RESUMMATION OF DOUBLE LOGARITHMIC TERMS $\ln^2(1/x)$ IN THE POLARIZED NONSINGLET STRUCTURE FUNCTION g_1 AT SMALL x VIA DGLAP-LIKE APPROACH

Dorota Kotlorz

Department of Physics, Technical University of Opole
Ozimska 75, 45-370 Opole, Poland
e-mail: dstrozik@po.opole.pl

and Andrzej Kotlorz

Department of Mathematics, Technical University of Opole
Luboszycka 3, 45-036 Opole, Poland

(Received July 2, 2001)

An alternative equation, resumming of the $\ln^2 1/x$ terms for the polarized nonsinglet structure function $g_1^{\rm NS}$ at small x is presented. Construction of the DGLAP-like formula for the auxiliary function, corresponding to the $g_1^{\rm NS}$ at rescaled Q^2 variable is shown. Predictions of this approach for the $g_1^{\rm NS}$ function at small x in a case of a flat as well as a dynamical input are given. The role of the fixed coupling constant and the running one is also discussed.

PACS numbers: 12.38 Bx

1. Introduction

Our knowledge about the structure functions of the nucleon is still incomplete because of lack of understanding how these structure functions behave in the small Bjørken x region. Neither present experimental data nor the theoretical QCD description give a full and unique picture of an exact shape of the quark and gluon distributions in the nucleon. Perturbative QCD analysis, based on the DGLAP evolution equations [1] is in good agreement with experimental measurements. This agreement concerns unpolarized [2,3] and polarized [4] structure functions of the nucleon within NLO approximation in the large and moderately small Bjørken x region. Unfortunately, the lack of experimental data in the small x region ($x < 10^{-3}$) makes the satisfactory determination of the Bjørken and Ellis–Jaffe sum rules [5] practically

impossible. This causes *e.g.* that the question "how is the spin of the nucleon made of partons?" remains still open. From recent papers [6, 7] we know that the small x behavior of both unpolarized and polarized structure functions is controlled by the double logarithmic terms $\alpha_s \ln^2 1/x$. However, this singular behavior of the structure functions at low x is better visible in the spin dependent case. For the unpolarized, nonsinglet structure function $F_2^{\rm NS}(x,Q^2) = F_2^p(x,Q^2) - F_2^n(x,Q^2)$ the QCD singular behavior at small x is overridden by the leading Regge contribution [8]. Next in the unpolarized, singlet case, the structure function at low x is driven by BFKL pomeron [9] because gluons play the dominant role. Thus the growth of structure functions of the nucleon, governed by leading double logarithmic terms $\alpha_s^n \ln^{2n}(x)$ becomes best visible for spin dependent functions. Therefore, the polarized structure functions of the nucleon may be a sensitive test of the perturbative QCD analysis in the small x region. The double logarithmic $\ln^2 x$ effects for spin dependent structure functions within the renormalization group approach have been discussed in [10]. The double logarithmic $\ln^2 x$ effects go beyond the standard LO or even NLO QCD evolution of parton distributions and correspond to the ladder diagrams with quark and gluon exchanges along the ladder [11]. One has also to take into account nonladder diagrams [11] but in the nonsinglet case they may be neglected as nonleading [6, 7]. Thus the nonsinglet, polarized structure function $g_1^{\rm NS}(x,Q^2) = g_1^p(x,Q^2) - g_1^n(x,Q^2)$ is a convenient function both for QCD analysis (because of its simplicity) and future experimental tests at HERA [12] concerning the determination of the Bjørken sum rule. The small x behavior of the nonsinglet polarized as well as the spin-averaged structure functions and the running coupling effects in the corresponding evolution equations have been recently presented in [13]. Theoretical predictions for $g_1^{\rm NS}$ at small x, incorporating the double $\ln^2 x$ effects have been also presented in [7, 14–16]. In these papers the perturbative QCD analysis is based on the unintegrated spin dependent quark distributions $f(x,k^2)$. It means that the sum of double logarithmic $\ln^2 x$ terms is represented by a appropriate integral equation for the unintegrated structure function $f(x,k^2)$.

In this paper we present an alternative approach of the double $\ln^2 x$ resummation for $g_1^{\rm NS}$ at small x. Our formalism is based on the usual quark distribution functions (and not on $f(x,k^2)$ function) and moreover generates the $\ln^2 x$ terms via DGLAP-like evolution equation for the rescaled quark transverse momentum squared $\mu^2 = k^2/x$. The purpose of this paper is to compare these two methods of the double $\ln^2 x$ resummation for the polarized nonsinglet structure function $g_1^{\rm NS}$ at low x. In the next section we briefly recall the origin of the double logarithmic $\ln^2 x$ effects at low x, incorporating the evolution equation based on the unintegrated function $f(x,k^2)$.

In Section 3 we introduce alternative formalism, in which after rescaling the kinematic variable $\mu^2 = k^2/x$ we get DGLAP-like evolution equation in μ^2. This equation for the polarized quark distributions (and hence for the $g_1^{\rm NS}$ function) generates the double logarithmic $\ln^2 x$ terms. Section 4 contains numerical results for the spin dependent nonsinglet structure function $g_1^{\rm NS}(x,Q^2)$ in our approach. We compare the both mentioned above methods and also compare their predictions for $g_1^{\rm NS}$ with SMC 1997 small x data. Finally in Section 5 we summarize our results.

