$\overset{(1)}{H}_{B} = \left[\overset{(1)}{\sigma}, \overset{(0)}{H}_{B}\right] = \frac{\delta^{R}\overset{(1)}{\sigma}}{\delta\Phi^{\alpha}}\omega^{\alpha\beta}\frac{\delta^{L}\overset{(0)}{H}_{B}}{\delta\Phi^{\beta}} , \tag{43}$$

where $\Phi^\alpha = \left(z^A, \eta, \mathcal{P}\right)$, and $\omega^{\alpha\beta} = \left[\Phi^\alpha, \Phi^\beta\right]$. The expansion of the right hand-side of (43) according to the antighost number

$$\frac{\delta^R \overset{(1)}{\sigma}}{\delta\Phi^\alpha}\omega^{\alpha\beta}\frac{\delta^L \overset{(0)}{H}_B}{\delta\Phi^\beta} = u^A\frac{\delta^L H_0}{\delta z^A} + \cdots, \tag{44}$$

leads to a trivial deformation of the BRST-invariant Hamiltonian (of the same type with (39)). Moreover, it can be shown that (40) deforms the remaining structure functions also in a trivial manner. In conclusion, the trivial solutions (40) produce trivial interactions.

In practical applications, it is commonly required that the deformations should be local, *i.e.*, $\overset{(1)}{\Omega}$, $\overset{(2)}{\Omega}$, $\overset{(1)}{H}_B$, $\overset{(2)}{H}_B$, *etc.* have to be local functionals. Let $F_1 = \int d^{D-1}x f_1$ and $F_2 = \int d^{D-1}x f_2$ be two local functionals. If the Dirac bracket is local, then $[F_1, F_2]$ is local, namely, there exists a local $[f_1, f_2]$ (but defined up to a $(D-1)$-dimensional divergence), such that $[F_1, F_2] = \int d^{D-1}x\, [f_1, f_2]$ (if the Dirac bracket itself is nonlocal, the deformations will also be nonlocal). Thus, the equations (17)–(18), *etc.* and (22)–(23), *etc.* can be written as

$$\begin{aligned}
2\overset{(0)}{s}\overset{(1)}{\omega} &= \partial^k \overset{(1)}{j}_k\,, &(45)\\
2\overset{(0)}{s}\overset{(2)}{\omega} + \left[\overset{(1)}{\omega}, \overset{(1)}{\omega}\right] &= \partial^k \overset{(2)}{j}_k\,, &(46)\\
&\vdots\\
\overset{(0)}{s}\overset{(1)}{h}_B + \left[\overset{(0)}{h}_B, \overset{(1)}{\omega}\right] &= \partial^k \overset{(1)}{m}_k\,, &(47)\\
\overset{(0)}{s}\overset{(2)}{h}_B + \left[\overset{(1)}{h}_B, \overset{(1)}{\omega}\right] + \left[\overset{(0)}{h}_B, \overset{(2)}{\omega}\right] &= \partial^k \overset{(2)}{m}_k\,, &(48)\\
&\vdots
\end{aligned}$$

in terms of the integrands $\overset{(k)}{h}_B$ and $\overset{(k)}{\omega}$. Even if the Dirac bracket is local, there might however appear obstructions if one insists on the locality of deformations. For instance, even if $\left[\overset{(1)}{\Omega}, \overset{(1)}{\Omega}\right]$ is $\overset{(0)}{s}$-exact, it is not granted that it is the BRST variation of a local functional. Such locality problems appear also in the Lagrangian deformation procedure [5]. The analysis of such obstructions can be done with the help of cohomological techniques in terms of the cohomological group $H(s|\tilde{d})$, where $\tilde{d} = dx^i\partial_i$ represents the spatial part of the exterior space-time derivative. However, in the case of most important applications [5, 11–19], the Lagrangian BRST deformation procedure leads to local interactions. Thus, we expect that the Hamiltonian BRST deformation treatment also outputs local vertices in practical applications of interest.

4. Examples

4.1. Chern–Simons model

Let us exemplify the prior procedure in the case of Abelian Chern–Simons model in three dimensions. We start with the Lagrangian action

$$S_0\left[A_\mu^a\right]=\tfrac{1}{2}\int d^3x\varepsilon^{\mu\nu\rho}k_{ab}A_\mu^aF_{\nu\rho}^b\,, \tag{49}$$

where k_{ab} is a non-degenerate, symmetric, and constant matrix, while

$$F_{\nu\rho}^b=\partial_\nu A_\rho^b-\partial_\rho A_\nu^b\equiv\partial_{[\nu}A_{\rho]}^b\,.$$

