MULTISTEP DIRECT REACTIONS OF 14 MeV NEUTRONS

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Multistep direct cross sections including the contribution of incoherent particle–hole and collective excitations in the continuum, are calculated and combined with multistep compound and compound nucleus cross sections to give a complete description of neutron inelastic scattering by niobium at incident energy of 14.1 MeV. The multistep direct reactions are enhanced by using the non-DWBA matrix elements that include the biorthogonally conjugated distorted waves. The results reveal contributions from two- and three-step reactions in agreement with experiment.

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1. Introduction

In the quantum-mechanical theories of multistep direct (MSD) reactions [1–3], the direct excitation of the collective degrees of freedom is not treated explicitly. Therefore any collective effects have been masked by adjusting the parameters of the incoherent MSD reaction or taken into account by introducing *ad hoc* corrections [4,5]. Subsequent simultaneous analyses of nucleon inelastic scattering and charge-exchange reactions showed that a good description of both types of reactions is possible only at the expense of inconsistent parametrization of the MSD reaction model [6]. Further attempts to explicitly include all the phonon, multi-phonon and giant resonance states observed by experiment, resulted in particularly strong collective contributions in the continuum [7]. However, the simple additive method of [7] leads

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to double counting of the excitation strength associated with certain values of the multipolarity λ. The problem of double-counting was overcome by applying the Energy-Weighted Sum Rules (EWSR's) to determine the extent to which the strength of a given multipolarity is exhausted by the incoherent particle–hole or collective excitation modes, respectively [8,9]. It was thereby established, that the double-differential cross section of a one-step direct reaction $(d^2\sigma/dEd\Omega)_{1\mathrm{SD}}$ includes contributions from (i) coherent excitations of collective one-phonon vibrations of multipolarity $\lambda \leq 4$ and (ii) incoherent excitations of particle–hole pairs of orbital angular momentum transfer $l > 4$. Such a differentiation is in accord with the results of studies of the giant isoscalar resonances. On the one hand, no one-phonon states of multipolarity higher than hexadecapole have been identified experimentally, on the other, collective excitations of higher multipolarity appear rather smeared out and show up as part of the continuum [10]. The closed form expression for the one-step direct (1SD) reaction cross section obtained by Marcinkowski and Mariański [9] has been used in calculations of cross sections of MSD reactions in the framework of the theory of Feshbach, Kerman, and Koonin [1] (FKK). The results revealed substantial contributions from multi-phonon, multi-particle–hole and the multitude of mixed particle–hole–phonon excitations to the continuum at incident energies of 20 MeV and 26 MeV. Important contributions from two-, three- and four-step reactions were found [11,12]. In this paper we apply this method to the analysis of neutron inelastic scattering by niobium at incident energy of 14.1 MeV [13] in order to show how the multistep process develops with projectile energy.

2. Calculations and results

The cross section for a 1SD reaction obtained by Marcinkowski and Mariański reads [9],

$$\left(\frac{d^2\sigma}{dEd\Omega}\right)_{1\mathrm{SD}} = \sum_{n_{\lambda\leq 4}} \beta^2_{n_\lambda}\left(\frac{d\sigma}{d\Omega}\right)^{\mathrm{DWBA-macr}}_{n_\lambda} f_\lambda[\hbar\omega_{n_\lambda},\Gamma]$$

$$+\sum_{\alpha,\beta}\sum_{l>4}(2l+1)Pg_\pi^{p_\pi+h_\pi}g_\nu^{p_\nu+h_\nu}UR_2(l)V^2_{\alpha,\beta}\left\langle\left(\frac{d\sigma}{d\Omega}\right)^{\mathrm{DWBA-micr}}_{l}\right\rangle . \quad (1)$$

The macroscopic DWBA cross sections in the first, (vib) term of the r.h.s. of Eq. (1) were calculated with form factors $F_\lambda = -R\partial U/\partial R$ obtained by using the complex optical potential of [14]. The β_{n_λ} are deformation parameters of the one-phonon states. The f_λ is the energy distribution function, assumed to be Gaussian with width adjusted to the experimental energy resolution for the one-phonon levels, or Lorentzian with width typical of the giant resonances. These cross sections are due to the isoscalar electric excitation of the one-phonon states, 2^+_1 at 0.93 MeV with $\beta_2 = 0.13$,