2. Double logarithmic $\ln^2 x$ resummation for the nonsinglet, polarized structure function $g_1^{\rm NS}(x,Q^2)$ using the unintegrated function $f(x,k^2)$

It has been noticed lately [6,7] that the spin dependent structure function g_1 in the small x region is dominated by $\ln^2(1/x)$ terms. This singular behavior, implied by QCD, is for the polarized structure functions the leading one. Comparatively, for unpolarized, nonsinglet structure functions of the nucleon, the QCD evolution behavior at small x is screened by the leading Regge contribution. The Regge theory [17], which concerns the Regge limit: $x \to 0$ predicts the following behavior of parton distributions at small x and $Q^2 \leq 1\,{\rm GeV}^2$:

$$\begin{aligned} x\Sigma &\sim \text{const. (Pomeron)}, \\ q_{\rm NS} &\sim x^{-0.5}\ \text{(Reggeon } A_2 : \rho - \omega), \\ \Delta\Sigma,\ \Delta q_{\rm NS} &\sim x^0 \div x^{0.5}\ \text{(Reggeon } A_1), \end{aligned} \tag{2.1}$$

where Σ, $q_{\rm NS}$, $\Delta\Sigma$, $\Delta q_{\rm NS}$ denote, respectively, singlet unpolarized, nonsinglet unpolarized, singlet polarized, nonsinglet polarized quark distributions. The shape of all spin dependent distributions $\Delta\Sigma$, $\Delta q_{\rm NS}$ is mostly governed by QCD evolutions with dominating $\ln^2 x$ terms at small x. The full contribution to the double $\ln^2 x$ resummation comes from the ladder diagram with quark and gluon exchanges along the ladder — *cf.* Fig. 1 and the nonladder bremsstrahlung diagrams [11]. In contrast to the singlet polarized function, for the nonsinglet one the contribution of nonladder diagrams is negligible. Thus examining the polarized, nonsinglet structure function $g_1^{\rm NS}(x,Q^2)$, we should consider only mentioned above ladder diagrams. The nonsinglet part of the spin dependent structure function has the form:

$$g_1^{\rm NS}(x,Q^2) = g_1^p(x,Q^2) - g_1^n(x,Q^2), \tag{2.2}$$

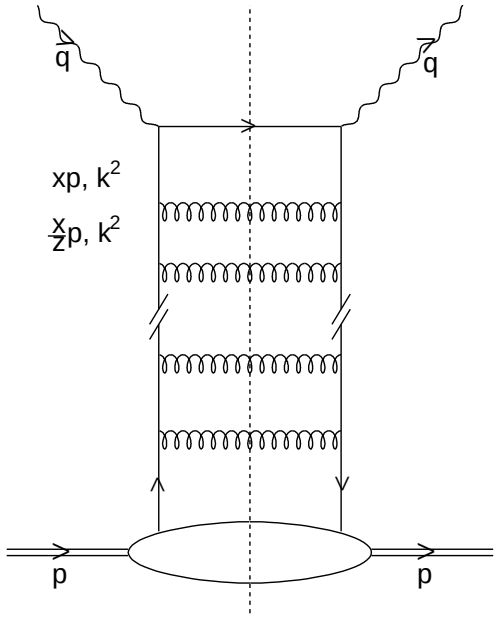

Fig. 1. A ladder diagram generating double logarithmic $\ln^2(1/x)$ terms in the non-singlet spin structure function g_1.

where g_1^p and g_1^n are spin dependent structure functions of proton and neutron, respectively. Let us remind the meaning of g_1. In the Bjørken limit

$$g_1(x) = \frac{1}{2} \sum_{i=u,d,s,..} e_i^2 \Delta q_i(x)\,, \tag{2.3}$$

$$\Delta q_i(x) = q_{i+}(x) - q_{i-}(x)\,, \tag{2.4}$$

where e_i is a charge of the i-flavor quark, $q_{i+}(x)$ ($q_{i-}(x)$) is the density distribution function of the i-quark with the spin parallel (antiparallel) to the parent nucleon. Function $g_1(x, Q^2)$ is connected with the helicity of the nucleon (*i.e.* spin projection on the momentum direction). Thus the integral

$$\langle \Delta q_i \rangle = \int_0^1 \Delta q_i(x) dx \tag{2.5}$$

is simply a part of the nucleon helicity, carried by a quark of i-flavor ($i = u, d, s, \ldots$). Polarized distribution functions of quarks are defined as:

$$\Delta q = \Delta q_{\rm val} + \Delta q_{\rm sea}\,. \tag{2.6}$$

Finally,

$$g_1^{\rm NS} = \frac{1}{6}(\Delta u_{\rm val} - \Delta d_{\rm val}) = \frac{1}{6}(\Delta u - \Delta d)\,. \tag{2.7}$$

Solutions of the equation for the unintegrated polarized nonsinglet structure function $f^{\rm NS}(x,Q^2)$, which gives the $\ln^2 x$ resummation are presented in [7,14–16]. $\ln^2 x$ as the only contribution at small x behavior of $g_1^{\rm NS}$ is examined in [7] while in [14–16] the unified description of $g_1^{\rm NS}$ incorporating both the DGLAP evolution and $\ln^2 x$ effects is presented. These all approaches are based on the unintegrated distribution function $f(x,Q^2)$, which is related to the $g_1(x,Q^2)$ via

$$g_1\left(x,Q^2\right) = g_1^{(0)}(x) + \int\limits_{k_0^2}^{Q^2(1/x-1)} \frac{dk^2}{k^2} f\left(x\left(1+\frac{k^2}{Q^2}\right),k^2\right), \tag{2.8}$$

where

$$g_1^{(0)}(x) = \int\limits_0^{k_0^2} \frac{dk^2}{k^2} f\left(x,k^2\right)\,. \tag{2.9}$$

Let us recall from [14] that the resummation of the double logarithmic terms $\ln^2 x$ in the limit of a very small x ($x \to 0$) is given in the case of the polarized, nonsinglet structure function by

$$f_{\rm NS}\left(x,k^2\right) = f_{\rm NS}^{(0)}\left(x,k^2\right) + \frac{\alpha_s(k^2)}{2\pi}\int\limits_x^1 \frac{dz}{z}\int\limits_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2}\Delta P_{qq}^{(0)}(z) f_{\rm NS}\left(\frac{x}{z},k'^2\right)\,. \tag{2.10}$$

