BOOST-INVARIANT PARTICLE PRODUCTION IN TRANSPORT EQUATIONS*

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The boost-invariant tunneling of particles along the hyperbolas of constant invariant time $\tau = \sqrt{t^2 - z^2}$ is included in the transport equations describing formation of the quark–gluon plasma in strong color fields. The non-trivial solutions of the transport equations exist if the boost-invariant distance between the tunneling particles, measured in the quasirapidity space $\eta = 1/2 \ln((t+z)/(t-z))$, is confined to a finite interval $\Delta\eta$. For realistic values of $\Delta\eta$ the solutions of the transport equations show similar characteristics to those found in the standard approach, where the tunneling takes place at constant time t. In the limit $\Delta\eta \to \infty$, the initial color fields decay instantaneously.

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1. Introduction

In this paper we solve transport equations describing production of the quark–gluon plasma in strong color fields. A novel feature of our approach is the implementation of the boost-invariant tunneling along the hyperbolas of constant invariant time $\tau = \sqrt{t^2 - z^2}$ into the framework of the relativistic kinetic theory. In this way we generalize the previous results of Refs. [1–4].

In the standard WKB description of the tunneling process [5, 6], the particles tunnel at fixed time t. They emerge from the vacuum at a certain distance from each other, and their longitudinal momenta vanish (in the center-of-mass frame of a pair). In Refs [1–4, 7], the WKB results were used to fix the longitudinal-momentum dependence of the production rates of quarks and gluons. The effect of the finite distance appearing between the tunneling particles is more difficult to include. One assumes usually that

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this distance is small, and the non-local character of the tunneling process is neglected. This approximation is not always valid. For large transverse mass, the distance between the tunneling particles may be quite substantial and should be taken into account.

The main difficulty connected with the non-local features of the tunneling concerns the causal properties of sequential decays. In this case one cannot define which pair is produced earlier or later, which leads to ambiguity in the determination of the decay probabilities. Nevertheless, at very high energies a solution to this problem exists [8]. Assuming that the pairs are produced at fixed invariant time $\tau = \sqrt{t^2 - z^2}$, one introduces a Lorentz-invariant sequence of pair production, and the requirements of causality can be naturally fulfilled. The tunneling of particles along the hyperbolas of constant invariant time leads to finite longitudinal momenta of the created particles. Thus, the production rates in the kinetic equations should include also this extra effect.

The boost-invariant description of the tunneling process was the main ingredient of the simulation program used to describe the space–time evolution of the color-flux tubes [9,10]. In particular, this program was applied in the investigations of intermittency [11] and soft photon emission [12]. In the present paper the boost-invariant tunneling along the hyperbolas of constant invariant time is included in the kinetic equations. In contrast to the simulation studies [11,12], which were restricted to the case of the elementary color fields, we now deal with stronger fields. Similarities and differences between the present approach and Refs. [1,2] are discussed in detail.

2. Pair production in strong chromoelectric fields

2.1. Boost-invariant tunneling

A semi-classical boost-invariant description of pair production in chromoelectric fields was introduced in Ref. [8]. In this approach the tunneling particles move along the hyperbola of constant invariant time

$$\tau = \sqrt{t^2 - z^2}. \tag{1}$$

If the virtual particles start their motion at $z = 0$, see Fig. 1, the end points of the tunneling trajectory may be determined from the energy–momentum conservation laws

$$E_f = \sqrt{m_\perp^2 + p_{f||}^2} = F\, z_f, \qquad p_{f||} = F\,(t_f - \tau)\,. \tag{2}$$

Here E_f and $p_{f||}$ are the energy and the longitudinal momentum of the particle at the space–time point where it emerges from the vacuum, $m_\perp$ is

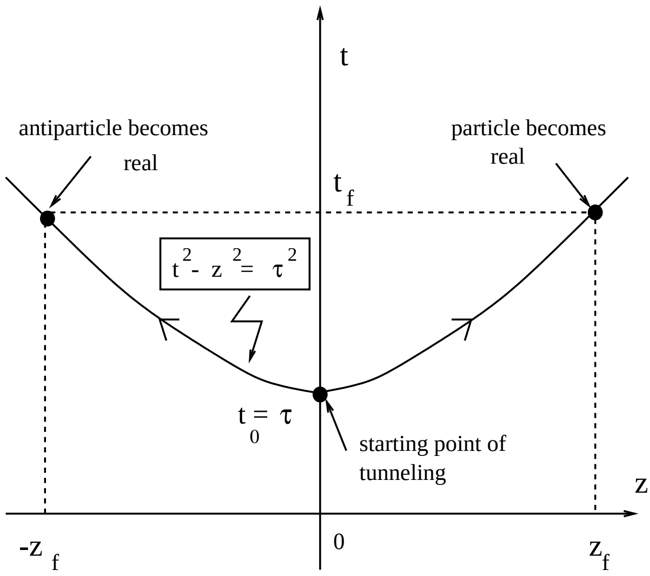

Fig. 1. Boost-invariant tunneling of particles along the trajectory of constant invariant time $\tau = \sqrt{t^2 - z^2}$

