RADIATIVE $\rho^+ \to \pi^+\gamma$ DECAY IN LIGHT CONE QCD

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We investigate the radiative $\rho^+ \to \pi^+\gamma$ decay in the framework of the light cone QCD sum rules. We estimate the coupling constant $g_{\rho\pi\gamma}$ for this decay and using this value of the coupling constant, we calculate the decay width of the $\rho^+ \to \pi^+\gamma$ decay. Our result is in good agreement with the experimental value.

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The determination of the fundamental parameters of hadrons from experimental data, in particular the coupling constants and form factors, requires some information about the physics at large distances. However, such nonperturbative information cannot be obtained directly from the fundamental QCD Lagrangian. Therefore, one has to employ some specific nonperturbative method for the determination of the parameters of hadrons. Among the various nonperturbative methods, QCD sum rules [1] have proved to be very useful in studying the properties of low-lying hadrons. In the traditional QCD sum rules method [1], hadronic parameters are connected with the QCD parameters through a few condensates of the nontrivial QCD vacuum structure in a nonperturbative way. Further progress has been achieved by an alternative method known as the QCD sum rules on the light cone. The light cone QCD sum rules are based on the operator product expansion on the light cone, which is an expansion over the twist of the operators rather than dimensions as in the traditional QCD sum rules. In this expansion, the main contribution comes from the lowest twist operators. The matrix elements of the nonlocal operators between a hadronic state and the vacuum define the hadronic wave functions which are the principle nonperturbative inputs into the sum rules. The applications of the light cone QCD sum rules to study hadronic properties can be found in [2–5] and references therein.

In the present work, we utilize the light cone QCD sum rules to study the $\rho^+ \to \pi^+\gamma$ decay which is described by magnetic dipole M1 transition amplitude [6].

The radiative transitions of the type $V \to P\gamma$ where V and P belong to the lowest multiplets of Vector (V) and Pseudoscalar (P) mesons and $VP\gamma$-couplings have been a subject of continuous interest in low-energy hadron physics [6]. The studies of these decays and $VP\gamma$-couplings were important to establish the basis of the quark model and SU(3) symmetry as well as to understand the symmetry-breaking effects [6]. On the other hand, $VP\gamma$-couplings also plays an important role in the photoproduction reactions of vector mesons on nucleons. At sufficiently high energies and low momentum transfers, electromagnetic production of vector mesons on nucleon targets has been explained by Pomeron exchange models [7]. However, at low energies near threshold scalar and pseudoscalar meson exchange mechanisms become important [8]. For the photoproduction of the ρ^0 meson, the effective coupling constant $g_{\rho\pi\gamma}$ is among the physical inputs for the calculation of the pseudoscalar exchange amplitude contributing to the photoproduction of ρ^0 meson. This coupling constant is introduced by choosing an effective Lagrangian describing the $VP\gamma$-vertex, which also defines this coupling constant, and it is then determined by using the experimental value of the decay width $\Gamma(\rho^0 \to \pi^0\gamma)$ of the radiative $\rho^0 \to \pi^0\gamma$ decay. However, in this decay the four-momentum of π^0 is time-like, $p^2 > 0$, whereas in the pseudoscalar exchange amplitude contributing to the photoproduction of ρ^0 meson it is space-like, $p^2 < 0$. Therefore, it will be of interest to study the effective coupling constant $g_{\rho\pi\gamma}$ from another point of view as well. In this work, we employ the light cone QCD sum rules to study the radiative $\rho^+ \to \pi^+\gamma$ decay and using isospin invariance estimate the coupling constant $g_{\rho\pi\gamma}$.

In order to study the $\rho \to \pi\gamma$ coupling constant, we consider the two point correlation function with photon

$$T_\mu(p,q) = i\int d^4x d^4y \mathrm{e}^{ipx}\langle\gamma(q)|T\{j^\rho_\mu(0)j_5(x)\}|0\rangle\,, \tag{1}$$

where j^ρ_μ and j_5 denote the interpolating currents for ρ^+ and π^+ mesons. We introduce these interpolating currents as [1]

$$\begin{aligned} j^\rho_\mu &= \overline{d}^a\gamma_\mu u^a\,, \\ j_5 &= \overline{d}^b i\gamma_5 u^b\,, \end{aligned} \tag{2}$$