Performing the canonical analysis and eliminating the second-class constraints (the independent variables are A_0^a, π_a^0, and A_k^a), we infer the first-class constraints

$$G_{1a}\equiv\pi_a^0\approx 0\,,\quad G_{2a}\equiv-\tfrac{1}{2}\varepsilon^{0ik}k_{ab}F_{ik}^b\approx 0\,, \tag{50}$$

and the first-class Hamiltonian

$$H_0=-2\int d^2xA_0^aG_{2a}\,. \tag{51}$$

The non-vanishing fundamental Dirac brackets read as $\left[A_0^a,\pi_b^0\right]=\delta^a{}_b$, $\left[A_k^a,A_j^b\right]=\frac{1}{2}\varepsilon_{0kj}k^{ab}$, hence the BRST charge takes the form

$$\overset{(0)}{\Omega}=\int d^2x\left(\pi_a^0\eta_1^a-\tfrac{1}{2}\varepsilon^{0ik}k_{ab}F_{ik}^b\eta_2^a\right)\,, \tag{52}$$

where k^{ab} is the inverse of k_{ab}, and (η_1^a,η_2^a) stand for the fermionic ghost number one ghosts. Thus, the BRST operator $\overset{(0)}{s}$ splits as $\overset{(0)}{s}=\delta+\gamma$. Then, we have

$$\delta A_0^a = 0\,,\qquad \delta\pi_a^0=0\,,\quad \delta A_k^a=0\,,\qquad \delta\eta_1^a=\delta\eta_2^a=0\,, \tag{53}$$

$$\delta\mathcal{P}_{1a} = -\pi_a^0\,,\quad \delta\mathcal{P}_{2a}=\tfrac{1}{2}\varepsilon^{0ik}k_{ab}F_{ik}^b\,, \tag{54}$$

$$\gamma A_0^a = \eta_1^a\,,\qquad \gamma\pi_a^0=0\,,\quad \gamma A_k^a=\tfrac{1}{2}\partial_k\eta_2^a\,,\quad \gamma\eta_1^a=\gamma\eta_2^a=0\,, \tag{55}$$

$$\gamma\mathcal{P}_{1a} = \gamma\mathcal{P}_{2a}=0\,. \tag{56}$$

In (54) and (56), $\mathcal{P}_{1a}$ and $\mathcal{P}_{2a}$ stand for fermionic antighosts corresponding to the ghosts η_1^a, respectively, η_2^a. The pure ghost and antighost numbers of the variables from the BRST complex are valued like

$$\mathrm{pgh}\left(z^A\right) = 0\,, \quad \mathrm{pgh}\left(\eta^\Gamma\right) = 1\,, \quad \mathrm{pgh}\left(\mathcal{P}_\Gamma\right) = 0\,, \tag{57}$$

$$\mathrm{antigh}\left(z^A\right) = 0\,, \ \mathrm{antigh}\left(\eta^\Gamma\right) = 0\,, \ \mathrm{antigh}\left(\mathcal{P}_\Gamma\right) = 1\,, \tag{58}$$

where

$$z^A = \left(A_\mu^a, \pi_a^0\right)\,, \quad \eta^\Gamma = \left(\eta_1^a, \eta_2^a\right)\,, \quad \mathcal{P}_\Gamma = \left(\mathcal{P}_{1a}, \mathcal{P}_{2a}\right). \tag{59}$$

Now, we solve the equation (45). In view of this, we develop $\overset{(1)}{\omega}$ according to the antighost number

$$\overset{(1)}{\omega} = \overset{(1)}{\omega}_0 + \overset{(1)}{\omega}_1 + \cdots + \overset{(1)}{\omega}_J\,, \quad \mathrm{antigh}\left(\overset{(1)}{\omega}_J\right) = J\,, \quad \mathrm{gh}\left(\overset{(1)}{\omega}_J\right) = 1, \tag{60}$$

where the last term in (60) can be assumed to be annihilated by γ, *i.e.*, $\gamma \overset{(1)}{\omega}_J = 0$. Thus, in order to compute the first-order deformation of the BRST charge, we need to know $H(\gamma)$. Analysing the definitions (55)–(56), we remark that the cohomology of γ will be generated by F_{ij}^a, π_a^0, $\mathcal{P}_\Gamma$ and their spatial derivatives, as well as by the undifferentiated ghosts η_2^a (the ghosts η_2^a are γ-closed, but their spatial derivatives are γ-exact, while the ghosts η_1^a are trivial in the cohomology of γ as they are γ-exact). Consequently, the general solution of the equation $\gamma\alpha = 0$ can be written as

$$\alpha = \alpha_M\left(\left[F_{ij}^a\right], \left[\pi_a^0\right], \left[\mathcal{P}_\Gamma\right]\right) \mathrm{e}^M\left(\eta_2^a\right) + \gamma\beta\,, \tag{61}$$

where $\mathrm{e}^M\left(\eta_2^a\right)$ constitutes a basis in the (finite-dimensional) space of the polynomials in the ghosts η_2^a, while the notation $\alpha[q]$ signifies that α depends on q and its spatial derivatives up to a finite order. As $\mathrm{pgh}\left(\overset{(1)}{\omega}_J\right) = J+1$, from (61) it results that we can represent $\overset{(1)}{\omega}_J$ under the form

$$\overset{(1)}{\omega}_J = \frac{1}{(J+1)!}\mu_{a_1\cdots a_{J+1}}\eta_2^{a_1}\cdots\eta_2^{a_{J+1}}. \tag{62}$$