the transverse mass, and F is a constant force acting on the particle (in this Section we shall concentrate our discussion on the case $F > 0$ only). Using the rapidity variable y we may write

$$E_f = m_\perp \cosh y_f \ , \qquad p_{f||} = m_\perp \sinh y_f \ . \tag{3}$$

In the analogous way we define the quasirapidity variable η, which gives

$$t_f = \tau \cosh \eta_f \ , \qquad z_f = \tau \sinh \eta_f . \tag{4}$$

Equations (2), (3) and (4) yield [8]

$$\sinh y_f = \frac{m_\perp}{2F\tau}, \qquad \eta_f = 2\ y_f . \tag{5}$$

An interesting feature of Eq. (5) is that the kinematical quantities characterizing the tunneling particles depend on τ, *i.e.*, pairs with different momenta are created at different invariant times. This is an effect of the boundary conditions which describe expansion of the system. For very large τ, when the boundary of the field is far away from the center of the system we recover the standard results: $p_{f||} = 0$, $z_f = m_\perp/F$ and $t_f = \tau$.

Although the boundary conditions change the kinematics of the tunneling process, the probability of tunneling per unit volume of space–time does not change [8]. Thus, we may use the well established formula for the production rate [5,6,13,14]

$$\frac{dN}{d^4x\ d^2p_\perp} = \frac{F}{4\pi^3}\left|\ln\left(1 \mp \exp\left(-\frac{\pi m_\perp^2}{F}\right)\right)\right|. \tag{6}$$

Here the minus sign is appropriate for fermions (in our case for quarks and antiquarks) and the plus sign should be used for bosons (in our case for gluons). Eq. (6) gives the rate integrated over the longitudinal momentum. The $p_{||}$-dependence may be taken into account by the following modification of (6)

$$\frac{dN}{d\Gamma} \equiv p^0 \frac{dN}{d^4x\ d^3p} = \frac{F}{4\pi^3}\left|\ln\left(1 \mp \exp\left(-\frac{\pi m_\perp^2}{F}\right)\right)\right| \delta\left(y - \eta + \frac{1}{2}\eta_f\right). \tag{7}$$

The production rate (7) is boost invariant and reduces to the previous formula if divided by p^0 and integrated over $p_{||}$. Moreover, the rapidity of the particles which are produced at $\eta = \eta_f$ is simply $y_f = \frac{1}{2}\eta_f$, the result required by the condition of the boost-invariant tunneling discussed above. In the limit of large invariant times or large color fields we find $\eta_f \longrightarrow 0$, hence the standard formula for tunneling is recovered: the longitudinal momenta of particles tunneling at $z = 0$ are zero.

It is important to emphasize that the production rate (6) was derived for a constant force F [5,6,13,14]. The use of this formula in the case of color fields which are changing in time (due to the screening effect) requires justification. The work in this direction has been done in Refs. [15–19], where a complete field-theoretic treatment of this problem was performed in electrodynamics. The results of Ref. [19] show that the use of the semiclassical production term in the transport equation is valid if a separation of different time scales can be achieved: the time scales associated with quantum phase oscillations and amplitudes of pair creation should be much smaller than the time scales associated with the oscillations of the fields. In the present paper we assume that such a separation is possible.

2.2. Boost-invariant variables w and v

In the next Sections we shall use the boost-invariant variables introduced in Refs [20,21]

$$u = \tau^2 = t^2 - z^2, \quad w = tp_{||} - zE, \quad \boldsymbol{p}_\perp, \tag{8}$$

and also

$$v = Et - p_{||}\ z = \sqrt{w^2 + m_\perp^2 u}\,. \tag{9}$$

From these two equations one can easily find the energy and the longitudinal momentum of a particle

$$E = p^0 = \frac{vt + wz}{u}\,, \quad p_{||} = \frac{wt + vz}{u}\,. \tag{10}$$

The invariant measure in the momentum space is

$$dP = d^2p_\perp \frac{dp_{||}}{p^0} = d^2p_\perp \frac{dw}{v}\,. \tag{11}$$

In addition we have

$$w = \tau m_\perp \sinh\left(y - \eta\right), \qquad v = \tau m_\perp \cosh\left(y - \eta\right). \tag{12}$$