The source of the double logarithmic terms $\ln^2 x$ in $g_1\left(x,Q^2\right)$ is the double integration in the formula for function $f\left(x,k^2\right)$:

$$f(x,k^2) \sim \frac{\alpha_s}{2\pi}\int\limits_x^1 \frac{dz}{z}\int\limits_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2}\,, \tag{2.11}$$

where the upper limit in the integral over the transverse momentum k'^2 is z-dependent ($= k^2/z$). Thus, double logarithmic terms come from the integration over the longitudinal momentum fraction z together with the integration over k'^2 with z-dependent upper limit:

$$f(x,k^2) \sim \ln^2\left(\frac{1}{x}\right) = \ln^2 x\,. \tag{2.12}$$

The analytical solution of (2.10) in the case of the fixed coupling constant α_s [15] shows the singular small x behavior of the polarized, nonsinglet structure function g_1 *i.e.* :

$$g_1^{\mathrm{NS}}\left(x,Q^2\right) \sim x^{-\lambda_{\mathrm{NS}}}, \tag{2.13}$$

$$\lambda_{\mathrm{NS}} = 2\sqrt{\frac{\alpha_s}{2\pi}\Delta P_{qq}^{(0)}(x)}, \tag{2.14}$$

where $\Delta P_{qq}^{(0)}(x)$ is the splitting function and in the limit $x \to 0$ is equal to 4/3. Hence for the fixed $\alpha_s = 0.18$, $\lambda_{\mathrm{NS}} = 0.39$ and as it has been already mentioned above, the singular small x shape of g_1^{NS}, implied by QCD, dominates the REGGE behavior:

$$\text{REGGE}: \quad g_1^{\mathrm{NS}}(x,Q^2) \sim x^{-\alpha_{\mathrm{NS}}(0)} \quad \alpha_{\mathrm{NS}}(0) \leq 0, \tag{2.15}$$

$$\text{QCD}: \quad g_1^{\mathrm{NS}}(x,Q^2) \sim x^{-\lambda_{\mathrm{NS}}} \qquad \lambda_{\mathrm{NS}} \sim 0.5\,. \tag{2.16}$$

In the case of the running coupling constant, numerical results for g_1^{NS}, based on the Regge flat input parametrization $g_1^{\mathrm{NS}}(x, Q_0^2 = 1\,\mathrm{GeV}^2)$ are given in [7,14,15]. In [16] the same method is used for dynamical input parametrization, where in contrast to Regge flat parametrizations, $g_1^{\mathrm{NS}}(x, Q_0^2 = 1\,\mathrm{GeV}^2)$ is singular for small x. The generation of the double logarithmic $\ln^2 x$ terms is also possible via DGLAP-like evolution equation for g_1 function with the rescaled transverse momentum squared. This alternative method is presented in the next section.

3. DGLAP-like evolution equation for g_1^{NS}, generating $\ln^2 x$ terms at low x

We study the origin of the double logarithmic $\ln^2 x$ terms in $g_1^{\mathrm{NS}}(x,Q^2)$ using a simple rescaling of Q^2 variable: $Q^2 \to \mu^2 = Q^2/x$, what leads to the DGLAP-like equation for g_1^{NS} with evolution in a new scale μ^2. We focus on the small x region so our initial equation for further investigations is that which contains only dominating $\ln^2 x$ part. First let us consider the case with a fixed coupling $\bar{\alpha_s} = \mathrm{const.}$, where

$$\bar{\alpha_s} = \frac{2\alpha_s}{3\pi}\,. \tag{3.1}$$

Thus the starting equation is:

$$f_{\mathrm{NS}}\left(x,k^2\right) = f_{\mathrm{NS}}^{(0)}\left(x,k^2\right) + \bar{\alpha_s}\int\limits_x^1 \frac{dz}{z}\int\limits_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2} f_{\mathrm{NS}}\left(\frac{x}{z},k'^2\right)\,. \tag{3.2}$$

After a simple substitution

$$\mu^2 = \frac{k^2}{x}\,, \qquad x' = \frac{x}{z}\,, \qquad \mu'^2 = \frac{k'^2}{x'}\,, \tag{3.3}$$

equation (3.2) takes the form:

$$f_{\rm NS}\left(x, x\mu^2\right) = f_{\rm NS}^{(0)}\left(x, x\mu^2\right) + \bar{\alpha}_s \int\limits_x^1 \frac{dz}{z} \int\limits_{k_0^2 \frac{z}{x}}^{\mu^2} \frac{d\mu'^2}{\mu'^2} f_{\rm NS}\left(\frac{x}{z}, \frac{x}{z}\mu'^2\right). \tag{3.4}$$

After introduction an auxiliary function $\varphi\left(x, \mu^2\right)$

$$\varphi\left(x, \mu^2\right) \equiv f_{\rm NS}\left(x, x\mu^2\right) \tag{3.5}$$

and applying Heaviside's Θ function:

$$\Theta(t) = \begin{cases} 1 & \text{for} \quad t > 0 \\ 0 & \text{for} \quad t \leq 0 \end{cases} \tag{3.6}$$

we get:

$$\varphi\left(x, \mu^2\right) = \varphi^{(0)}\left(x, \mu^2\right) + \bar{\alpha}_s \int\limits_x^1 \frac{dz}{z} \int\limits_{k_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2}\, \Theta\left(\mu'^2 - k_0^2 \frac{z}{x}\right) \varphi\left(\frac{x}{z}, \mu'^2\right), \tag{3.7}$$

or

$$\varphi_\Theta\left(x, \mu^2\right) = \varphi_\Theta^{(0)}\left(x, \mu^2\right) + \bar{\alpha}_s \int\limits_x^1 \frac{dz}{z} \int\limits_{k_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \varphi_\Theta\left(\frac{x}{z}, \mu'^2\right), \tag{3.8}$$

where

$$\varphi_\Theta\left(x, \mu^2\right) \equiv \Theta\left(\mu^2 - \frac{k_0^2}{x}\right) \varphi\left(x, \mu^2\right). \tag{3.9}$$

Equation (3.8) has an exact form of the DGLAP Q^2 evolution formula for the unintegrated distribution function $f(x, Q^2)$. The mentioned DGLAP evolution equation for polarized nonsinglet quark distributions Δp (and hence for $g_1^{\rm NS}$ function too) in the small x region is given by:

$$\frac{\partial \Delta p\left(x, Q^2\right)}{\partial \ln Q^2} = \bar{\alpha}_s \int\limits_x^1 \frac{dz}{z} \Delta p\left(\frac{x}{z}, Q^2\right). \tag{3.10}$$

