MASS DEPENDENCE OF HBT CORRELATIONS IN e^+e^- ANNIHILATION (II)

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This paper continues the study of the consequences of the Bjorken–Gottfried hypothesis for the HBT parameters measured in e^+e^- annihilation. It is shown that introducing a natural cut-off for transverse momenta of emitted particles, one can describe the observed ratio of transverse and longitudinal HBT radii for pions without destroying the good description of the mass dependence of the HBT parameters for heavier particles.

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1. Recently, we have investigated [1] the consequences of the hypothesis [2] that the generalized Bjorken–Gottfried relation [3], connecting the space–time position of a hadron produced in a high-energy collision to its 4-momentum. This relation is a consequence of the fast expansion of the system and can explain the mass-dependence of the interaction radii observed in e^+e^- annihilation at LEP 1 [4,5]. As discussed in detail in [2], this mass dependence is a manifestation of the well-known observation [6] that a correlation between the momentum and the emission point of a particle can drastically affect the results of the HBT correlation experiment. It was shown in [1] that this hypothesis explains quantitatively the observed mass effect. The anisotropy of the measured pion HBT radii [4] was, however, only qualitatively understood[1]. Although this problem is only peripheral

[1] This was pointed out to us by U. Heinz [7] who called our attention to the fact that, although the measured absolute values of the radii are rather uncertain, their ratios are much better determined.

with respect to our main interest (*i.e.* explaining the mass dependence of the measured HBT parameters), it seems interesting to check if our approach can accommodate this detail of $\pi\pi$ data.

In the present note we investigate this point and find that by introducing a natural cut-off on the transverse momenta of particles emitted from the expanding space-time "tube" (which is the source of particle emission in the Bjorken–Gottfried description of the production process) allows to describe correctly the ratio of longitudinal and transverse HBT radii without destroying the good description of their mass dependence.

2. The generalized Bjorken–Gottfried hypothesis, as formulated in [2], postulates the linear relation between the 4-momentum of the produced particle and the space–time position at which it is produced[2]:

$$q_\mu = \lambda x_\mu \, . \tag{1}$$

Relation (1) implies

$$\lambda = \frac{M_\perp}{\tau} \, , \tag{2}$$

where $M_\perp^2 = E^2 - q_{||}^2 = m^2 + q_\perp^2$, and

$$\tau = \sqrt{t^2 - z^2} \tag{3}$$

is the *longitudinal* proper time after the collision (t and z are time and longitudinal position of the production point).

Since this picture is purely classical, it represents only a qualitative idea, whose application to the description of the actual data requires an adequate reformulation taking into account the effects of the quantum nature of the system considered. In [2] we have proposed to use (1) and (2) as a guide-line for construction of the source function[3] $S(P,X)$ [9,10] related to the density matrix in momentum space by the Fourier transform

$$\rho\left(q = P + \tfrac{1}{2}Q,\, q' = P - \tfrac{1}{2}Q\right) = \int d^4X \mathrm{e}^{iQX} S(P,X) \, . \tag{4}$$

All the variables are four dimensional, so that both space and time integrations are involved. Thus specifying the source function fixes completely the single particle properties of the model.

Like the standard Wigner function [11], $S(P,X)$ gives approximately (as far as possible without contradicting quantum mechanics) the single-particle

[2] To our knowledge, the first application of relation (1) to a discussion of the quantum interference between identical particles was proposed (in a different context) by Csorgo and Zimanyi [8].

[3] It was called there a "generalized Wigner function".

distribution in momentum and in space–time. Therefore it has an intuitive meaning[4]. which can be exploited for implementation of the relations (1), (2).

Following [1] we thus postulate the source function in the factorized form

$$S(P,X) = F(\tau) S_{\|} S_{\perp} \,, \tag{5}$$

where

$$S_{\|} = \exp\left(\frac{1}{2\delta_{\|}^2}\left(P_+ - \frac{M_\perp}{\tau} X_+)\right)\left(P_- - \frac{M_\perp}{\tau} X_-)\right)\right) \,, \tag{6}$$

and

$$S_\perp = \exp\left(-\frac{X_\perp^2}{2r_\perp^2}\right) \exp\left(-\frac{P_\perp^2}{2\Delta^2}\right) \exp\left(-\frac{\left(\vec{P}_\perp - \frac{M_\perp}{\tau}\vec{X}_\perp\right)^2}{2\delta_\perp^2}\right) . \tag{7}$$

Here

$$X_\pm = t \pm z \,; \qquad P_\pm = P_0 \pm P_z \,, \tag{8}$$

so that

$$M_\perp^2 = P_+ P_- \,; \qquad \tau^2 = X_+ X_- \,. \tag{9}$$

We have used Gaussian forms in order to simplify the evaluation of the Fourier transform (4).