Equation (12) allows us to rewrite the production rate in the form

$$\frac{dN}{d\Gamma} = p^0 \frac{dN}{d^4x\ d^3p} = \frac{F}{4\pi^3}\left|\ln\left(1 \mp \exp\left(-\frac{\pi m_\perp^2}{F}\right)\right)\right| \delta\left(w - w_0\right) v\,, \tag{13}$$

where

$$w_0 = \tau m_\perp \sinh\left(y_f - \eta_f\right) = -\tau m_\perp \sinh\left(\frac{\eta_f}{2}\right) = -\frac{m_\perp^2}{2F}\,. \tag{14}$$

2.3. Finite-size corrections

Comparing Eqs (8) and (14) we find that the longitudinal momenta of the tunneling particles at $z = 0$ are $-m_\perp^2/(2F\tau)$. Thus, for large invariant times $p_{||} \to 0$. On the other hand, in the limit of small invariant times $p_{||}$ becomes infinite. As we shall see in more detail in Section 4, this divergence leads to infinite values of color currents at $\tau = 0$, and to infinite decay rate of the initial chromoelectric field. The physical origin of this singularity is the possibility of the creation of particles at infinite values of t and z, which correspond to small values of the invariant time τ. In practice, we always deal with finite systems and such tunneling cannot take place. In order to take into account such finite-size corrections we impose an additional condition on the tunneling process, namely [10]

$$2\eta_f = 4\mathrm{Arcsinh}\left(\frac{m_\perp}{2F\tau}\right) < \Delta\eta\,. \tag{15}$$

Here $\Delta\eta$ is a parameter which determines the space–time region in quasi-rapidity, $|\eta| < \Delta\eta$, where the tunneling is possible. The tunneling particles should fit into this region, hence the distance between the emerging members of a pair should be smaller than $\Delta\eta$. As a consequence, our final expression for the production rate is [1]

$$\begin{aligned} \frac{dN}{d\Gamma} &= \frac{F}{4\pi^3}\left|\ln\left(1 \mp \exp\left(-\frac{\pi m_\perp^2}{F}\right)\right)\right| \\ &\quad\times\theta\left[2\tau F\sinh\left(\frac{\Delta\eta}{4}\right) - m_\perp\right]\delta\left(w - w_0\right)v\,, \end{aligned} \tag{16}$$

where θ is the step function

$$\theta(x) = 1 \quad \text{for} \quad x > 0, \quad \theta(x) = 0 \quad \text{for} \quad x \le 0\,. \tag{17}$$

We note that condition (15) was also used in the simulation program [9,10]. In that case, the allowed region in the quasirapidity space was determined by the actual size of a decaying color flux tube. In the present study, the size of $\Delta\eta$ is suggested by the rapidity range accessible in the ultra-relativistic heavy-ion collisions at SPS and RHIC. In the following, we consider three typical values: $\Delta\eta = 4$, 6 and 8.

3. Semi-classical kinetic equations for quark–gluon plasma

In the *Abelian dominance approximation*, the equations for quarks, antiquarks and gluons have the form [2,22,23]

$$\left(p^\mu\partial_\mu \pm g\boldsymbol{\epsilon}_i\cdot\boldsymbol{F}^{\mu\nu}p_\nu\partial^p_\mu\right)G^\pm_i(x,p) = \frac{dN^\pm_i}{d\Gamma}, \tag{18}$$

$$\left(p^\mu\partial_\mu + g\boldsymbol{\eta}_{ij}\cdot\boldsymbol{F}^{\mu\nu}p_\nu\partial^p_\mu\right)\tilde{G}_{ij}(x,p) = \frac{d\tilde{N}_{ij}}{d\Gamma}, \tag{19}$$

where $G^+_i(x,p)$, $G^-_i(x,p)$ and $\tilde{G}_{ij}(x,p)$ are the phase-space densities of quarks, antiquarks and gluons, respectively. Here g is the strong coupling constant, and $i,j = (1,2,3)$ are color indices. The terms on the left-hand-side describe the free motion of the particles and the interaction of the particles with the mean field $\boldsymbol{F}_{\mu\nu}$. The terms on the right-hand-side describe production of quarks and gluons due to the decay of the field. The distribution functions $G^\pm_i$ and $\tilde{G}_{ij}$ include the spin degeneracy factors. In the numerical calculations we neglect the quark masses and assume that Eq. (18) holds for $N_f = 3$ flavors. We note that Eqs (18) and (19) do not include the thermalization effects. The latter can be taken into account in the relaxation time approximation (see for example Refs [24–26]).