With this choice, it is simply to see that the γ-invariant coefficient $\mu_{a_1\cdots a_{J+1}}$ belongs to $H_J\left(\delta|\tilde{d}\right)$, hence it is solution to the equation

$$\delta\mu_{a_1\cdots a_{J+1}} + \partial_k b^k_{a_1\cdots a_{J+1}} = 0\,, \tag{63}$$

for some $b^{k}_{a_1\cdots a_{J+1}}$. Using the result from [29] adapted to the Hamiltonian context, it follows that $H_J\left(\delta|\tilde{d}\right)$ vanishes for $J>1$, so we can write that $\overset{(1)}{\omega}=\overset{(1)}{\omega}_0+\overset{(1)}{\omega}_1$, with $\overset{(1)}{\omega}_1=\frac{1}{2}\mu_{ab}\eta_2^a\eta_2^b$, where μ_{ab} pertains to $H_1\left(\delta|\tilde{d}\right)$. From the latter equations in (54) we have that the general representative of $H_1\left(\delta|\tilde{d}\right)$ is of the type $\mu_{ab}=C^{c}{}_{ab}\mathcal{P}_{2c}$, where $C^{c}{}_{ab}$ are some constants, antisymmetric in the lower indices, $C^{c}{}_{ab}=-C^{c}{}_{ba}$. The reason for considering $C^{c}{}_{ab}$ to be constant results from the equation that must be obeyed by μ_{ab}, namely, $\delta\mu_{ab}=\partial_k\left(C^{c}{}_{ab}\varepsilon^{0kj}k_{cd}A_j^d\right)$. In this way, we obtained that $\overset{(1)}{\omega}_1=\frac{1}{2}C^{c}{}_{ab}\mathcal{P}_{2c}\eta_2^a\eta_2^b$. Equation (45) at antighost number zero reads as $\delta\overset{(1)}{\omega}_1+\gamma\overset{(1)}{\omega}_0=\partial_k n^k$, which further yields $\overset{(1)}{\omega}_0=C^{c}{}_{ad}k_{cb}\varepsilon^{0kj}A_k^aA_j^d\eta_2^b$. Consequently, we inferred that the complete first-order deformation of the BRST charge is pictured by

$$\overset{(1)}{\omega}=C^{c}{}_{ab}\left(\frac{1}{2}\mathcal{P}_{2c}\eta_2^a\eta_2^b+k_{cd}\varepsilon^{0kj}A_k^aA_j^b\eta_2^d\right)\,. \tag{64}$$

Simple computation leads to

$$\left[\overset{(1)}{\Omega},\overset{(1)}{\Omega}\right]=\int d^2x\left(-\frac{1}{3}C^{c}{}_{[ab}C^{m}{}_{n]c}\mathcal{P}_{2m}\eta_2^a\eta_2^b\eta_2^n-\varepsilon^{0ij}k_{ad}C^{c}{}_{[ne}C^{d}{}_{b]c}\eta_2^a\eta_2^bA_i^nA_j^e\right)\,. \tag{65}$$

The last relation shows that $\left[\overset{(1)}{\Omega},\overset{(1)}{\Omega}\right]$ cannot be written like an $\overset{(0)}{s}$-exact modulo $\tilde{d}$ local functional, as required by (46). For this reason it is necessary to have $\left[\overset{(1)}{\Omega},\overset{(1)}{\Omega}\right]=0$. This condition takes place if and only if $C^{c}{}_{[ab}C^{m}{}_{n]c}=0$, so if and only if these constants verify the Jacobi identity. This further implies that $\overset{(k)}{\Omega}=0$ for all $k\geq 2$. Thus, the deformed BRST charge, consistent to all orders in the deformation parameter, takes the final form

$$\Omega=\int d^2x\left(\pi_a^0\eta_1^a-\varepsilon^{0ik}k_{ca}\left(\frac{1}{2}F_{ik}^c-gC^{c}{}_{bd}A_i^bA_k^d\right)\eta_2^a+\frac{1}{2}gC^{c}{}_{ab}\mathcal{P}_{2c}\eta_2^a\eta_2^b\right), \tag{66}$$

and it is clearly a local functional.

Next, we derive the deformed BRST-invariant Hamiltonian. The BRST-invariant Hamiltonian for the free model is given by $\overset{(0)}{H}_B=H_0+2\int d^2x\eta_1^a\mathcal{P}_{2a}$, such that with the help of (64) we find

$$\left[\overset{(0)}{h}_B,\overset{(1)}{\omega}\right]=-2C^{c}{}_{ab}k_{cd}\varepsilon^{0ij}A_j^b\left(\eta_1^dA_i^a+\eta_2^d\partial_iA_0^a\right)-2C^{c}{}_{ab}\mathcal{P}_{2c}\eta_2^a\eta_1^b\,. \tag{67}$$

Under these circumstances, the solution to the equation (47) reads as

$$\overset{(1)}{h}_B = 2C^c{}_{ab}\left(k_{cd}\varepsilon^{0ij}A_0^d A_i^a A_j^b + A_0^b \mathcal{P}_{2c}\eta_2^a\right). \tag{68}$$

Straightforward computation gives $\left[\overset{(1)}{H}_B, \overset{(1)}{\Omega}\right] = 0$, hence equation (48) is satisfied with the choice $\overset{(2)}{h}_B = 0$. Therefore, the higher-order deformation equations for the BRST-invariant Hamiltonian are verified with $\overset{(3)}{H}_B = \overset{(4)}{H}_B = \cdots = 0$. Combining the last results, we can write down the complete deformed BRST-invariant Hamiltonian like

$$H_B = 2\int d^2x\left(-A_0^a\varepsilon^{0ik}k_{ca}\left(\tfrac{1}{2}F_{ik}^c - gC^c{}_{bd}A_i^b A_k^d\right) + \left(\eta_1^a - gC^a{}_{cb}A_0^b\eta_2^c\right)\mathcal{P}_{2a}\right), \tag{69}$$

hence it is also a local functional.