Relation between $\Delta p(x,Q^2)$ and the unintegrated distribution $f_p\left(x,Q^2\right)$ is as usual:

$$f_p\left(x,Q^2\right)=\frac{\partial \Delta p\left(x,Q^2\right)}{\partial \ln Q^2}\,, \tag{3.11}$$

what implies:

$$\Delta p\left(x,Q^2\right)=\Delta p_0(x)+\int\limits_{Q_0^2}^{Q^2}\frac{dQ'^2}{Q'^2}f_p\left(x,Q'^2\right)\,, \tag{3.12}$$

where $\Delta p_0(x)$ is a nonperturbative part of Δp

$$\Delta p_0(x)=\int\limits_{0}^{Q_0^2}\frac{dQ'^2}{Q'^2}f_p\left(x,Q'^2\right) \tag{3.13}$$

and $Q_0^2=1\,\mathrm{GeV}^2$ is the low scale of perturbative QCD. Hence the evolution equation (3.10) written for the unintegrated distribution function $f_p\left(x,Q^2\right)$ takes the form:

$$f_p\left(x,Q^2\right)=f_p^{(0)}\left(x,Q^2\right)+\bar{\alpha}_s\int\limits_x^1\frac{dz}{z}\int\limits_{Q_0^2}^{Q^2}\frac{dQ'^2}{Q'^2}f_p\left(\frac{x}{z},Q'^2\right) \tag{3.14}$$

and

$$f_p^{(0)}\left(x,Q^2\right)=\bar{\alpha}_s\int\limits_x^1\frac{dz}{z}\Delta p_0\left(\frac{x}{z}\right). \tag{3.15}$$

One can see from (3.8) and (3.14) that the double $\ln^2 x$ resummation equation written for $\varphi_\Theta\left(x,\mu^2\right)$ function is a DGLAP μ^2 evolution equation. The auxiliary function $\varphi_\Theta\left(x,\mu^2\right)$ may be, similarly as in (3.11), represented by an integrated function $u\left(x,\mu^2\right)$:

$$\varphi_\Theta\left(x,\mu^2\right)=\frac{\partial u\left(x,\mu^2\right)}{\partial \ln \mu^2} \tag{3.16}$$

and conversely

$$u\left(x,\mu^2\right)=u_0(x)+\int\limits_{k_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2}\varphi_\Theta\left(x,\mu'^2\right). \tag{3.17}$$

Thus equation (3.8), generating double $\ln^2 x$ effects can be rewritten as:

$$\frac{\partial u\left(x,\mu^2\right)}{\partial \ln \mu^2} = \bar{\alpha_s} \int\limits_x^1 \frac{dz}{z} u\left(\frac{x}{z},\mu^2\right). \tag{3.18}$$

Relation between the nonsinglet polarized structure function $g_1^{\mathrm{NS}}\left(x,Q^2\right)$ and the auxiliary function $u\left(x,\mu^2\right)$ is as follows:

$$g_1^{\mathrm{NS}}\left(x,Q^2=x\mu^2\right) = u\left(x,\mu^2\right). \tag{3.19}$$

In this way the problem of producing the $\ln^2 x$ terms for $g_1^{\mathrm{NS}}\left(x,Q^2\right)$ in the small x region via equation (2.10) has been reduced to the DGLAP evolution to the momentum scale $\mu^2 = Q^2/x$. It is not astonishing: appearing of the new evolution scale Q^2/x has its origin in the upper limit k^2/z of the integration over the transverse momentum in (2.10). This logarithmic integration over the transverse momentum up to the $z-$dependent limit k^2/z together with the logarithmic integration over the longitudinal momentum fraction z give double logarithmic $\ln^2 x$ terms. The mechanism of appearing of the $\ln^2 x$ effects in g_1^{NS} from DGLAP-like equations (3.18)–(3.19) is well visible just in a case of the fixed coupling constant $\bar{\alpha_s} = \mathrm{const}$. Then the Eq. (3.18) can be solved analytically. Using standard Mellin's method one can get the solution of Eq. (3.18) in the form (see Appendix A):

$$u\left(x,\mu^2\right) \sim \sum_{k=0}^{\infty} \frac{\left(\bar{\alpha_s} \ln\frac{1}{x} \ln\frac{\mu^2}{k_0^2}\right)^k}{k!k!} \tag{3.20}$$

and hence:

$$g_1^{\mathrm{NS}}\left(x,Q^2\right) = u\left(x,\frac{Q^2}{x}\right) \sim \sum_{k=0}^{\infty} \frac{\left[\bar{\alpha_s} \ln\frac{1}{x}\left(\ln\frac{1}{x}+\ln\frac{Q^2}{k_0^2}\right)\right]^k}{k!k!}, \tag{3.21}$$

what gives approximately the leading small x behavior:

$$g_1^{\mathrm{NS}}\left(x,Q^2\right) \sim x^{-2\sqrt{\bar{\alpha_s}}}. \tag{3.22}$$

Taking into account parton interactions through the introduction of the running coupling constant one can get from (3.18) an equation which incorporates the running couplings effects $\bar{\alpha_s} \to \bar{\alpha_s}(Q^2)$:

$$\frac{\partial u\left(x,\mu^2\right)}{\partial \ln \mu^2} = \bar{\alpha_s}\left(x\mu^2\right) \int\limits_x^1 \frac{dz}{z}\, u\left(\frac{x}{z},\mu^2\right). \tag{3.23}$$

However, more justified theoretically seems to be the introduction of the running coupling by the substitution $\bar{\alpha_s} \to \bar{\alpha_s}\left(Q^2/z\right)$, what gives:

$$\frac{\partial u\left(x,\mu^2\right)}{\partial \ln \mu^2} = \int\limits_x^1 \frac{dz}{z} \bar{\alpha_s}\left(\frac{x\mu^2}{z}\right) u\left(\frac{x}{z},\mu^2\right). \tag{3.24}$$

Our numerical analysis presented in the next section contains both the above "running coupling" prescriptions and the constant $\bar{\alpha_s}$ case as well.