[1] We neglect here the boundary effects: The tunneling processes can start from a place close to the edge of the system and may have not enough space to fit into the allowed region. Such situations should be eliminated by introducing an additional constraint.

The only non-zero components of the tensor $\boldsymbol{F}_{\mu\nu} = (F^3_{\mu\nu}, F^8_{\mu\nu})$ are those corresponding to the chromoelectric field $\boldsymbol{\mathcal{E}}$, which may be written as

$$\boldsymbol{\mathcal{E}} = \boldsymbol{F}^{30} = -2\frac{d\boldsymbol{h}}{du} = -\frac{1}{\tau}\frac{d\boldsymbol{h}}{d\tau}\,. \tag{20}$$

Here $\boldsymbol{h}$ is a function of the variable $u = \tau^2$ only (note that $\boldsymbol{\mathcal{E}}$ is invariant under Lorentz boosts along the z-axis). The quarks couple to the chromoelectric field $\boldsymbol{\mathcal{E}}$ through the charges [27]

$$\boldsymbol{\epsilon}_1 = \frac{1}{2}\left(1, \sqrt{\frac{1}{3}}\right), \quad \boldsymbol{\epsilon}_2 = \frac{1}{2}\left(-1, \sqrt{\frac{1}{3}}\right), \quad \boldsymbol{\epsilon}_3 = \left(0, -\sqrt{\frac{1}{3}}\right). \tag{21}$$

The gluons couple to $\boldsymbol{\mathcal{E}}$ through the charges $\boldsymbol{\eta}_{ij}$ defined by relation

$$\boldsymbol{\eta}_{ij} = \boldsymbol{\epsilon}_i - \boldsymbol{\epsilon}_j. \tag{22}$$

According to our discussion from the previous Section, the production rates of quarks and antiquarks in the chromoelectric field $\boldsymbol{\mathcal{E}}$ are

$$\frac{dN_i^{\pm}}{d\Gamma} = \mathcal{R}_i(\tau, p_\perp)\delta\left(w \mp w_i\right) v, \tag{23}$$

where we have defined

$$\mathcal{R}_i(\tau, p_\perp) \;=\; \frac{\Lambda_i}{4\pi^3}\left|\ln\left(1 - \exp\left(-\frac{\pi p_\perp^2}{\Lambda_i}\right)\right)\right| \theta\left[2\tau\Lambda_i \sinh\left(\frac{\Delta\eta}{4}\right) - p_\perp\right], \tag{24}$$

$$\Lambda_i \;=\; \left(g\left|\boldsymbol{\epsilon}_i \cdot \boldsymbol{\mathcal{E}}\right| - \sigma_q\right)\theta\left(g\left|\boldsymbol{\epsilon}_i \cdot \boldsymbol{\mathcal{E}}\right| - \sigma_q\right), \tag{25}$$

and

$$w_i = -\frac{p_\perp^2}{2\Lambda_i}\mathrm{sign}\left(\boldsymbol{\epsilon}_i \cdot \boldsymbol{\mathcal{E}}\right). \tag{26}$$

The quantity Λ_i describes the effective force acting on the tunneling quarks. The effect of the screening of the initial field by the tunneling particles is taken into account by the subtraction of the "elementary force" characterized by the string tension σ_q. Similarly, for gluons we have

$$\frac{d\tilde{N}_{ij}}{d\Gamma} = \tilde{\mathcal{R}}_{ij}(\tau, p_\perp)\delta\left(w - w_{ij}\right) v, \tag{27}$$

where

$$\tilde{\mathcal{R}}_{ij}(\tau,p_\perp) \;=\; \frac{\Lambda_{ij}}{4\pi^3}\left|\ln\left(1+\exp\left(-\frac{\pi p_\perp^2}{\Lambda_{ij}}\right)\right)\right|\theta\left[2\tau\Lambda_{ij}\sinh\left(\frac{\Delta\eta}{4}\right)-p_\perp\right], \tag{28}$$

$$\Lambda_{ij} \;=\; \left(g\left|\boldsymbol{\eta}_{ij}\cdot\boldsymbol{\mathcal{E}}\right|-\sigma_g\right)\theta\left(g\left|\boldsymbol{\eta}_{ij}\cdot\boldsymbol{\mathcal{E}}\right|-\sigma_g\right), \tag{29}$$

and

$$w_{ij} = -\frac{p_\perp^2}{2\Lambda_{ij}}\mathrm{sign}\left(\boldsymbol{\eta}_{ij}\cdot\boldsymbol{\mathcal{E}}\right). \tag{30}$$

We note that the string tension of a tube spanned by gluons is three times stronger than that of a quark tube, $\sigma_g = 3\sigma_q$ [3].