Taking into account (66) and (69), we can proceed to the identification of the new gauge theory. From the antighost-independent terms in (66) we observe that the deformation of the BRST charge implies the deformed first-class constraints

$$\gamma_{2a} \equiv -\varepsilon^{0ik}k_{ca}\left(\tfrac{1}{2}F_{ik}^c - gC^c{}_{bd}A_i^b A_k^d\right) \approx 0\,, \tag{70}$$

the remaining constraints being undeformed. The term $\frac{1}{2}\,g\,C^c{}_{ab}\mathcal{P}_{2c}\,\eta_2^a\,\eta_2^b$ shows that the modified constraint functions generate a Lie algebra in terms of the structure constants $C^c{}_{ab}$

$$[\gamma_{2a}, \gamma_{2b}] = gC^c{}_{ab}\gamma_{2c}\,. \tag{71}$$

On the other hand, the antighost-independent piece in (69)

$$H = -2\int d^2x A_0^a\varepsilon^{0ik}k_{ca}\left(\tfrac{1}{2}F_{ik}^c - gC^c{}_{bd}A_i^b A_k^d\right), \tag{72}$$

is precisely the first-class Hamiltonian of the interacting theory. The components linear in the antighosts from (69) indicate that the Dirac brackets among the new first-class Hamiltonian and deformed constraint functions are modified as

$$[H, \gamma_{2a}] = -2gC^c{}_{ab}A_0^b\gamma_{2c}\,. \tag{73}$$

In conclusion, the resulting coupled first-class theory is nothing but the non Abelian version of the Chern–Simons model in three dimensions, described by the local Lagrangian action

$$\bar{S}_0\left[A_\mu^a\right] = \int d^3x\varepsilon^{\mu\nu\rho}A_\mu^a\left(\tfrac{1}{2}k_{ab}F_{\nu\rho}^b - \tfrac{2}{3}gC_{abc}A_\nu^b A_\rho^c\right), \tag{74}$$

where $C_{abc} = C^d{}_{[bc} k_{a]d}$. As first-class constraints generate gauge transformations, from the deformations (70) and (71) we can state that the added interactions involved with (72) modify both the gauge transformations and their algebra.

4.2. Two-dimensional nonlinear theories

Next, we analyze the nontrivial deformations of a two-dimensional gauge theory, described by the Lagrangian action

$$S_0\left[H_\mu^a, \varphi_a, A_\mu^a, B_a^{\mu\nu}\right] = \int d^2x \left(H_\mu^a \partial^\mu \varphi_a + \tfrac{1}{2} B_a^{\mu\nu} \partial_{[\mu} A_{\nu]}^a\right). \tag{75}$$

The canonical analysis of this model yields (after the elimination of the second-class constraints) the first-class constraints

$$G_{1a} \equiv \pi_a^0 \approx 0\,,\ G_{2a} \equiv -\partial_1 B_a^{01} \approx 0\,,\ G_{3a} \equiv p_a^1 \approx 0\,,\ G_{4a} \equiv -\partial^1 \varphi_a \approx 0\,, \tag{76}$$

and the first-class Hamiltonian

$$H_0 = \int dx^1 \left(A_0^a G_{2a} + H_1^a G_{4a}\right), \tag{77}$$

where the non-vanishing Dirac brackets among the independent variables are expressed by

$$\left[A_0^a, \pi_b^0\right] = \delta^a{}_b,\ \left[\varphi_a, H_0^b\right] = \delta_a{}^b,\ \left[A_1^a, B_b^{01}\right] = \delta^a{}_b\,,\ \left[H_1^a, p_b^1\right] = \delta^a{}_b\,. \tag{78}$$

Consequently, the BRST charge and the BRST-invariant Hamiltonian take the form

$$\overset{(0)}{\Omega} = \int dx^1 \left(\pi_a^0 \eta_1^a + p_a^1 C_1^a - \left(\partial_1 B_a^{01}\right) \eta_2^a - \left(\partial^1 \varphi_a\right) C_2^a\right), \tag{79}$$

$$\overset{(0)}{H}_B = H_0 + \int dx^1 \left(\eta_1^a \mathcal{P}_{2a} + C_1^a P_{2a}\right), \tag{80}$$

where the indices 1 and 2 involved with the ghosts and antighosts simply correspond to the indices of the associated constraint functions in (76). Just like in the previous example, the 'free' BRST differential reduces to the first two pieces, $\overset{(0)}{s} = \delta + \gamma$. These two operators are defined on the generators from the BRST complex as

$$\begin{aligned}
\delta z^A &= 0\,, \qquad \delta \eta^\Gamma = 0\,, & (81)\\
\delta \mathcal{P}_{1a} &= -\pi_a^0\,,\ \ \delta P_{1a} = -p_a^1\,,\ \delta \mathcal{P}_{2a} = \partial_1 B_a^{01}\,,\ \delta P_{2a} = \partial^1 \varphi_a\,, & (82)\\
\gamma A_0^a &= \eta_1^a, \qquad \gamma \pi_a^0 = 0\,, \qquad \gamma \varphi_a = 0\,, \qquad \gamma H_0^a = -\partial^1 C_2^a\,, & (83)\\
\gamma A_1^a &= \partial_1 \eta_2^a\,,\ \gamma B_a^{01} = 0\,, \qquad \gamma H_1^a = C_1^a\,, \qquad \gamma p_a^1 = 0\,, & (84)\\
\gamma \eta^\Gamma &= 0\,, \qquad \gamma \mathcal{P}_\Gamma = 0\,, & (85)
\end{aligned}$$

where

$$z^A = \left(A_0^a, \pi_a^0, \varphi_a, H_0^a, A_1^a, B_a^{01}, H_1^a, p_a^1\right), \tag{86}$$

$$\eta^\Gamma = \left(\eta_1^a, C_1^a, \eta_2^a, C_2^a\right), \quad \mathcal{P}_\Gamma = \left(\mathcal{P}_{1a}, P_{1a}, \mathcal{P}_{2a}, P_{2a}\right). \tag{87}$$

Both the antighost and pure ghost numbers of the variables (87) coincide with the corresponding ones involved with (57)–(58).