4. Numerical predictions for $g_1^{\mathrm{NS}}(x,Q^2)$ based on the DGLAP-like equation, resumming the $\ln^2 x$ terms

We solve numerically equations (3.18), (3.23) and (3.24) which are DGLAP μ^2 evolution equations for the auxiliary function $u\left(x,\mu^2\right)$. The relation between $u\left(x,\mu^2\right)$ and the physical polarized nonsinglet structure function $g_1^{\mathrm{NS}}(x,Q^2)$ (2.7) is given by (3.19). In this way one can get the small x behavior of g_1^{NS} governed by the double logarithmic $\ln^2 x$ effects. We compare our predictions with those, received in the unintegrated $f(x,Q^2)$ approach (2.10) and described in [7]. Solving the DGLAP equations (3.18), (3.23) and (3.24) one should have an input parametrization of the u function at the low scale k_0^2. Because $u(x,Q^2)$ has a meaning of the physical g_1^{NS} function for the rescaled Q^2 variable as it is shown in (3.19), one can also write:

$$u\left(x,k_0^2\right) = g_1^{\mathrm{NS}}\left(x,xk_0^2\right). \tag{4.1}$$

The low scale k_0^2 introduced in (3.7) is equal to the usually used QCD cut-off parameter $k_0^2 = 1\,\mathrm{GeV}^2$. The dependence on z/x of the lower limit in the integration (3.4):

$$Q_0^2 = \frac{z}{x}k_0^2 \tag{4.2}$$

became "shifted" to the definition of $\varphi_{\Theta}\left(x,\mu^2\right)$ via (3.9). It means that for running coupling cases one should take into account in the evolution equations (3.23) and (3.24) the cut-off factor $\Theta(\mu^2 - k_0^2/x)$. Otherwise, the

running variable $x\mu^2$ in the coupling α_s becomes less than the low scale of QCD evolution $k_0^2 = 1\,\mathrm{GeV}^2$ what is definitely incorrect. There is no such a constraint in the fixed coupling constant. Taking into account that for perturbative QCD analysis the low cut-off parameter $k_0^2 = 1\,\mathrm{GeV}^2$ we put this value for the input scale of the u function too. This implies that for small x the input parametrizations $u(x, k_0^2)$ corresponds to the g_1^{NS} at the very low scale xk_0^2. We assume, that below the $k_0^2 = 1\,\mathrm{GeV}^2$ the behavior of the quark distributions is the same as at k_0^2. Therefore, we apply the standard parametrizations of the valence quarks (and hence of g_1^{NS}) for the auxiliary function u:

$$u\left(x, k_0^2\right) = g_1^{\mathrm{NS}}\left(x, k_0^2\right), \qquad k_0^2 = 1\,\mathrm{GeV}^2. \tag{4.3}$$

There are two basic kinds of input parametrizations of $g_1^{\mathrm{NS}}\left(x, k_0^2\right)$: the Regge one, which is flat at small x and the singular one, which behaves like x^{-a} ($a \sim 0.3$) at small x. In our numerical calculations we use two different input parametrizations: the Regge one, which is given by

$$\mathrm{REGGE}: \quad u\left(x, k_0^2\right) = \frac{2}{3} g_A (1-x)^3 = 0.838(1-x)^3, \tag{4.4}$$

where $g_A = 1.257$ is the axial vector coupling and the dynamical input GRSV [18]:

$$\begin{aligned} \mathrm{GRSV}: \quad & u\left(x, k_0^2\right) = 0.327 x^{-0.267} \\ & \times \left(1 - 0.583 x^{0.175} + 1.723x + 3.436 x^{3/2}\right)(1-x)^{3.486} \\ & + 0.027 x^{-0.624}\left(1 + 1.195 x^{0.529} + 6.164x + 2.726 x^{3/2}\right)(1-x)^{4.215}. \end{aligned} \tag{4.5}$$

For details about these parametrizations see also [16]. In all calculations $\Lambda_{\mathrm{QCD}} = 232\,\mathrm{MeV}$. Our numerical results are presented in Figs. 2–5.

In Fig. 2 the predictions for g_1^{NS} at small x, based on the Eq. (3.18) for Regge and GRSV inputs, respectively, are shown. We use two different values of fixed coupling: $\alpha_s = 0.18$ and $\alpha_s = 0.12$. The input parametrizations are also plotted. In Fig. 3 we confront the fixed coupling results of Eq. (3.18) for $\alpha_s = 0.18$ with those, based on Eqs. (3.23), (3.24), taking into account running coupling effects. Two groups of lines correspond to different inputs: Regge and GRSV. The effective slope λ_{NS} determined from (2.16) as

$$\lambda_{\mathrm{NS}} \sim \frac{\partial \ln g_1^{\mathrm{NS}}}{\partial \ln \frac{1}{x}} \tag{4.6}$$

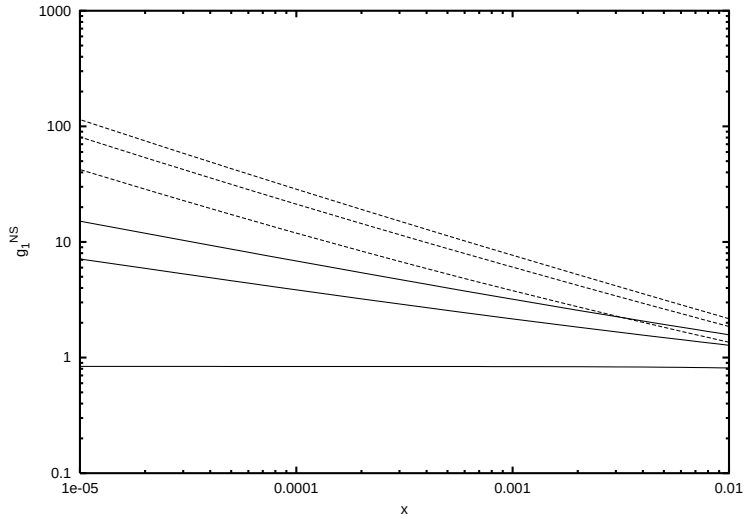

Fig. 2. The nonsinglet spin structure function of the proton $g_1^{\mathrm{NS}}(x,Q^2)$ received from (3.18). Solid lines correspond to Regge input, dashed lines — to GRSV input. In each group of lines (Regge or GRSV) the lowest (for small x) line is the input at $k_0^2 = 1\,\mathrm{GeV}^2$, the middle one concerns the fixed coupling $\alpha_s = 0.12$ and the upper one is for $\alpha_s = 0.18$. Evolution scale $Q^2 = 10\,\mathrm{GeV}^2$.