The implementation of the boost invariance in Eqs. (18) and (19) leads us to the following form of the transport equations [2]

$$\frac{\partial G_i^\pm}{\partial\tau} \mp g\boldsymbol{\epsilon}_i\cdot\frac{d\boldsymbol{h}}{d\tau}\frac{\partial G_i^\pm}{\partial w} = \tau\mathcal{R}_i\left(\tau,p_\perp\right)\delta\left(w\mp w_i(\tau,p_\perp)\right), \tag{31}$$

$$\frac{\partial\tilde{G}_{ij}}{\partial\tau} - g\boldsymbol{\eta}_{ij}\cdot\frac{d\boldsymbol{h}}{d\tau}\frac{\partial\tilde{G}_{ij}}{\partial w} = \tau\tilde{\mathcal{R}}_{ij}\left(\tau,p_\perp\right)\delta\left(w-w_{ij}(\tau,p_\perp)\right). \tag{32}$$

Their formal solution is

$$G_i^\pm\left(\tau,w,p_\perp\right) = \int\limits_0^\tau d\tau'\;\tau'\,\mathcal{R}_i\left(\tau',p_\perp\right)\delta\left(\Delta h_i\left(\tau,\tau'\right)\pm w - w_i(\tau',p_\perp)\right), \tag{33}$$

$$\tilde{G}_{ij}\left(\tau,w,p_\perp\right) = \int\limits_0^\tau d\tau'\;\tau'\;\tilde{\mathcal{R}}_{ij}\left(\tau',p_\perp\right)\delta\left(\Delta h_{ij}\left(\tau,\tau'\right)+w-w_{ij}(\tau',p_\perp)\right), \tag{34}$$

where

$$\Delta h_i\left(\tau,\tau'\right) \equiv g\boldsymbol{\epsilon}_i\cdot\left[\boldsymbol{h}\left(\tau\right)-\boldsymbol{h}\left(\tau'\right)\right], \tag{35}$$

$$\Delta h_{ij}\left(\tau,\tau'\right) \equiv g\boldsymbol{\eta}_{ij}\cdot\left[\boldsymbol{h}\left(\tau\right)-\boldsymbol{h}\left(\tau'\right)\right]. \tag{36}$$

One may notice that the distribution functions (33) and (34) satisfy the following symmetry relations

$$G_i^-\left(\tau,w,p_\perp\right) = G_i^+\left(\tau,-w,p_\perp\right), \quad \tilde{G}_{ij}\left(\tau,w,p_\perp\right) = \tilde{G}_{ji}\left(\tau,-w,p_\perp\right). \tag{37}$$

We note also that the time integrals in (33) and (34) reveal the non-Markovian character of the particle production mechanism: the behavior of the system at a time τ is determined by the whole evolution of the system in the time interval $0\le\tau'\le\tau$.

4. Color currents

Equations (18) and (19) show how the particles behave under the influence of the field. In order to obtain a self-consistent set of equations we should have also the dynamic equation for the field. It can be written in the following Maxwell form

$$\partial_\mu \boldsymbol{F}^{\mu\nu}(x) = \boldsymbol{j}^\nu(x) + \boldsymbol{j}^\nu_D(x), \tag{38}$$

where $\boldsymbol{j}^\nu$ is the *conductive current* (related to the simple fact that particles carry color charges $\boldsymbol{\epsilon}_i$ and $\boldsymbol{\eta}_{ij}$) and $\boldsymbol{j}^\nu_D$ is the *displacement current* (induced by the tunneling of quarks and gluons from the vacuum).

4.1. Conductive current

The form of the conductive current is standard

$$\boldsymbol{j}^\nu(x) = g \int dP\, p^\nu \left[N_f \sum_{i=1}^{3} \boldsymbol{\epsilon}_i \left(G_i^+(x,p) - G_i^-(x,p) \right) + \sum_{i,j=1}^{3} \boldsymbol{\eta}_{ij} \tilde{G}_{ij}(x,p) \right]. \tag{39}$$

Substituting the quark and gluon distribution functions (33) and (34) into Eq. (39), and using Eqs. (10) and (37) we find that $\boldsymbol{j}^\nu(x)$ has the following space–time structure

$$\boldsymbol{j}^\nu(x) = \left[\boldsymbol{j}^0(x), 0, 0, \boldsymbol{j}^3(x) \right] = [z,0,0,t] \boldsymbol{\mathcal{J}}(\tau), \tag{40}$$

where

$$\boldsymbol{\mathcal{J}}(\tau) = \frac{2g}{u} \int d^2p_\perp \frac{dw\ w}{v} \left[N_f \sum_{i=1}^{3} \boldsymbol{\epsilon}_i\, G_i^+(\tau,w,p_\perp) + \sum_{i>j}^{3} \boldsymbol{\eta}_{ij}\, \tilde{G}_{ij}(\tau,w,p_\perp) \right]. \tag{41}$$