The first factor in $S_\perp$ represents a standard cylindrically symmetric "tube" of radius $r_\perp$ in configuration space[5]. The second factor introduces a natural limit on the transverse momenta of particles emitted from the tube[6]. The remaining one introduces a correlation between the momentum and the emission point of the particle, as required by the generalized Bjorken–Gottfried condition (1). It represents the key point of our model, as it is this factor which is responsible for the mass dependence of the HBT radii [1, 2, 6].

The parameters $\delta_{\|}$ and $\delta_\perp$ parametrize the correlation lengths and the function $F(\tau)$ gives the distribution of the proper time τ at which the particles are created.

[4] It should be realized that, in contrast to the standard Wigner function which relates the particle wave functions at different positions but at *the same time*, the source function relates the particle production amplitudes at different positions and *at different times.* (as is clearly seen from (4)). Consequently some care is needed in order to assess its physical interpretation.

[5] To simplify the argument, we ignore the rapidity and z dependence of the single particle spectrum. This seems a reasonable approximation in the central rapidity region at high energy and can be easily removed, if necessary.

[6] In the previous version of the model [1] this effect was neglected.

Substituting the formulae of this section into (4) one obtains the expression for the single-particle density matrix in momentum space in the form of integrals over the transverse position and pseudorapidity. These expressions were explicitly written down and the integrals evaluated in [1]. The results are summarized below.

$$\rho(q,q') = \int \tau d\tau F(\tau)\rho_{||}\rho_{\perp}\,, \tag{10}$$

$$\begin{aligned}\rho_{\perp}(\vec{q}_{\perp},\vec{q}\,'_{\perp}) &= 2\pi r_{\mathrm{eff}}^2 \exp\left(-\frac{\vec{P}^2}{2}\left(\frac{1}{\omega^2}+\frac{1}{\Delta^2}\right)-\frac{\vec{Q}_{\perp}^2 r_{\mathrm{eff}}^2}{2}\right) \\ &\quad\times \exp\left(-i\frac{M_{\perp}\tau v^2}{\omega^2}\vec{P}_{\perp}\vec{Q}_{\perp}\right),\end{aligned} \tag{11}$$

where

$$\omega^2 = M_{\perp}^2 v^2 + \delta_{\perp}^2\,; \quad v^2 = \frac{r_{\perp}^2}{\tau^2}\,; \quad r_{\mathrm{eff}}^2 = \frac{r_{\perp}^2\delta_{\perp}^2}{\omega^2} \tag{12}$$

$$\rho_{||} = 2\exp\left(\frac{M_{\perp}^2}{\delta_{||}^2}\right) K_0(s) \tag{13}$$

with

$$s^2 = \frac{M_{\perp}^4}{\delta_{||}^4} - \tau^2 Q_t^2 - i\frac{\tau M_{\perp}}{\delta_{||}^2}(m_{\perp}^2 - m_{\perp}'^2)\,, \tag{14}$$

where

$$P = \tfrac{1}{2}(q+q')\,; \quad Q = q - q'\,; \quad Q_t^2 = Q_0^2 - Q_{||}^2\,. \tag{15}$$

3. The single particle distribution is given by the diagonal elements of the density matrix. From the formulae of the previous section we thus obtain

$$\begin{aligned}\rho(q) &\equiv \frac{dn}{dy d^2 q_{\perp}} \\ &= 2\pi r_{\perp}^2\delta_{\perp}^2 \exp\left(\frac{m_{\perp}^2}{\delta_{||}^2}\right) K_0\left(\frac{m_{\perp}^2}{\delta_{||}^2}\right)\exp\left(-\frac{q_{\perp}^2}{2\Delta^2}\right) I(q_{\perp}^2)\,,\end{aligned} \tag{16}$$

where

$$I(q_{\perp}^2) = \int \tau d\tau F(\tau)\bar{\omega}^{-2}\exp\left(-\frac{q_{\perp}^2}{2\bar{\omega}^2}\right) \tag{17}$$