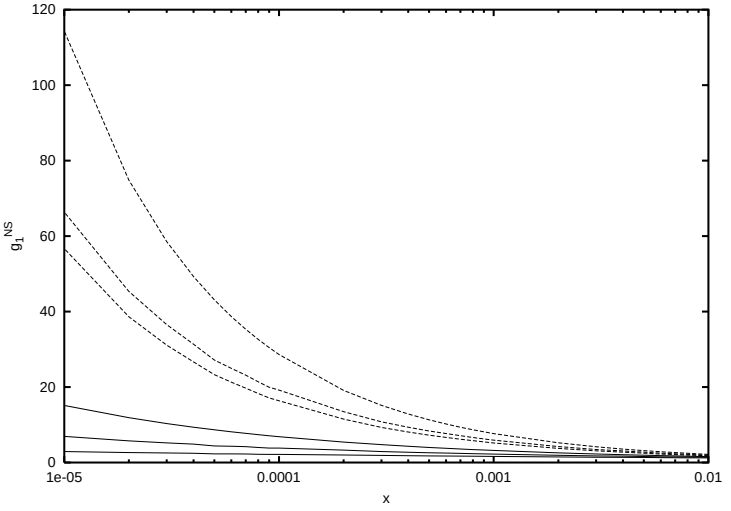

Fig. 3. The nonsinglet spin structure function of the proton $g_1^{\mathrm{NS}}(x,Q^2 = 10\,\mathrm{GeV}^2)$. Solid lines correspond to Regge input, dashed lines — to GRSV input. In each group of plots (Regge or GRSV) the lowest line concerns the running coupling $\alpha_s(Q^2/z)$ case (3.24), the middle one corresponds to $\alpha_s(Q^2)$ case (3.23) while the upper plot is for fixed coupling $\alpha_s = \mathrm{const.} = 0.18$ (3.18).

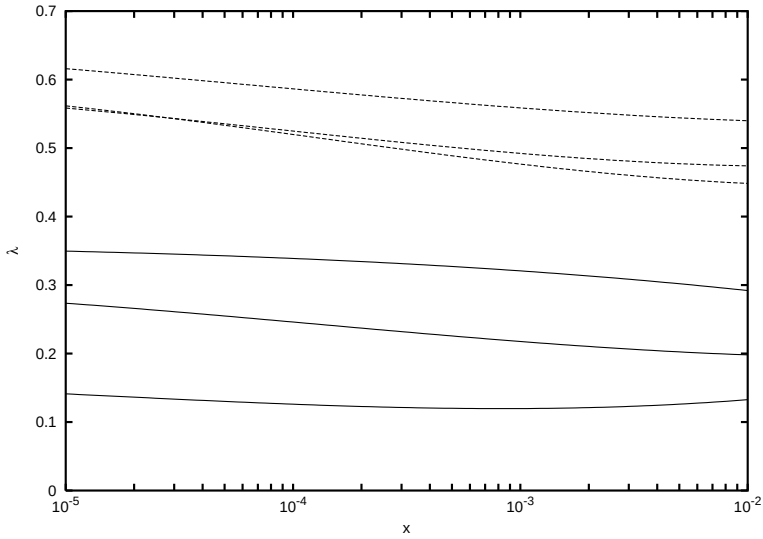

Fig. 4. Slope $\lambda_{\rm NS}$ defined in (4.6) for different inputs (Regge — solid, GRSV — dashed) and different α_s. For each group of lines (Regge or GRSV) the upper plot corresponds to fixed $\alpha_s = 0.18$, in the middle lies line for the running $\alpha_s(Q^2)$, and the lowest plot is for the "more running" case: $\alpha_s(Q^2/z)$, $Q^2 = 10\,{\rm GeV}^2$.

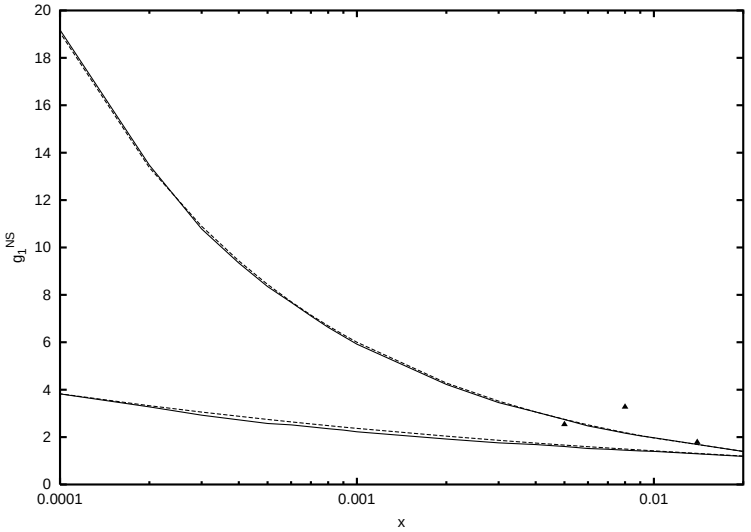

Fig. 5. Comparison of $g_1^{\rm NS}$ predictions based on DGLAP-like Eq. (3.23) — solid with that based on $f(x,Q^2)$ approach (2.10) — dashed at $Q^2 = 10\,{\rm GeV}^2$. Triangles show the recent small x SMC data 1997 [4].

for both input parametrizations and all mentioned above α_s cases: $\alpha_s = \text{const.} = 0.18$, $\alpha_s(Q^2)$, $\alpha_s(Q^2/z)$ is presented in Fig. 4. The comparison of small x predictions for $g_1^{\rm NS}$ based on the DGLAP-like $\ln^2 x$ approach with those based on the unintegrated function $f(x, Q^2)$ and Eq. (2.10) is shown in Fig. 5. Both parametrizations are used. The running coupling effects $\alpha_s(Q^2)$ are included. We also plot few small x recent SMC 1997 data [4]. In all plots $Q^2 = 10\,\text{GeV}^2$.

From Figs. 2–5 one can read the following conclusions:

1. The $\ln^2 x$ resummation gives steep growth of the structure function in the small x region. It is well known effect [6,7] that for $x \leq 10^{-2}$ the $\ln^2 x$ terms dominate over the LO (or even NLO) evolution.

2. The growth of the structure function in the small x region is, of course, much steeper for the dynamical parametrization than for the flat one. Singular inputs at k_0^2 intensify the QCD evolution effects while in the case of flat parametrizations the singular small x behavior of structure functions is completely and only generated by QCD evolution.