2_3^+ at 2.49 MeV with $\beta_2 = 0.8$ and 3_1^- at 2.30 MeV with $\beta_3 = 0.18$, as well as the excitation of the dipole, quadrupole and the Low Energy component of the Octupole (LEOR) giant Resonances in ^{92}Zr. The LEOR exhausts 30% of the octupole strength [10]. The weak coupling model multiplets were used for ^{93}Nb. The indices α and β denote neutrons ν or protons π in the second (ph) term of the r.h.s. of Eq. (1). The sum over α and β contains two terms for nucleon scattering, namely those corresponding to the excitation of a neutron ph-pair and a proton ph-pair. $P = 1/2$ is the parity distribution and $R_2(l)$ is the spin distribution of the density of the $1p1h$ states, $\omega_{1,1} = g_\pi^{p+h} g_\nu^{p+h} U$ [15]. The latter was taken with $g_\pi = Z/13$ and $g_\nu = N/13$ as the single-particle state densities for protons and neutrons, respectively. The microscopic incoherent DWBA cross sections in (ph) were calculated with a real effective interaction of Yukawa form with 1 fm range. The strength was assigned the standard values $V_{\pi\pi} = V_{\nu\nu} = 12.7$ MeV and $V_{\pi\nu} = V_{\nu\pi} = 43.1$ MeV [16], slightly increased due to the dependence on the incident energy [17]. The microscopic cross sections were averaged over the final particle–hole states $(j_p j_h^{-1})_{\rm lm}$ of the shell model contained in 1 MeV intervals. Both the macroscopic and the microscopic cross sections were calculated with the DWUCK-4 code [18]. The spectroscopic amplitude $(2j_h + 1)^{1/2}$ was used in the microscopic option of DWUCK-4.

The multistep cross section of the FKK theory is obtained by multiple folding of the one-step cross section [1,19],

$$\begin{aligned}\left(\frac{d^2\sigma}{dEd\Omega}\right)_{\rm MSD} &= \int \frac{m_1 E_1}{(2\pi)^2\hbar^2} dE_1 d\Omega_1 \\ &\times \int \frac{m_2 E_2}{(2\pi)^2\hbar^2} dE_2 d\Omega_2 \ldots \int \frac{m_{M-1} E_{M-1}}{(2\pi)^2\hbar^2} dE_{M-1} d\Omega_{M-1} \\ &\times \left(\frac{d^2\sigma}{dE_M d\Omega_M}\right)_{\rm 1SD} \ldots \left(\frac{d^2\sigma}{dE_2 d\Omega_2}\right)_{\rm 1SD} \left(\frac{d^2\sigma}{dE_1 d\Omega_1}\right)_{\rm 1SD} . \end{aligned} \tag{2}$$

E_i and m_i are the energy and mass of the scattered nucleon after the i-th stage of the reaction. The final states in the (ph) components of the 1SD cross sections in Eq. (2) are assumed to be $1p1h$ states independent on the reaction stage M. On the other hand, each phonon in the (vib) cross section in Eq. (2) results in multi-phonon states [20] built on the final phonon states of the preceding reaction stage. The energies of the multi-phonon states are sums of energies of the constituent phonons. Therefore, it is important to include into the (vib) component only one-phonon states. A fine integration grid $\Delta E_i \leq 1$ MeV is required in Eq. (2) in order to get rid of spurious one-phonon peaks from the multi-phonon spectra and to saturate the shapes and magnitudes of the calculated cross sections [11].

The cross sections describing the successive transitions in Eq. (2) are derived, except for the last one, from non-DWBA matrix elements. The non-DWBA matrix elements are a result of the biorthogonality of the distorted waves [21]. They can be expressed in terms of the normal DWBA matrix elements including the inverse elastic S-matrix factor [22]. Therefore the corresponding partial l-wave cross sections are multiplied by the modulus squared of the inverse elastic matrix element S_l^{-2}. These enhancing factors apply not only to the excitation of the incoherent ph-pairs but also to those that add coherently to a collective vibration, since the distorted waves, whether the excited states are single-particle ones or collective, are the same eigenfunctions of the complex optical potential and form a complete set with the adjoint distorted waves. Thus $(M-1)$ out of the M 1SD cross sections in Eq. (2) contain a sum of the enhanced $(S^{-2}\mathrm{vib}) = \sum_{\lambda\leq 4}(S_\lambda)^{-2}\sigma_\lambda(\mathrm{vib})$ and $(S^{-2}\mathrm{ph}) = \sum_{l>4}(S_l)^{-2}\sigma_l(\mathrm{ph})$ cross sections. As a result the multistep cross sections of Eq. (2) contain the following combinations of the two terms [11,12]:

2SD, $(S^{-2}\mathrm{vib},\mathrm{vib}) + (S^{-2}\mathrm{ph},\mathrm{vib}) + (S^{-2}\mathrm{vib},\mathrm{ph}) + (S^{-2}\mathrm{ph},\mathrm{ph})$,

3SD, $(S^{-2}\mathrm{vib},S^{-2}\mathrm{vib},\mathrm{vib}) + (S^{-2}\mathrm{ph},S^{-2}\mathrm{vib},\mathrm{vib}) + (S^{-2}\mathrm{vib},S^{-2}\mathrm{ph},\mathrm{vib})$
$+(S^{-2}\mathrm{vib},S^{-2}\mathrm{vib},\mathrm{ph}) + (S^{-2}\mathrm{ph},S^{-2}\mathrm{ph},\mathrm{vib}) + (S^{-2}\mathrm{ph},S^{-2}\mathrm{vib},\mathrm{ph})$
$+(S^{-2}\mathrm{vib},S^{-2}\mathrm{ph},\mathrm{ph}) + (S^{-2}\mathrm{ph},S^{-2}\mathrm{ph},\mathrm{ph})$,

4SD, *etc.*

where for simplicity we have omitted the limits of the summations. The elastic scattering matrix elements, $\mid S_l \mid^2 = (1-T_l)$,are expressed in terms of the partial wave transmission coefficients T_l of the optical potential of [14]. In the calculations of the MSD cross sections, we have only considered contributions from processes where the leading particle in the intermediate stages is a neutron. Thus we have included the terms (n,n'',n'), (n,n'',n''',n') and (n,n'',n''',n'''',n') ignoring terms such as $(n,p,n'),(n,N,N',n')$ and (n,N,N',N'',n'), where N is either a neutron or a proton. It is expected that at the incident energies considered here the cross sections for charge-exchange reactions or proton inelastic scattering are significantly smaller than those for neutron scattering, so the contribution from the latter terms should be negligible.

The results for the incident energy of 14 MeV are shown in Figs. 1–3, as a function of emission energy.

The corresponding partial cross sections, obtained after integrating over angle and outgoing energy, are included in Table I. The collective one-phonon cross sections (vib) exhaust the dipole, quadrupole and octupole EWSR's limits and the incoherent one-step (ph) excitations observe the limits for the transferred orbital angular momenta l>4. The decomposition of the 2SD, 3SD and 4SD cross sections into the mixed contributions of (vib) and (ph) shown in Table I does not agree with the conclusions of [23], according

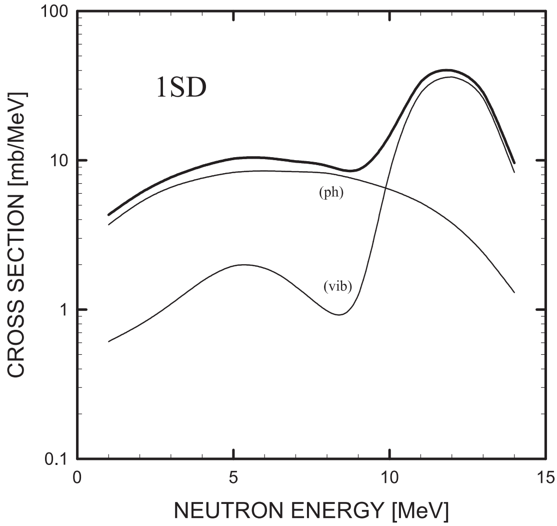

Fig. 1. The calculated cross section for the 1SD component of the ^{93}Nb$(n,n')^{93}$Nb reaction, at incident neutron energy of 14 MeV (thick solid line). The contributions due to excitation of one-phonon collective states of multipolarity $\lambda \leq 4$ and to incoherent excitation of ph-pairs of transferred orbital angular momenta $l > 4$ are shown separately as thin lines.