and

$$\bar{\omega}^2 = m_\perp^2 v^2 + \delta_\perp^2 \,. \tag{18}$$

To obtain information on the two-particle correlation function, one has to make further assumptions. We follow the standard treatment [9,10,12], assuming that one can evaluate the two-particle correlation function as if there were no other correlations between particles except for those induced by quantum interference. Under this condition the normalized two-particle correlation function is given by

$$C(q_1,q_2) = \frac{|\rho(q_1,q_2)|^2}{\rho(q_1)\rho(q_2)} \,. \tag{19}$$

Eq. (10) represents the density matrix as an integral over the proper time τ at which the particles are produced. In the present paper, following [1,2], we shall accept the approximation that the production happens in a very narrow interval of τ, so that the integration over τ simply amounts to introducing a fixed value $\tau = \tau_0$ in the formulae of the previous section. In this way the unknown function $F(\tau)$ is replaced by one parameter, τ_0 which fixes the overall scale of the problem. We take $\tau_0 = 0.9$ fm (small deviations from this value result in proportional changes in the obtained theoretical values of the HBT radii).

The other parameters are: v, Δ, $\delta_\perp$ and $\delta_{||}$, each with a clear physical meaning. As shown in [1], the results are rather insensitive to the value of $\delta_{||}$. Therefore we have restricted our analysis to the case when

$$\delta_\perp = \delta_{||} \equiv \delta \tag{20}$$

which turns out to be sufficient to describe the data[7].

The remaining three parameters were determined in two steps. First, δ and Δ were determined by the requirement that the transverse momentum distribution obtained from the data sample of $\approx 3 \times 10^5$ Z^0 hadronic decays measured in the DELPHI experiment [13] should be correctly described by formula (16) [8]. For each value of v, this procedure allows to determine fairly well both Δ and δ. In this way we are left with (practically) one free parameter, *i.e.* v. Once the correct description of the transverse momentum distribution is achieved, the two-particle correlation function (10) is calculated for several values of v and the corresponding radii $R_\perp$ and $R_{||}$

[7] It should be emphasized that our purpose is not to find the best values of the parameters of the model but only to show that there exist a set of their reasonable values which is not inconsistent with the pion data and can at the same time explain the observed mass-dependence of the HBT radii.

[8] A detailed description of the data is given in [1].

determined, following exactly the procedure used in [1] to which we refer the reader for a detailed description.

The final value of v was chosen by requiring

$$\frac{R_{||}}{R_{\perp}} = 1.4 \tag{21}$$

which roughly corresponds to the average ratio of the measured radii: 1.36 ± 0.04 calculated from the published data [4].

The calculations were also done for kaons, protons and Λ's. Since the available data samples for these particles are much smaller than those for pions, their $q_{\perp}^2$ distributions are not discriminative enough to pin-point reliably the model parameters. Therefore the parameters were taken the same as for pions (given in Table I). This assumption is to be verified once better data are available but we have checked that it reproduces reasonably well the main characteristics of the transverse momentum distributions of kaons, protons and Λ's.

TABLE I

The model parameters

v	δ (GeV)	Δ (GeV)
0.94	0.233	0.421

The mass dependence of the calculated $R_{||}$ and $R_{\perp}$ is plotted in Fig. 1 where also the available LEP data [4, 5] is shown.

One sees that the inequality $R_{||} > R_{\perp}$ is generally satisfied in the model, although the difference between the two radii at higher masses is not as large as in the case of pions. The data points in this figure at the pion mass represent the results for $R_{||}$ and $R_{\perp}$, whereas the points at higher masses correspond to the correlation radius R_0 determined in 1-dimensional analyses (the only available data for heavy particles). The points at kaon mass represent measurements for both K_s^0 and $K^{\pm}$ pairs. The measurements for Λ pairs come from spin analysis (except for the second ALEPH point with small error) where there is no need for a reference sample. The correspondence between R_0 and the two radii $R_{||}$ and $R_{\perp}$ is not obvious (at least experimentally), but the trend of the data is reasonably well reproduced by the model. More accurate data on kaons would be of great help to further elucidate this point.