The use of the explicit form of the distribution functions G_i and $\tilde{G}_{ij}$ in Eq. (41) gives

$$\begin{aligned} \boldsymbol{\mathcal{J}}(\tau) \ =\ & -\frac{2gN_f}{u} \sum_{i=1}^{3} \boldsymbol{\epsilon}_i \int_0^\tau d\tau'\ \tau' \int d^2p_\perp \frac{\mathcal{R}_i(\tau',p_\perp)\left[\Delta h_i - w_i(p_\perp,\tau')\right]}{\sqrt{\left[\Delta h_i - w_i(p_\perp,\tau')\right]^2 + p_\perp^2 u}} \\ & -\frac{2g}{u} \sum_{i>j}^{3} \boldsymbol{\eta}_{ij} \int_0^\tau d\tau'\ \tau' \int d^2p_\perp \frac{\tilde{\mathcal{R}}_{ij}(\tau',p_\perp)\left[\Delta h_{ij} - w_{ij}(p_\perp,\tau')\right]}{\sqrt{\left[\Delta h_{ij} - w_{ij}(p_\perp,\tau')\right]^2 + p_\perp^2 u}}. \end{aligned} \tag{42}$$

4.2. Displacement current

The structure of the displacement current is less obvious. One can find the form of $\boldsymbol{j}^\nu_D$ through the analysis of the energy and momentum conservation laws for both the matter (quarks and gluons) and the field

$$\partial_\mu T^{\mu\nu}_{\rm matter}(x) + \partial_\mu T^{\mu\nu}_{\rm field}(x) = 0. \tag{43}$$

The $\nu = 0$ component of this equation gives for the field part

$$\partial_\mu T^{\mu 0}_{\rm field} = \frac{\partial}{\partial t}\left(\frac{1}{2}\boldsymbol{\mathcal{E}}^2\right) = \boldsymbol{\mathcal{E}}\cdot\frac{\partial \boldsymbol{\mathcal{E}}}{\partial t} = -\boldsymbol{F}^{30}\cdot\frac{\partial \boldsymbol{F}^{03}}{\partial t}, \tag{44}$$

and for the matter part

$$\begin{aligned}\partial_\mu T^{\mu 0}_{\rm matter} &= -g\boldsymbol{F}^{\mu\nu}\cdot\int dP\, p^0\, p_\nu\, \partial^p_\mu\left[N_f\sum_{i=1}^{3}\boldsymbol{\epsilon}_i\left(G^+_i - G^-_i\right) + \sum_{i,j=1}^{3}\boldsymbol{\eta}_{ij}\tilde{G}_{ij}\right]\\ &\quad + \int dP\, p^0\left[N_f\sum_{i=1}^{3}\left(\frac{dN^+_i}{d\Gamma} + \frac{dN^-_i}{d\Gamma}\right) + \sum_{i,j=1}^{3}\frac{d\tilde{N}_{ij}}{d\Gamma}\right],\end{aligned} \tag{45}$$

where we used Eqs. (18) and (19). Using the last results we may write

$$\begin{aligned}\boldsymbol{F}^{30}\cdot\frac{\partial \boldsymbol{F}^{03}}{\partial t} &= \boldsymbol{F}^{30}\cdot g\int dP\, p^3\left[N_f\sum_{i=1}^{3}\boldsymbol{\epsilon}_i\left(G^+_i - G^-_i\right) + \sum_{i,j=1}^{3}\boldsymbol{\eta}_{ij}\tilde{G}_{ij}\right]\\ &\quad + \boldsymbol{F}^{30}\cdot\int dP\,\left[N_f\sum_{i=1}^{3}\frac{p^0\boldsymbol{\epsilon}_i}{\boldsymbol{\epsilon}_i\cdot\boldsymbol{F}^{30}}\left(\frac{dN^+_i}{d\Gamma} + \frac{dN^-_i}{d\Gamma}\right)\right.\\ &\quad \left.+\sum_{i>j}^{3}\frac{p^0\boldsymbol{\eta}_{ij}}{\boldsymbol{\eta}_{ij}\cdot\boldsymbol{F}^{30}}\left(\frac{d\tilde{N}_{ij}}{d\Gamma} + \frac{d\tilde{N}_{ji}}{d\Gamma}\right)\right].\end{aligned} \tag{46}$$