3. The effective slope $\lambda_{\rm NS}$ (4.6) is contained between 0.14 for the non-singular input and the running coupling $\alpha_s(Q^2/z)$ and 0.6 for the singular input and the fixed coupling $\alpha_s = 0.18$. This shows apart from the above conclusion in the point 2 that for the running coupling constant $\alpha_s(Q^2)$, $\lambda_{\rm NS}$ is smaller than for the fixed one. Besides, introduction of the $\alpha_s = \alpha_s(Q^2/z)$ causes the slope $\lambda_{\rm NS}$ still smaller:

$$\lambda_{\rm NS}(\alpha_s = \text{const.} = 0.18) > \lambda_{\rm NS}\left(\alpha_s = \alpha_s\left(Q^2\right)\right) > \lambda_{\rm NS}\left(\alpha_s = \alpha_s\left(\frac{Q^2}{z}\right)\right). \quad (4.7)$$

For comparison, the Regge theory predicts in the small x region $\alpha_{A_1}(0) \leq 0$ and $g_1^{\rm NS}$ behaves as $x^{-\alpha_{A_1}(0)} \sim \text{const.}$

4. Comparing our results with those, based on the unintegrated function $f(x, Q^2)$ and the equation (2.10) one can see agreement of these two approaches. It is not astonishing because both the methods concern in fact the same problem: resummation of the double logarithmic terms $\ln^2 x$ for the $g_1^{\rm NS}$ function in the small x region. Our approach based on the DGLAP-like equation (3.18) enables us to keep the constance upper scale of evolution μ^2 instead of Q^2/z like in (2.10). In this way this approach becomes similar to the standard DGLAP Q^2 evolution and we can adopt the methods used in it to get new — $\ln^2 x$ effects.

5. SMC recent data [4] for very small x (3 points shown in Fig. 5) confirm the growth of the polarized nonsinglet structure function $g_1^{\rm NS}$ in the small x region. It seems that the singular inputs make possible the better agreement of theoretical predictions with experimental data.

5. Summary and conclusions

In this paper we have presented the double logarithmic $\ln^2 x$ terms resummation for the spin dependent nonsinglet structure function $g_1^{\rm NS}$. In our approach we have solved the DGLAP-like equation for the auxiliary function $u\left(x,\mu^2\right)$ which corresponds to the physical function $g_1^{\rm NS}\left(x,Q^2=x\mu^2\right)$ at the rescaled μ^2 variable. Our calculations have been performed for the simple nonsingular Regge parametrization and for the dynamical one as well. Apart from the fixed coupling constant case, the running coupling effects have been taken into account. Besides the effective slope, controlling the small x behavior of $g_1^{\rm NS}$ has been estimated. Its value has been found to lie between 0.14 for the nonsingular input and the running coupling $\alpha_s\left(Q^2/z\right)$, and 0.6 for the singular input and the fixed coupling $\alpha_s=0.18$. We found that the $\ln^2 x$ effects govern the small x increase of the structure function $g_1^{\rm NS}\left(x,Q^2\right)$. This growth is larger in the case of the fixed coupling constant than for the running one and, of course, is larger for the singular input $g_1^{\rm NS}\left(x,k_0^2\right)$ than for the flat (*e.g.* Regge) one. The equation we have considered is not applicable for the large x ($x>10^{-2}$) region. Our formalism is, however, correct in the very interesting small x region. Presented results confirm that the $\ln^2 x$ effects are very significant for $x\leq 10^{-2}$. The spin dependent structure functions of the nucleon are a sensitive test of the perturbative QCD analysis in the small x region. However, lack of experimental data in the very low x region $\left(x\leq 10^{-3}\right)$ causes the satisfactory verification of the theoretical QCD predictions impossible in this region. Also the predictions incorporating the double logarithmic $\ln^2 x$ effects in $g_1^{\rm NS}$ are still awaiting for their crucial probe.

We are grateful to Jan Kwieciński for help in preparing this paper.

Appendix A

Analytical solution of the DGLAP-like evolution equation for $g_1^{\rm NS}$ generating double logarithmic $\ln^2 x$ effects at small x

We solve the equation (3.18) with the fixed coupling $\bar{\alpha_s}=0.038$ using the standard Mellin method. The Mellin transformation defines the n-moment of $u\left(x,\mu^2\right)$ function by:

$$u^n\left(\mu^2\right)\equiv\int\limits_0^1 x^{n-1}u\left(x,\mu^2\right)dx\,. \tag{A.1}$$

Thus in moment space the evolution equation for small x (3.18) is simply given by

$$\frac{du^n\left(\mu^2\right)}{d\ln\mu^2} = \frac{\bar{\alpha_s}}{n} u^n\left(\mu^2\right) . \tag{A.2}$$

The solution of (A.2) is straightforward

$$u^n\left(\mu^2\right) = u_0^n \left[\frac{\mu^2}{k_0^2}\right]^{\bar{\alpha_s}/n} , \tag{A.3}$$

where u_0^n is the n-moment of the input $u\left(x, k_0^2\right)$ function:

$$u_0^n = \int\limits_0^1 x^{n-1} u\left(x, k_0^2\right) dx . \tag{A.4}$$

For the nonsingular input parametrization

$$u\left(x, k_0^2\right) \sim \text{const.} , \tag{A.5}$$

u_0^n has a form:

$$u_0^n = \frac{\text{const.}}{n} . \tag{A.6}$$

Employing the inverse Mellin transformation:

$$u\left(x, \mu^2\right) = \frac{1}{2\pi i} \int\limits_{c-i\infty}^{c+i\infty} x^{-n} u^n\left(\mu^2\right) dn , \tag{A.7}$$

which gives

$$u\left(x, \mu^2\right) = \frac{\text{const.}}{2\pi i} \int\limits_{c-i\infty}^{c+i\infty} \frac{x^{-n}}{n} \left[\frac{\mu^2}{k_0^2}\right]^{\bar{\alpha_s}/n} dn . \tag{A.8}$$

One can get the solution $u\left(x, \mu^2\right)$ of the form:

$$u\left(x, \mu^2\right) = \text{const.} \sum_{k=0}^{\infty} \frac{\left(\bar{\alpha_s} \ln\frac{1}{x} \ln\frac{\mu^2}{k_0^2}\right)^k}{k!k!} \tag{A.9}$$