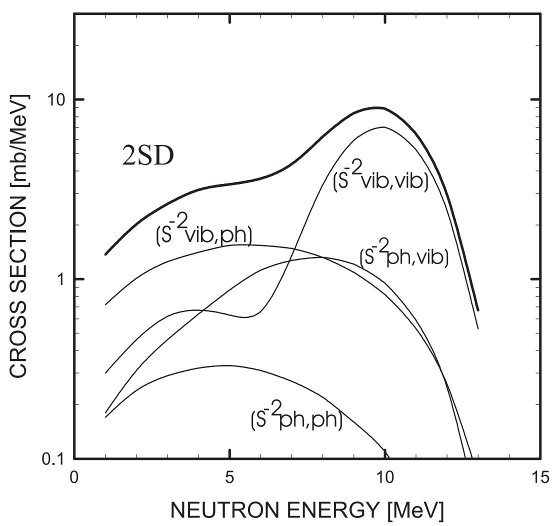

Fig. 2. The same as in Fig. 1 but for the 2SD (thick solid line). The four contributions resulting from Eq. (2) are shown as thin lines.

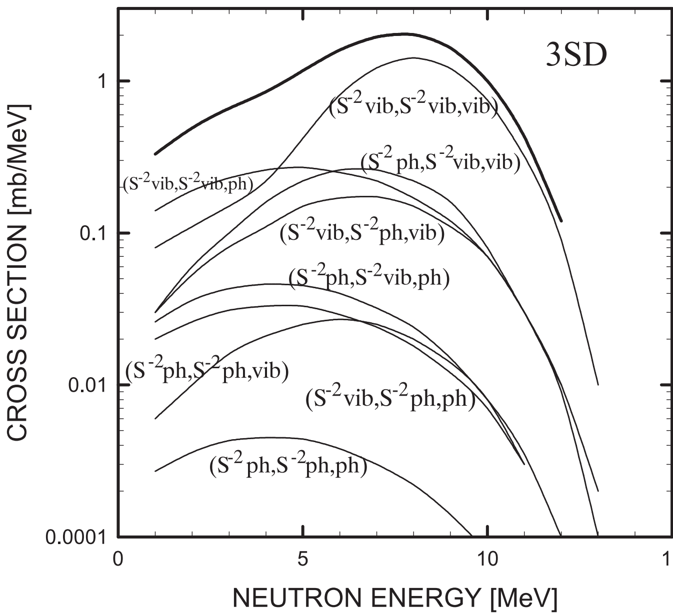

Fig. 3. The same as in Fig. 1 but for the 3SD (thick solid line). The eight contributions resulting from Eq. (2) are also shown as thin lines.

TABLE I

The decomposition of the MSD cross sections for the ^{93}Nb$(n,n')^{93}$Nb reaction at 14 MeV.

σ(1SD)	(mb)	σ(2SD)	(mb)	σ(3SD)	(mb)
(ph)	82	$(S^{-2}\,\mathrm{vib},\mathrm{vib})$	29	$(S^{-2}\,\mathrm{vib},S^{-2}\,\mathrm{vib},\mathrm{vib})$	6.8
(vib)	119	$(S^{-2}\,\mathrm{vib},\mathrm{ph})$	13	$(S^{-2}\,\mathrm{vib},S^{-2}\,\mathrm{vib},\mathrm{ph})$	2.0
		$(S^{-2}\,\mathrm{ph},\mathrm{vib})$	9	$(S^{-2}\,\mathrm{ph},S^{-2}\,\mathrm{vib},\mathrm{vib})$	1.6
		$(S^{-2}\,\mathrm{ph},\mathrm{ph})$	3	$(S^{-2}\,\mathrm{vib},S^{-2}\,\mathrm{ph},\mathrm{vib})$	1.2
				$(S^{-2}\,\mathrm{ph},S^{-2}\,\mathrm{vib},\mathrm{ph})$	0.3
				$(S^{-2}\,\mathrm{vib},S^{-2}\,\mathrm{ph},\mathrm{ph})$	0.2
				$(S^{-2}\,\mathrm{ph},S^{-2}\,\mathrm{ph},\mathrm{vib})$	0.2
				$(S^{-2}\,\mathrm{ph},S^{-2}\,\mathrm{ph},\mathrm{ph})$	0.03
Total	201		54		12.33

to which the cross sections for the MSD reactions are dominated by the multi-ph contributions, while the mixed terms and the multi-phonon ones are negligible. On the contrary, we observe that the relative contributions due to the mixed multi-ph-phonon as well as the multi-phonon excitations increase with increasing number of reaction steps. This is not surprising since the excitation energy available at each stage of the multistep reaction decreases allowing for the low energy phonon excitations mainly. On the whole, the results show substantial contributions from multistep processes. The integrated 2SD and 3SD neutron emission cross sections at the incident energy of 14 MeV are 54 mb and 12 mb respectively, compared to the 201 mb of the 1SD cross section which is the sum of (ph) = 82 mb and (vib) = 119 mb.