In order to determine the deformations of the BRST charge and BRST-invariant Hamiltonian, we follow the same line like previously. We start with the expansion (60) and assume that its last representative can be taken to be annihilated by γ. In this case, the cohomology of γ will be generated by φ_a, π_a^0, B_a^{01}, p_a^1, $\mathcal{P}_\Gamma$ and their spatial derivatives, as well as by the undifferentiated ghosts η_2^a and C_2^a, hence the general solution of the equation $\gamma\alpha = 0$ is given by

$$\alpha = \alpha_M\left(\left[\varphi_a\right], \left[\pi_a^0\right], \left[B_a^{01}\right], \left[p_a^1\right], \left[\mathcal{P}_\Gamma\right]\right) \mathrm{e}^M\left(\eta_2^a, C_2^a\right) + \gamma\beta, \tag{88}$$

where $\mathrm{e}^M\left(\eta_2^a, C_2^a\right)$ is a basis in the (finite-dimensional) space of the polynomials in the ghosts. In this situation we have again that $\mathrm{pgh}\left(\overset{(1)}{\omega}_J\right) = J+1$, such that (88) implies that we can take

$$\overset{(1)}{\omega}_J = \sum_{k=0}^{J+1} m_{a_1\cdots a_k b_1\cdots b_{J-k+1}} \eta_2^{a_1}\cdots\eta_2^{a_k} C_2^{b_1}\cdots C_2^{b_{J-k+1}}, \tag{89}$$

so the γ-invariant coefficients $m_{a_1\cdots a_k b_1\cdots b_{J-k+1}}$ pertain to $H_J\left(\delta|\tilde{d}\right)$, or, in other words, they must obey the equation

$$\delta m_{a_1\cdots a_k b_1\cdots b_{J-k+1}} + \partial_i c^i_{a_1\cdots a_k b_1\cdots b_{J-k+1}} = 0. \tag{90}$$

It is easy to see that $H_J\left(\delta|\tilde{d}\right)$ vanishes again for $J > 1$, such that $\overset{(1)}{\omega} = \overset{(1)}{\omega}_0 + \overset{(1)}{\omega}_1$, where

$$\overset{(1)}{\omega}_1 = m_{ab}\eta_2^a\eta_2^b + n_{ab}C_2^aC_2^b + u_{ab}\eta_2^aC_2^b, \tag{91}$$

with m_{ab}, n_{ab} and u_{ab} from $H_1\left(\delta|\tilde{d}\right)$. The general representative of $H_1\left(\delta|\tilde{d}\right)$ can be written under the form

$$\chi_{ab} = \lambda\frac{\delta W_{ab}}{\delta\varphi_c}P_{2c} + \sigma\left(M^c{}_{ab}\mathcal{P}_{2c} + \frac{\delta M^c{}_{ab}}{\delta\varphi_d}B_{c01}P_{2d}\right), \tag{92}$$

where the coefficients W_{ab} and ${M^c}_{ab}$ depend on φ_a, the latter ones are antisymmetric in the lower indices, ${M^c}_{ab} = -{M^c}_{ba}$, while λ and σ are some constants. On account of (92), we simply infer that

$$\delta\chi_{ab} = \partial^1\left(\lambda W_{ab} + \sigma {M^c}_{ab} B_{c01}\right)$$

which confirms that χ_{ab} verifies the equation (90). Moreover, we observe that χ_{ab} is γ-invariant. For subsequent purpose, we restrict ourselves to the choices

$$\begin{aligned} m_{ab} &= -\frac{1}{2}\left({M^c}_{ab}\mathcal{P}_{2c} + \frac{\delta {M^c}_{ab}}{\delta\varphi_d} B_{c01} P_{2d}\right),\ u_{ab} = -\frac{\delta W_{ab}}{\delta\varphi_c} P_{2c}\,, \\ n_{ab} &= 0 \text{ and } {M^c}_{ab} = \frac{\delta W_{ab}}{\delta\varphi_c} \end{aligned}$$

which further lead to

$$\overset{(1)}{\omega}_1 = -\frac{1}{2}\left(\frac{\delta W_{ab}}{\delta\varphi_c}\mathcal{P}_{2c} + \frac{\delta^2 W_{ab}}{\delta\varphi_c\delta\varphi_d} B_{c01} P_{2d}\right)\eta_2^a\eta_2^b - \frac{\delta W_{ab}}{\delta\varphi_c} P_{2c}\eta_2^a C_2^b\,.$$