Going back to the function $g_1^{\rm NS}\left(x, Q^2 = x\mu^2\right)$, one can find from (A.9) through (3.19) the following expression:

$$g_1^{\rm NS}(x,Q^2) \equiv u\left(x, \frac{Q^2}{x}\right) = \text{const.}\, J_0\left(2\sqrt{\bar{\alpha_s} \ln\frac{1}{x} \ln\frac{Q^2}{xk_0^2}}\right), \qquad \text{(A.10)}$$

where $J_0(y)$ denotes modified Bessel function:

$$J_0(y) = \sum_{k=0}^{\infty} \frac{\left(\frac{y}{2}\right)^{2k}}{k!k!}. \qquad \text{(A.11)}$$

This $\ln^2 x$ terms resummation (A.10) gives in the small x region the effective behavior of $g_1^{\rm NS}$:

$$g_1^{\rm NS}(x,Q^2) = x^{-\lambda_{\rm NS}}, \qquad \text{(A.12)}$$

where

$$\lambda_{\rm NS} \sim 2\sqrt{\bar{\alpha_s}}. \qquad \text{(A.13)}$$

For the fixed $\bar{\alpha_s} = 0.038$ the effective slope $\lambda_{\rm NS}$ is equal to 0.39.

REFERENCES

[1] V.N. Gribov, L.N. Lipatov, *Sov. J. Nucl. Phys.* **15**, 438 and 675 (1972); Yu.L. Dokshitzer, *Sov. Phys. JETP* **46**, 641 (1977); G. Altarelli, G. Parisi, *Nucl. Phys.* **B126**, 298 (1977).

[2] H1 collaboration: A. de Roeck *et al.*, 1995, Proc. Workshop on DIS and QCD, Paris, France, 24–28 April, 1995.

[3] ZEUS collaboration: B. Foster, Proc. Workshop on DIS and QCD, Paris, France, 24–28 April, 1995; *Z. Phys.* **C69**, 607 1996.

[4] SMC, D. Adams *et al., Phys. Lett.* **B329**, 399 (1994); D. Adams *et al., Phys. Lett.* **B336**, 125 (1994); D. Adams *et al., Phys. Rev.* **D56**, 5330 (1997); B. Adeva *et al., Phys. Lett.* **B412**, 414 (1997); SMC, B. Adeva *et al., Phys. Lett.* **B302**, 533 (1993); D. Adams *et al., Phys. Lett.* **B357**, 248 (1995); D. Adams *et al., Phys. Lett.* **B396**, 338 (1997).

[5] J.D. Bjørken, *Phys. Rev.* **148**, 1467 (1966); J.D. Bjørken, *Phys. Rev.* **D1**, 1376 (1970); J. Ellis, R.L. Jaffe, *Phys. Rev.* **D9**, 1444 (1974); J. Ellis, R.L. Jaffe, *Phys. Rev.* **D10**, 1669 (1974).

[6] J. Bartels, B.I. Ermolaev, M.G. Ryskin, *Z. Phys.* **C70**, 273 (1996); J. Bartels, B.I. Ermolaev, M.G. Ryskin, *Z. Phys.* **C72**, 627 (1996).

[7] J. Kwieciński, *Acta Phys. Pol.* **B27**, 893 (1996).

[8] A. Donnachie, P.V. Landshoff, *Phys. Lett.* **296B**, 257 (1992).

[9] E.A. Kuraev, L.N. Lipatov, V. Fadin, *Zh. Eksp. Teor. Fiz.* **72**, 373 (1977); E.A. Kuraev, L.N. Lipatov, V. Fadin, *Sov. Phys. JETP* **45**, 199 (1977); Ya.Ya. Balitzkij, L.N. Lipatov, *Yad. Fiz.* **28**, 1597 (1978); Ya.Ya. Balitzkij, L.N. Lipatov, *Sov. J. Nucl. Phys.* **28**, 822 (1978)); L.N. Lipatov, *Perturbative QCD*, Ed. A.H. Mueller, World Scientific, Singapore 1989, p.441; J.B. Bronzan, R.L. Sugar, *Phys. Rev.* **D17**, 585 (1978); T. Jaroszewicz, *Acta Phys. Pol.* **B11**, 965 (1980).

[10] J. Blümlein, A. Vogt, *Acta Phys. Pol.* **B27**, 1309 (1996); J. Blümlein, A. Vogt, *Phys. Lett.* **B386**, 350 (1996); J. Blümlein, S. Riemersma, A. Vogt, *Nucl. Phys. B (Proc. Suppl.)* **51C**, 30 (1996); J. Blümlein, S. Riemersma, *Acta Phys. Pol.* **B28**, 577 (1997); Y. Kiyo *et al., Z. Phys.* **C74**, 631 (1997).

[11] R. Kirschner, L.N. Lipatov, *Nucl. Phys.* **B213**, 122 (1983); R. Kirschner, *Z. Phys.* **C67**, 459 (1995).

[12] A. De Roeck *et al., Eur. Phys. J.* **C6**, 121 (1999); A. De Roeck, *Acta Phys. Pol.* **B29**, 1343 (1998).

[13] B.I. Ermolaev, M. Greco, S.I. Troyan, `hep-ph/9906276`; `hep-ph/0009037`; `hep-ph/0104082`; `hep-ph/0106317`; *Nucl. Phys.* **B571**, 137 (2000); *Nucl. Phys.* **B594**, 71 (2001).

[14] B. Badełek, J. Kwieciński, *Phys. Lett.* **B418**, 229 (1998).

[15] J. Kwieciński, *Acta Phys. Pol.* **B29**, 1201 (1998).

[16] D. Kotlorz, *Acta Phys. Pol.* **B31**, 1721 (2000).

[17] P.D.B. Collins, *An Introduction to Regge Theory and High Energy Physics*, Cambridge University Press, Cambridge 1977.

[18] M. Glück, E. Reya, M. Stratmann, W. Vogelsang, *Phys. Rev.* **D53**, 4775 (1996); M. Glück, E. Reya, A. Vogt, *Z. Phys.* **C67**, 433 (1995).