where u, d are up and down quark fields, respectively, and a, b are the color indices. The overlap amplitudes of these interpolating currents with the meson states are defined as

$$\begin{aligned}\langle 0|j_\mu^\rho|\rho\rangle &= \lambda_\rho u_\mu\,,\\ \langle 0|j_5|\pi\rangle &= \lambda_\pi\,,\end{aligned}\tag{3}$$

where u_μ is the polarization vector of ρ meson. The coupling constant $g_{\rho\pi\gamma}$ for the $\rho \to \pi\gamma$ decay is defined as follows:

$$\langle \pi\gamma|\rho\rangle = i\frac{e}{m_\rho} g_{\rho\pi\gamma}\varepsilon^{\alpha\beta\gamma\delta} p_\alpha u_\beta q_\gamma \epsilon_\delta\,,\tag{4}$$

where ϵ_μ and q_μ are the polarization vector and momentum of the photon, respectively, and e is the electric charge.

In accordance with the QCD sum rules method strategy, we evaluate the two point correlation function both from a phenomenological point of view, and also from a theoretical approach in terms of QCD degrees of freedom. We then equate these two representations and construct the corresponding sum rule for the coupling constant $g_{\rho\pi\gamma}$.

The theoretical part of the sum rule for the coupling constant $g_{\rho\pi\gamma}$ is obtained in terms of QCD degrees of freedom by calculating the two point correlator in the deep Euclidean region where p^2 and $(p+q)^2$ are large and negative. In this calculation the full light quark propagator with both perturbative and nonperturbative contributions is used, and it is given as [9]

$$\begin{aligned} iS(x,0) &= \langle 0|T\{\overline{q}(x)q(0)\}|0\rangle \\ &= i\frac{\not x}{2\pi^2x^4} - \frac{\langle\overline{q}q\rangle}{12} - \frac{x^2}{192}m_0^2\langle\overline{q}q\rangle \\ &\quad -ig_s\frac{1}{16\pi^2}\int_0^1 du\left\{\frac{\not x}{x^2}\sigma_{\mu\nu}G^{\mu\nu}(ux) - 4iu\frac{x_\mu}{x^2}G^{\mu\nu}(ux)\gamma_\nu\right\} + \ldots\,,\end{aligned}\tag{5}$$

where terms proportional to light quark mass m_u or m_d are neglected. After straightforward computation we obtain

$$\begin{aligned} T_\mu(p,q) &= \int d^4x e^{ipx}\{-B\langle\gamma(q)|\overline{u}(x)\gamma_\mu\gamma_5 u(0)|0\rangle \\ &\quad -\tfrac{1}{2}A\left[\varepsilon_{\alpha\mu\sigma\rho}x_\alpha + ix_\rho g_{\mu\sigma} - ix_\sigma g_{\mu\rho}\right]\langle\gamma(q)|\overline{u}(x)\sigma_{\rho\sigma}u(0)|0\rangle\}\,,\end{aligned}\tag{6}$$

where$A = \frac{i}{2\pi^2x^4}$ and $B = -\frac{1}{12}\langle\overline{u}u\rangle - \frac{m_0^2}{192}\langle\overline{u}u\rangle x^2$. In order to evaluate the two point correlation function further, we need the matrix elements $\langle\gamma(q)|\overline{q}\gamma_\alpha\gamma_5 q|0\rangle$ and $\langle\gamma(q)|\overline{q}\sigma_{\alpha\beta}q|0\rangle$. These matrix elements are defined in terms of the photon wave functions [10–12]

$$\begin{aligned}
\langle\gamma(q)|\overline{q}\gamma_\alpha\gamma_5 q|0\rangle &= \frac{f}{4}e_q e\varepsilon_{\alpha\beta\rho\sigma}\epsilon^\beta q^\rho x^\sigma \int\limits_0^1 du \mathrm{e}^{iuqx}\psi(u)\,, \\
\langle\gamma(q)|\overline{q}\sigma_{\alpha\beta} q|0\rangle &= ie_q\langle\overline{q}q\rangle \int\limits_0^1 du \mathrm{e}^{iuqx} \\
&\quad \times\{(\epsilon_\alpha q_\beta - \epsilon_\beta q_\alpha)\left[\chi\phi(u) + x^2[g_1(u) - g_2(u)]\right] \\
&\quad + [qx(\epsilon_\alpha x_\beta - \epsilon_\beta x_\alpha) + \epsilon x(x_\alpha q_\beta - x_\beta q_\alpha)]\, g_2(u)\}\,, \qquad (7)
\end{aligned}$$