By means of the equation $\delta \overset{(1)}{\omega}_1 + \gamma \overset{(1)}{\omega}_0 = \partial_k n^k$, we then find that

$$\overset{(1)}{\omega}_0 = W_{ab}\left(A^{a1}C_2^b + \eta_2^a H_0^b\right) - \frac{\delta W_{ab}}{\delta\varphi_c} B_{c01}\eta_2^a A^{b1},$$

so the complete first-order deformation of the BRST charge reads as

$$\begin{aligned} \overset{(1)}{\omega} &= -\frac{1}{2}\left(\frac{\delta W_{ab}}{\delta\varphi_c}\mathcal{P}_{2c} + \frac{\delta^2 W_{ab}}{\delta\varphi_c\delta\varphi_d} B_{c01} P_{2d}\right)\eta_2^a\eta_2^b - \frac{\delta W_{ab}}{\delta\varphi_c} P_{2c}\eta_2^a C_2^b \\ &\quad + W_{ab}\left(A^{a1}C_2^b + \eta_2^a H_0^b\right) - \frac{\delta W_{ab}}{\delta\varphi_c} B_{c01}\eta_2^a A^{b1}. \end{aligned} \tag{93}$$

After some computation, we arrive at

$$\left[\overset{(1)}{\Omega}, \overset{(1)}{\Omega}\right] = \int dx^1 \left(t_{abc} w^{abc} + \frac{\delta t_{abc}}{\delta\varphi_d} {v_d}^{abc} + \frac{\delta^2 t_{abc}}{\delta\varphi_n\delta\varphi_d} {z_{dn}}^{abc}\right), \tag{94}$$

where we performed the notations

$$\begin{aligned} t_{abc} &\equiv W_{e[a}\frac{\delta W_{bc]}}{\delta\varphi_e}\,, &(95)\\ w^{abc} &= -2C_2^a\eta_2^b A^{c1} + H_0^a\eta_2^b\eta_2^c\,, &(96)\\ {v_d}^{abc} &= \tfrac{1}{3}\mathcal{P}_{2d}\eta_2^a\eta_2^b\eta_2^c - B_{d01}\eta_2^a\eta_2^b A^{c1} + P_{2d}C_2^a\eta_2^b\eta_2^c\,, &(97)\\ {z_{dn}}^{abc} &= \tfrac{1}{3}B_{d01}P_{2n}\eta_2^a\eta_2^b\eta_2^c\,. &(98) \end{aligned}$$

From (94) it follows that $\left[\overset{(1)}{\Omega}, \overset{(1)}{\Omega}\right]$ is not $\overset{(0)}{s}$-exact modulo $\tilde{d}$, as required, therefore it should vanish. This is attained if and only if

$$t_{abc} = 0 \tag{99}$$

which is nothing but the Jacobi identity for a nonlinear gauge algebra. In consequence, we can take the second- and higher-order deformations of the BRST charge to vanish, $\overset{(k)}{\Omega} = 0$, $k \geq 2$.

Next, we pass to analyse the deformations of the BRST-invariant Hamiltonian (80). In view of this, we find that

$$\begin{aligned}\left[\overset{(0)}{H}_B, \overset{(1)}{\Omega}\right] = \overset{(0)}{s}\Bigg(\int dx^1\Bigg(&\left(\frac{\delta W_{ab}}{\delta\varphi_c}\mathcal{P}_{2c} + \frac{\delta^2 W_{ab}}{\delta\varphi_c\delta\varphi_d}B_{c01}P_{2d}\right)\eta_2^a A_0^b \\ &+\frac{\delta W_{ab}}{\delta\varphi_c}P_{2c}\left(\eta_2^a H_1^b - A_0^a C_2^b\right) + \frac{\delta W_{ab}}{\delta\varphi_c}B_{c01}A_0^a A^{b1} \\ &-W_{ab}\left(A^{a1}H_1^b + A_0^a H_0^b\right)\Bigg)\Bigg),\end{aligned} \tag{100}$$

so the first-order deformation, which is controlled by the equation (47), will be expressed by

$$\begin{aligned}\overset{(1)}{h}_B = &-\left(\frac{\delta W_{ab}}{\delta\varphi_c}\mathcal{P}_{2c} + \frac{\delta^2 W_{ab}}{\delta\varphi_c\delta\varphi_d}B_{c01}P_{2d}\right)\eta_2^a A_0^b \\ &-\frac{\delta W_{ab}}{\delta\varphi_c}P_{2c}\left(\eta_2^a H_1^b - A_0^a C_2^b\right) - \frac{1}{2}\frac{\delta W_{ab}}{\delta\varphi_c}B_c^{\mu\nu}A_\mu^a A_\nu^b + W_{ab}A^{a\mu}H_\mu^b.\end{aligned} \tag{101}$$

Direct computation yields $\left[\overset{(1)}{H}_B, \overset{(1)}{\Omega}\right] = 0$, hence equation (48) is fulfilled with the choice $\overset{(2)}{h}_B = 0$. Further, all higher-order deformations of the BRST-invariant Hamiltonian can be taken to vanish, $\overset{(3)}{H}_B = \overset{(4)}{H}_B = \cdots = 0$.