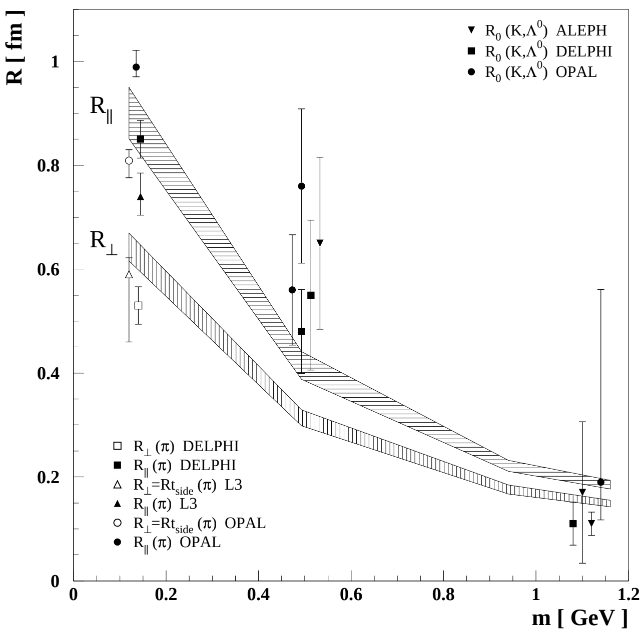

Fig. 1. $R_{\parallel}$ and $R_{\perp}$ calculated from the model (shaded bands) for π, K, p, Λ. Data points at $m = m_\pi$ represent results of 2- and 3-dimensional analysis of LEP data [4]. For 3-dim results the R_{Tside} was chosen as the representative geometrical transverse dimension of the pions source. Points at higher masses represent 1-dim source radius R_0 [5]

4. In conclusion, we have found that the correlation between the momentum and the production point of a produced hadron, suggested by the Gottfried–Bjorken hypothesis of the in-out cascade, seems to account for the observed correlation between identical particles observed in e^+e^- annihilation. In particular, it can account for the experimentally measured anisotropy of the two-pion correlation function. The mass dependence of the "source radius" is also adequately described. Large uncertainties, both in the theoretical determination of the model parameters, and in the experimental data do not allow, however, to obtain more quantitative conclusions.

Several comments are in order.

(i) The mass dependence of the effective HBT correlation radii calculated from the model and shown in Fig. 1 was obtained under the assumption that the parameters of the model do not depend on particle masses. For v, δ and Δ this assumption can be — in principle — verified, once better data are available. No such direct check is in sight for τ_0, however. Nevertheless, since the observed mass dependence of the

HBT correlation radii does agree — at least approximately — with the experimental observations, one may take it as an argument that *also* τ_0 *does not depend on particle mass*:

$$\tau_0 \sim \text{const.}\,(M_\perp)\,. \tag{22}$$

As emphasized in [1] this is a rather non-trivial conclusion, as it indicates that — within the Bjorken–Gottfried hypothesis (1) — all particles are emitted at, roughly, the same proper time τ_0. As discussed in [1], this property is not shared by some other models of particle production [14].

(ii) In the present paper, studying the two-particle distribution, we considered — following the approach employed in experimental analyses [4] — the boost invariant variable $Q_\perp^2$ and the variable $Q_{||}^2$ evaluated in the longitudinal center-of-mass system. Assuming boost invariance and azimuthal symmetry of the distributions, one finds that a complete analysis would involve 4 variables. As it is convenient to choose them boost invariant, one could use for instance the two transverse momenta $|p_{1\perp}|$ and $|p_{2\perp}|$, the relative azimuthal angle $\phi_1 - \phi_2$ and the relative rapidity $y_1 - y_2$. Such an analysis is under way [16].

(iii) Another interpretation of the experimentally observed HBT parameters was given in [17]. The authors take the point of view that the observed HBT radii do indeed correspond to the actual size of the particle emission region which is thus strongly dependent on the particle mass. They argue that this dependence may be understood from the uncertainty principle. This approach is rather different from ours. In our description the parameters characterizing the particle emission region are *mass independent* and the observed change in the HBT radii comes *solely* from the momentum-position correlation as expressed in the assumed Bjorken–Gottfried condition (1).

(iv) The Bjorken–Gottfried in–out mechanism is the simplest implementation of the idea that hadrons created in a high-energy collision emerge from a rapidly expanding "tube" (*cf.* [18]). The velocity of the longitudinal expansion is determined by the boost invariance of the system and that of transverse expansion is given by the parameter v. It may thus be interesting to extend this analysis to heavy ion collisions where qualitatively similar mass effects are expected.

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