where the parameter χ is the magnetic susceptibility of the quark condensate and e_q is the quark charge, $\psi(u)$ and $\phi(u)$ stand for the leading twist-2 photon wave functions, while $g_1(u)$ and $g_2(u)$ are the two-particle photon wave functions of twist-4. The dimensional constant f is for normalization purposes [10]. In further analysis the path ordered gauge factor is omitted since we work in the fixed point gauge [13].

In order to construct the phenomenological part of the two point correlation function in Eq. (1), we note that the two point function $\mathrm{T}_\mu(p,q)$ satisfies a dispersion relation and we saturate this dispersion relation by inserting a complete set of one hadron states into the correlation function and obtain

$$T_\mu(p,q) = \frac{\langle\pi\gamma|\rho\rangle\langle\rho|j^\rho_\mu|0\rangle\langle 0|j_5|\pi\rangle}{[(p+q)^2 - m_\rho^2](p^2 - m_\pi^2)} + \ldots\,, \qquad (8)$$

where the contributions from the higher states and the continuum starting from some threshold s_0 are denoted by dots. In order to take these contributions into account we invoke the quark-hadron duality prescription and replace the hadron spectral density with the spectral density calculated in QCD.

After evaluating the Fourier transform for the M1 structure and then performing the double Borel transformation with respect to the variables $Q_1^2 = -p^2$ and $Q_2^2 = -(p+q)^2$, we finally obtain the following sum rule for the coupling constant $g_{\rho\pi\gamma}$

$$\begin{aligned}
g_{\rho\pi\gamma} &= \frac{3m_\rho(e_u+e_d)\langle\overline{u}u\rangle}{\lambda_\rho\lambda_\pi}\mathrm{e}^{m_\pi^2/M_1^2}\mathrm{e}^{m_\rho^2/M_2^2} \\
&\quad \times\left\{-\chi\phi(u_0)M^2 f_0(s_0/M^2) + 4\left(g_1(u_0) - g_2(u_0)\right)\right\}\,, \qquad (9)
\end{aligned}$$

where the function $f_0(s_0/M^2) = 1 - \mathrm{e}^{-s_0/M^2}$ is the factor used to subtract the continuum, s_0 being the continuum threshold, and

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2} \, , \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} \tag{10}$$

with M_1^2 and M_2^2 are the Borel parameters.

For the numerical evaluation of the sum rule we use the values $\langle \overline{u}u \rangle = -0.014$ GeV^3 [14], $m_\rho = 0.770$ GeV, $m_\pi = 0.140$ GeV [15], and $\chi = -3.3$ GeV^{-2} [10]. The overlap amplitude λ_π for the π meson state is given by the relation $\lambda_\pi = f_\pi \frac{m_\pi^2}{m_u + m_d}$ [16]. We use the experimental value $f_\pi = 0.132$ GeV and $m_u + m_d = 0.014$ GeV, and obtain this amplitude as $\lambda_\pi = 0.18$ GeV^2. We note that neglecting the electron mass the e^+e^- decay width of ρ^0 meson is given as $\Gamma(\rho^0 \to e^+e^-) = \frac{4\pi\alpha^2}{3} \frac{\lambda_\rho^2}{m_\rho^3}$. Then using the value obtained from the experimental leptonic decay width of ρ^0 [15], we obtain the value $\lambda_\rho = 0.118$ GeV^2 for the overlap amplitude of the ρ^0 meson. By isospin invariance we obtain the overlap amplitude for ρ^+ meson as $\lambda_\rho = 0.17$ GeV^2. In order to analyze the dependence of the coupling constant $g_{\rho\pi\gamma}$ on the Borel parameters M_1^2 and M_2^2, we study independent variations of M_1^2 and M_2^2. We find that the sum rule is quite stable for $M_1^2 = 0.5$ GeV^2 and for 0.6 GeV$^2 \leq M_2^2 \leq 1.4$ GeV^2. These limits on M_2^2 determine the allowed interval for the vector channel [17]. Moreover, we study the dependence of the sum rule on the threshold parameter s_0. The variation of the coupling constant $g_{\rho\pi\gamma}$ as a function of Borel parameters M_2^2 for different values of s_0 for $M_1^2 = 0.5$ GeV^2 is shown in Fig. 1 from which we conclude that the variation is very stable. The sources contributing to the uncertainty in the coupling constant are those due to variations in M_1^2, M_2^2, s_0 and the uncertainties in the estimated values of the vacuum condensate and the magnetic susceptibility. If we take these uncertainties into account by a conservative estimate, we obtain the coupling constant $g_{\rho\pi\gamma}$ as $g_{\rho\pi\gamma} = 0.64 \pm 0.05$. This value of the coupling constant is consistent with its value used in the analysis of ρ^0 photoproduction reactions through pseudoscalar exchange amplitude which is $g_{\rho\pi\gamma} = 0.54$ [18]. If we use $\langle \gamma\pi|\rho\rangle$ amplitude given in Eq. (2), then the decay width for $\rho^+ \to \pi^+\gamma$ is obtained as