Putting together the results inferred so far, we obtain that the complete form of the deformed BRST charge and deformed BRST-invariant Hamiltonian for the model under study, consistent to all orders in the deformation parameter, are expressed by

$$\begin{aligned}\Omega = \int dx^1 \Bigg(&\pi_a^0\eta_1^a + p_a^1 C_1^a + \left(-\partial^1\varphi_a - gW_{ab}A^{b1}\right)C_2^a \\ &+\left(-\partial_1 B_a^{01} + gW_{ab}H_0^b - g\frac{\delta W_{ab}}{\delta\varphi_c}B_{c01}A^{b1}\right)\eta_2^a \\ &-\frac{1}{2}g\frac{\delta W_{ab}}{\delta\varphi_c}\mathcal{P}_{2c}\eta_2^a\eta_2^b - gP_{2c}\eta_2^a\left(\frac{1}{2}\frac{\delta^2 W_{ab}}{\delta\varphi_c\delta\varphi_d}B_{d01}\eta_2^b + \frac{\delta W_{ab}}{\delta\varphi_c}C_2^b\right)\Bigg),\end{aligned} \tag{102}$$

respectively,

$$
\begin{aligned}
H_B \; = \; & \int dx^1 \left(A_0^a G_{2a} + H_1^a G_{4a} + g W_{ab} A^{a\mu} H_\mu^b - \frac{g}{2} \frac{\delta W_{ab}}{\delta \varphi_c} B_c^{\mu\nu} A_\mu^a A_\nu^b \right. \\
& + \left(\eta_1^c + g \frac{\delta W_{ab}}{\delta \varphi_c} \eta_2^a A_0^b \right) \mathcal{P}_{2c} + \left(C_1^c + g \frac{\delta^2 W_{ab}}{\delta \varphi_c \delta \varphi_d} B_{d01} \eta_2^a A_0^b \right. \\
& \left. \left. + g \frac{\delta W_{ab}}{\delta \varphi_c} \left(\eta_2^a H_1^b + C_2^a A_0^b \right) \right) P_{2c} \right) .
\end{aligned} \tag{103}
$$

The above quantities allow us to identify the resulting interacting theory. The antighost-independent pieces in (102) furnish the deformed first-class constraints

$$
\gamma_{2a} \; \equiv \; -\partial_1 B_a^{01} + g W_{ab} H_0^b - g \frac{\delta W_{ab}}{\delta \varphi_c} B_{c01} A^{b1} \approx 0 \,, \tag{104}
$$

$$
\gamma_{4a} \; \equiv \; -\partial^1 \varphi_a - g W_{ab} A^{b1} \approx 0 \,, \tag{105}
$$

the others being unaffected. The terms linear in the antighosts show that some of the Dirac brackets among the new first-class constraints are also deformed, namely,

$$
[\gamma_{2a}, \gamma_{2b}] \; = \; -g \frac{\delta W_{ab}}{\delta \varphi_c} \gamma_{2c} - g \frac{\delta^2 W_{ab}}{\delta \varphi_c \delta \varphi_d} B_{d01} \gamma_{4c} \,, \tag{106}
$$

$$
[\gamma_{4a}, \gamma_{2b}] \; = \; -g \frac{\delta W_{ab}}{\delta \varphi_c} \gamma_{4c} \,. \tag{107}
$$

On the other hand, with the help of the components in (103) independent of ghosts and antighosts, we read the deformed first-class Hamiltonian

$$
H = \int dx^1 \left(A_0^a G_{2a} + H_1^a G_{4a} + g W_{ab} A^{a\mu} H_\mu^b - \frac{g}{2} \frac{\delta W_{ab}}{\delta \varphi_c} B_c^{\mu\nu} A_\mu^a A_\nu^b \right) , \tag{108}
$$

while the terms linear in the antighosts offer the Dirac brackets among the modified first-class constraints and first-class Hamiltonian of the type

$$
[H, G_{1a}] \; = \; \gamma_{2a} , \; [H, G_{3a}] = \gamma_{4a} \,, \tag{109}
$$

$$
[H, \gamma_{2a}] \; = \; g \left(\frac{\delta^2 W_{ab}}{\delta \varphi_c \delta \varphi_d} B_{d01} A_0^b + \frac{\delta W_{ab}}{\delta \varphi_c} H_1^b \right) \gamma_{4c} + g \frac{\delta W_{ab}}{\delta \varphi_c} A_0^b \gamma_{2c} \,, \tag{110}
$$

$$
[H, \gamma_{4a}] \; = \; g \frac{\delta W_{ab}}{\delta \varphi_c} A_0^b \gamma_{4c} \,. \tag{111}
$$

In this manner, the coupled model describes nothing but a two-dimensional nonlinear gauge theory, pictured by the local Lagrangian action

$$\begin{aligned}\bar{S}_0\left[H^a_\mu, \varphi_a, A^a_\mu, B^{\mu\nu}_a\right] &= \int d^2x \left(H^a_\mu \left(\partial^\mu \varphi_a + gW_{ab}A^{b\mu}\right)\right. \\ &\quad \left. + \frac{1}{2}B^{\mu\nu}_a \left(\partial_{[\mu} A^a_{\nu]} + g\frac{\delta W_{bc}}{\delta \varphi_a}A^b_\mu A^c_\nu\right)\right), \quad (112)\end{aligned}$$

subject to the deformed gauge transformations

$$\bar{\delta}_\epsilon H^a_\mu = \left(\delta^a{}_c\partial^\nu + g\frac{\delta W_{bc}}{\delta\varphi_a}A^{b\nu}\right)\epsilon^c_{\mu\nu} + g\left(\frac{\delta W_{bc}}{\delta\varphi_a}H^b_\mu - \frac{\delta^2 W_{bc}}{\delta\varphi_a\delta\varphi_d}B_{d\mu\nu}A^{b\nu}\right)\epsilon^c, \quad (113)$$