Similarly, we may analyze the $\nu = 3$ component of Eq. (43). The conclusion is that the field equations (38) represent a sufficient condition for the conservation of energy and momentum if the displacement current has the structure

$$\boldsymbol{j}^\nu_D(x) = \left[\boldsymbol{j}^0_D(x), 0, 0, \boldsymbol{j}^3_D(x)\right] = [z, 0, 0, t]\boldsymbol{\mathcal{J}}_D(\tau) \tag{47}$$

where

$$\begin{aligned}\boldsymbol{\mathcal{J}}_D(\tau) &= \frac{N_f}{\tau^2}\sum_{i=1}^{3}\frac{\boldsymbol{\epsilon}_i}{\boldsymbol{\epsilon}_i\cdot\boldsymbol{\mathcal{E}}}\int dw\, d^2p_\perp\left(\frac{dN^+_i}{d\Gamma} + \frac{dN^-_i}{d\Gamma}\right)\\ &\quad + \frac{1}{\tau^2}\sum_{i>j}^{3}\frac{\boldsymbol{\eta}_{ij}}{\boldsymbol{\eta}_{ij}\cdot\boldsymbol{\mathcal{E}}}\int dw\, d^2p_\perp\left(\frac{d\tilde{N}_{ij}}{d\Gamma} + \frac{d\tilde{N}_{ji}}{d\Gamma}\right).\end{aligned} \tag{48}$$

Integration over w in Eq. (48) can be easily done

$$\begin{aligned}\boldsymbol{\mathcal{J}}_D(\tau) &= \frac{2N_f}{\tau^2}\sum_{i=1}^{3}\frac{\boldsymbol{\epsilon}_i}{\boldsymbol{\epsilon}_i\cdot\boldsymbol{\mathcal{E}}}\int d^2p_\perp\sqrt{w_i^2+p_\perp^2 u}\,\mathcal{R}_i(\tau,p_\perp)\\ &\quad+\frac{2}{\tau^2}\sum_{i>j}^{3}\frac{\boldsymbol{\eta}_{ij}}{\boldsymbol{\eta}_{ij}\cdot\boldsymbol{\mathcal{E}}}\int d^2p_\perp\sqrt{w_{ij}^2+p_\perp^2 u}\,\tilde{\mathcal{R}}_{ij}(\tau,p_\perp)\,. \qquad (49)\end{aligned}$$

We note that the form of Eqs (40) and (47) implies that both the conductive and displacement currents are conserved separately

$$\partial_\nu\,\boldsymbol{j}^\nu(x)=0\,, \qquad \partial_\nu\,\boldsymbol{j}_D^\nu(x)=0\,. \qquad (50)$$

We also note that the integral over $p_\perp$ is restricted by condition (15), implicitly included in $\mathcal{R}_i$ and $\tilde{\mathcal{R}}_{ij}$. If we did not use our finite-size correction, the displacement current would diverge at small τ, $\boldsymbol{\mathcal{J}}_D(\tau)\sim\tau^{-2}$, and the field equation would be singular, compare Eq. (51). In the standard case (with zero longitudinal momenta of the tunneling particles and with no finite-size corrections), the quantities w_i and w_{ij} are zero, hence $\boldsymbol{\mathcal{J}}_D(\tau)\sim\tau^{-1}$ for small τ, and the field equation is regular in the limit $\tau\to 0$

5. Field equations

With the all substitutions required by the boost invariance, the field equation (38) may be written as

$$\frac{d\boldsymbol{\mathcal{E}}(\tau)}{d\tau} = -\tau\left[\boldsymbol{\mathcal{J}}(\tau)+\boldsymbol{\mathcal{J}}_D(\tau)\right] \qquad (51)$$

or

$$\frac{d^2\boldsymbol{h}(\tau)}{d\tau^2} = \frac{1}{\tau}\frac{d\boldsymbol{h}(\tau)}{d\tau}+\tau^2\left[\boldsymbol{\mathcal{J}}(\tau)+\boldsymbol{\mathcal{J}}_D(\tau)\right]. \qquad (52)$$

This is an integro-differential equation for the function $\boldsymbol{h}(\tau)$, because the conductive current $\boldsymbol{\mathcal{J}}(\tau)$ depends not only on $\boldsymbol{h}(\tau)$ but also on all the values of $\boldsymbol{h}(\tau')$ for $0\le\tau'\le\tau$. Eq. (52) has to be solved numerically step by step for given initial values. These are taken in the form [2, 3]

$$\boldsymbol{h}(0)=0\,, \quad \frac{1}{\tau}\frac{d\boldsymbol{h}}{d\tau}(0) = -\boldsymbol{\mathcal{E}}_0 = -\sqrt{\frac{2\sigma_g}{\pi r^2}}k\boldsymbol{q}\,. \qquad (53)$$