$$\Gamma(\rho^+ \to \pi^+\gamma) = \frac{\alpha}{24} \frac{(m_\rho^2 - m_\pi^2)^3}{m_\rho^5} g_{\rho\pi\gamma}^2 \quad . \tag{11}$$

Therefore, from our analysis we determine $\Gamma(\rho^+ \to \pi^+\gamma)$ decay widths for $\rho^\pm$ mesons as $\Gamma(\rho^\pm \to \pi^\pm\gamma) = (86 \pm 14)$ KeV. Our result is in good agreement with the measured decay width [15], which is $\Gamma(\rho^\pm \to \pi^\pm\gamma) = (68 \pm 7)$ KeV.

In our analysis, we use the values for the overlap amplitudes λ_ρ and λ_π the values we obtain by relating these to the experimentally measured quantities. However, a QCD sum rule analysis [16] for these amplitudes yield the

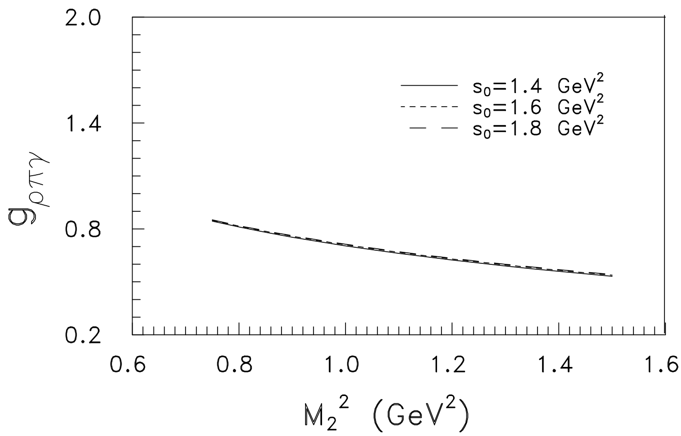

Fig. 1. The coupling constant $g_{\rho\pi\gamma}$ as a function of the Borel parameter M_2^2 for different values of the threshold parameters s_0 with M_1^2=0.5 GeV^2.

result λ_ρ=0.17 GeV^2, λ_π=0.17 GeV^2 which are very close to the values we obtain. Moreover, an independent phenomenological analysis [19] gives the values λ_ρ=0.17 GeV^2, λ_π=0.20 GeV^2 for these amplitudes. If we use these latter values for the overlap amplitudes in our analysis, we then obtain for the coupling constant and the decay width the values $g_{\rho\pi\gamma} = 0.58 \pm 0.04$ and $\Gamma(\rho^\pm \to \pi^\pm\gamma) = (71 \pm 10)$ KeV. We note that the electromagnetic decays $V \to P\gamma$ of vector mesons in the flavor SU(3) sector was studied previously [16] by employing the method of QCD sum rules in the presence of the external electromagnetic field, and the results $g_{\rho\pi\gamma} = 0.59$ and $\Gamma(\rho^\pm \to \pi^\pm\gamma) = 68$ KeV were obtained. The values obtained in our analysis are consistent with these results. Therefore, our results which are obtained utilizing the light cone QCD sum rules supplements the study of this decay using QCD sum rules in external field method.

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