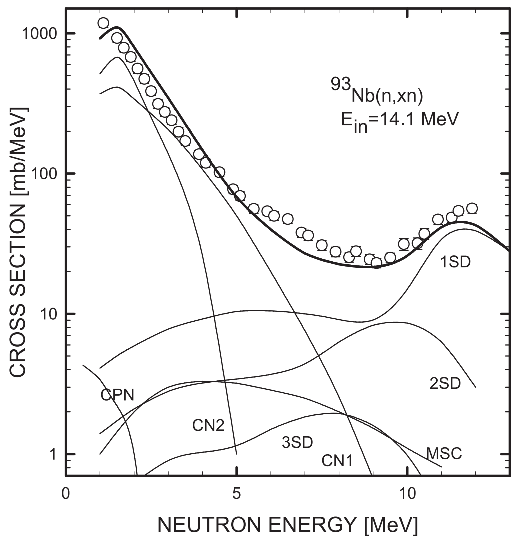

Fig. 4. Comparison of the calculated cross sections with the spectrum of neutrons from the ^{93}Nb$(n,xn)^{93}$Nb reaction measured at incident energy of 14.1 MeV [13]. The thick line is a sum of all contributions shown. The labels CN1 and CN2 denote the primary and secondary neutrons evaporated from the compound nucleus, respectively. CPN denotes secondary neutrons preceded by evaporation of a proton and MSC labels the sum of emissions from the three steps of the preequilibrium compound reaction. The cross sections of the 2SD and 3SD reactions were calculated according to Eqs. (1) and (2).

The cross sections for CN emission of the low energy neutrons were calculated according to the theory of Hauser and Feshbach. The multistep compound (MSC) reaction cross sections were calculated in the framework of the theory of FKK [1,19], allowing for the gradual absorption [24] of incident flux into the quasi-bound states of the MSC reaction chain. The radial overlap integral of the single-particle wave functions in the MSC cross section was calculated with constant wave functions within the nuclear volume. The overlap integral was subsequently corrected by a factor of $\frac{1}{2}$ [25] in order to approximate the result of the microscopic calculation. The resulting cross sections are thus about $\frac{1}{4}$ of those obtained in previous analyses [25].

The results of the MSD, MSC and CN calculations are compared with the inclusive neutron spectrum measured at incident energy of 14.1 MeV [13] in Fig. 4. The overall agreement between theory and experiment is very good over the entire energy range. The structures observed in the calculated 2SD and 3SD spectra can be identified as those arising from the superposition of multiphonon excitations onto a smooth background of ph-excitations. The peak in the 1SD spectrum at the highest outgoing energy is due to the three low energy one-phonon states given above. The