$$\bar{\delta}_\epsilon\varphi_a = -gW_{ab}\epsilon^b, \quad (114)$$

$$\bar{\delta}_\epsilon A^a_\mu = \left(\delta^a{}_c\partial_\mu + g\frac{\delta W_{bc}}{\delta\varphi_a}A^b_\mu\right)\epsilon^c, \quad (115)$$

$$\bar{\delta}_\epsilon B^{\mu\nu}_a = gW_{ab}\epsilon^{b\mu\nu} - g\frac{\delta W_{ab}}{\delta\varphi_c}B^{\mu\nu}_c\epsilon^b. \quad (116)$$

Now, it is clear that the deformation procedure modifies the action, the gauge transformations, as well as their algebra.

5. Conclusion

To conclude with, in this paper we have presented a Hamiltonian BRST approach to the construction of consistent interactions among fields with gauge freedom. Our procedure reformulates the problem of constructing Hamiltonian consistent interactions as a deformation problem of the BRST charge and BRST-invariant Hamiltonian of a given "free" theory. We have derived the general equations that govern the Hamiltonian BRST deformation method and proved that they possess solutions. Next, we have written down the local version of these equations and discussed on the locality of their solutions. Finally, the general theory was exemplified in the case of the Chern-Simons model in three dimensions and of a two-dimensional nonlinear gauge theory. In connection with these models, we explicitly obtained the deformed first-class constraints, first-class Hamiltonian, and accompanying Hamiltonian gauge algebra. We think that our approach together with the general results in [26] might be successfully applied, among others, to the computation of local BRST cohomologies for those theories whose Lagrangian version is more intricate than the Hamiltonian one.

This work has been supported by the Romanian National Council for Academic Scientific Research (CNCSIS) grant.

REFERENCES

[1] R. Arnowitt, S. Deser, *Nucl. Phys.* **49**, 133 (1963).

[2] J. Fang, C. Fronsdal, *J. Math. Phys.* **20**, 2264 (1979).

[3] F.A. Berends, G.H. Burgers, H. Van Dam, *Nucl. Phys.* **B260**, 295 (1985); *Z. Phys.* **C24**, 247 (1984).

[4] A.K.H. Bengtsson, *Phys. Rev.* **D32**, 2031 (1985).

[5] G. Barnich, M. Henneaux, *Phys. Lett.* **B311**, 123 (1993).

[6] I.A. Batalin, G.A. Vilkovisky, *Phys. Lett.* **B102**, 27 (1981).

[7] I. A. Batalin, G.A. Vilkovisky, *Phys. Rev.* **D28**, 2567 (1983).

[8] I.A. Batalin, G.A. Vilkovisky, *J. Math. Phys.* **26**, 172 (1985).

[9] M. Henneaux, *Nucl. Phys. B (Proc. Suppl.)* **18A**, 47 (1990).

[10] M. Henneaux, C. Teitelboim, *Quantization of Gauge Systems*, Princeton Univ. Press, Princeton 1992.

[11] G. Barnich, M. Henneaux, R. Tatar, *Int. J. Mod. Phys.* **D3**, 139 (1994).

[12] M. Henneaux, *Phys. Lett.* **B368**, 83 (1996).

[13] M. Henneaux, B. Knaepen, *Phys. Rev.* **D56**, 6076 (1997).

[14] X. Bekaert, M. Henneaux, *Int. J. Theor. Phys.* **38**, 1161 (1999).

[15] X. Bekaert, M. Henneaux, A. Sevrin, *Phys. Lett.* **B468**, 228 (1999).

[16] X. Bekaert, M. Henneaux, A. Sevrin, *Nucl. Phys. B (Proc. Suppl.)* **88**, 27 (2000).

[17] X. Bekaert, M. Henneaux, A. Sevrin, `hep-th/0004049`.

[18] C. Bizdadea, M.G. Mocioaca, S.O. Saliu, *Phys. Lett.* **B459**, 145 (1999).

[19] C. Bizdadea, L. Saliu, S.O. Saliu, *Phys. Scripta* **61**, 307 (2000).

[20] E.S. Fradkin, G.A. Vilkovisky, *Phys. Lett.* **B55**, 224 (1975).

[21] I.A. Batalin, G.A. Vilkovisky, *Phys. Lett.* **B69**, 309 (1977).

[22] E.S. Fradkin, T.E. Fradkina, *Phys. Lett.* **B72**, 343 (1978).

[23] I.A. Batalin, E.S. Fradkin, *Phys. Lett.* **B122**, 157 (1983).

[24] M. Henneaux, *Phys. Rep.* **126**, 1 (1985).

[25] G. Barnich, *Mod. Phys. Lett.* **A9**, 665 (1994).

[26] G. Barnich, M. Henneaux, *J. Math. Phys.* **37**, 5273 (1996).

[27] R. Ferraro, M. Henneaux, M. Puchin, *J. Math. Phys.* **34**, 2757 (1993).

[28] J.M.L. Fisch, M. Henneaux, *Phys. Lett.* **B226**, 80 (1989).

[29] M. Henneaux, B. Knaepen, C. Schomblond, *Commun. Math. Phys.* **186**, 137 (1997).