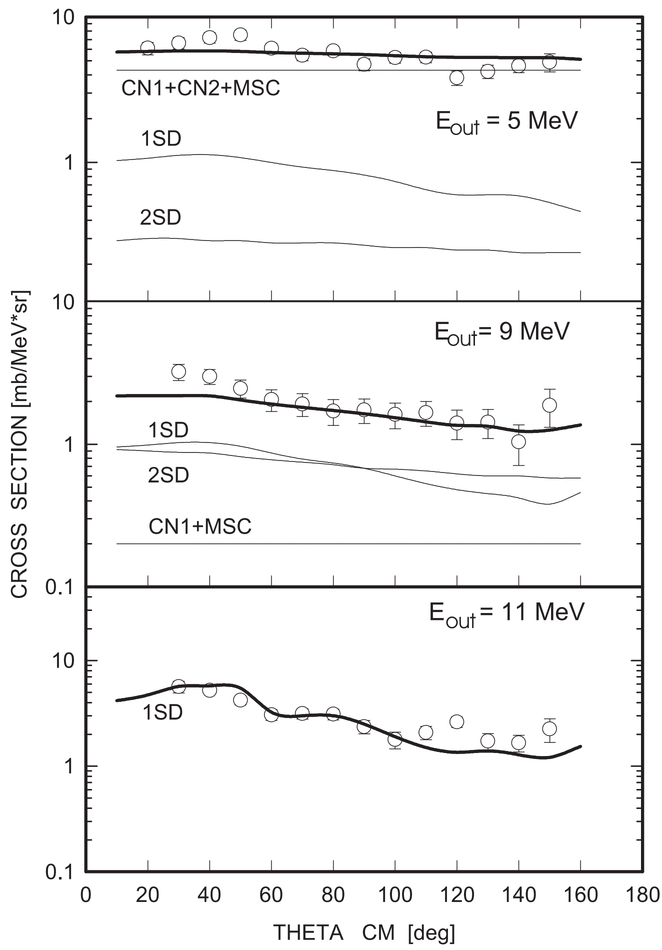

Fig. 5. Comparison of the calculated double-differential cross sections with the angular distributions of neutrons from the ^{93}Nb$(n,xn)^{93}$Nb reaction measured at incident energy of 14.1 MeV [13]. The thick lines are the sums of the contributions shown. The outgoing neutron energies are given. The labels CN1, CN2 and MSC are the same as in Fig. 4.

energy distribution of these states was assumed to be Gaussian with width $\Gamma = 2$ MeV to account for the experimental energy resolution and the spread of the weak coupling multiplets. The one-phonon structure, folded in Eq. (2) into a two- and three-phonon one, can then be observed in the 2SD and 3SD spectra at energies twice and trice the excitation energy of the one-phonon states, respectively. It is also worth noting that the contribution of the MSC component is insignificant compared to that of the MSD. This is partly due to the approximation of the microscopic calculation of the radial overlap integral and partly due to the gradual absorption.

The experimental angular distributions are also satisfactorily reproduced at all the outoing energies, as shown in Fig. 5. Only 1SD and 2SD reactions contribute significantly at the incident energy of 14 MeV. The cross sections for the ^{93}Nb(n,xn)^{93}Nb reaction have also been described by using the basis of collective states of the RPA. Due to the complexity of the calculations involved, only the first two steps of the reaction were calculated [26]. The results obtained at the incident energy of 14 MeV were 202 mb for the 1SD reactions and 57 mb for the 2SD ones, in excellent agreement with the results of the present work (see Table I). The shapes of the emission spectra of [26] also resemble the ones obtained in the present work, as shown in Fig. 6. One could therefore argue that the new 1SD cross section of Eq. (1) in conjunction with Eq. (2) present a closed form approximation of the RPA cross sections.

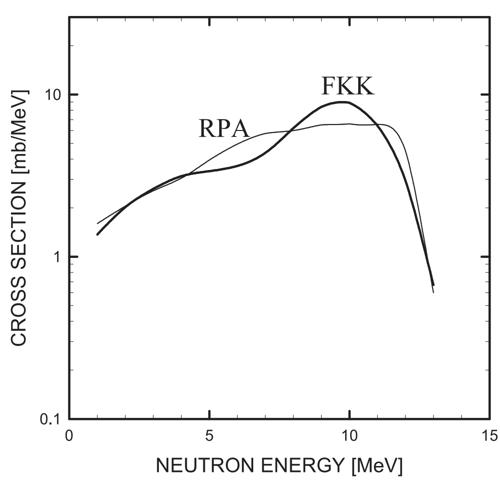

Fig. 6. Comparison of the 2SD cross section calculated according to Eqs. (1) and (2) for the $^{93}\mathrm{Nb}(n,n')^{93}\mathrm{Nb}$ reaction at incident energy of 14 MeV (labelled FKK) with the corresponding cross section obtained by Lenske *et al.* using the RPA basis states [26].

3. Conclusions

The MSD cross sections calculated in the framework of the FKK theory, using (i) the new 1SD cross section of Eq. (1) and (ii) the non-DWBA matrix elements in Eq. (2) are able to reproduce the data for neutron inelastic scattering by niobium. The results of the calculations show important contributions from two-step reactions at 14 MeV. The contribution of the 3SD reactions amounts to only 6% of the 1SD cross section and therefore can be neglected. The use of non-DWBA matrix elements in Eq. (2) leads to an enhancement of the MSD cross sections. In fact, the non-DWBA MSD cross sections are larger than the corresponding normal DWBA MSD cross sections by a factor of $(3.5)^{M-1}$. Furthermore, the enhanced non-DWBA MSD cross sections support the gradual absorption of the incoming flux into the quasibound compound states of increasing complexity [24,27,28]. On the other hand, gradual absorption combined with more accurate microscopic overlap integrals result in reduced MSC cross sections. Therefore, the MSD reactions dominate over the MSC ones even at an incident energy as low as 14 MeV.

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