Here the Gauss law has been used to determine the initial strength of the chromoelectric field $\boldsymbol{\mathcal{E}}_0$ in terms of the transverse radius of the color-flux-tube (πr^2= 1 fm^2), the string tension ($\sigma_g=3\sigma_q=3$ GeV/fm), and the number of color charges k. Since the exchange of color charges at the initial stage of a heavy-ion collision leads to the color fields spanned by gluons [28], we assume that $\boldsymbol{q}$ is one of the gluon color charges $\boldsymbol{\eta}_{ij}$. In practice we take $\boldsymbol{q} = \boldsymbol{\eta}_{12}$, so only the third component of the chromoelectric field, $\boldsymbol{\mathcal{E}}^{(3)}$, is present in our numerical calculations. The solution of Eq. (52) is independent of the initial condition for $\boldsymbol{h}\,(\tau)$ because of the cancellations connected with the gauge transformation which leaves $\boldsymbol{\mathcal{E}}$ unchanged.

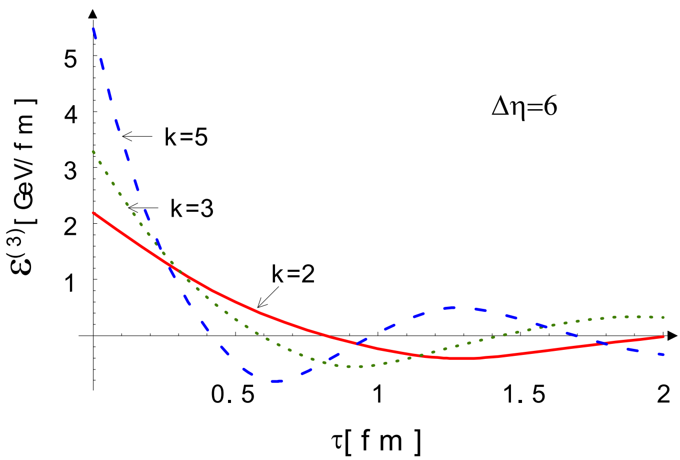

Fig. 2. Time dependence of the chromoelectric field for different values of k. The maximal quasirapidity interval allowed for tunneling is fixed, $\Delta\eta$=6.

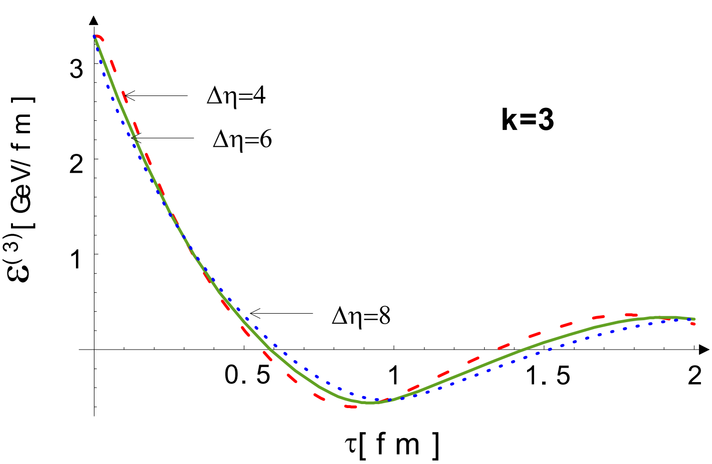

Fig. 3. Time dependence of the chromoelectric field for different values of $\Delta\eta$. The initial field strength is fixed, $k = 3$.

In Fig. 2 we plot the time dependence of the chromoelectric field, as calculated from Eqs (52) and (53) for different values of the parameter k. The maximal allowed quasirapidity interval $\Delta\eta$ is 6 in this case. We observe the field oscillations with the frequency growing with k. The shape of the oscillations is almost identical to those found before in Ref. [2]. Clearly, the modification of the tunneling process does not affect the field behavior in this case. In Fig. 3 we show the time dependence of the field for the fixed initial strength, $k = 3$, and for different values of $\Delta\eta$. In the three considered cases, $\Delta\eta = 4$, 6 and 8, the field oscillations are very similar. Looking in more detail, we observe that with increasing $\Delta\eta$, the chromoelectric field decreases faster at the very initial stage of the process, *i.e.*, for $0 < \tau < 0.1$ fm. Later the decrease of the field is weaker and for $\tau \sim 0.3$ fm the values of the chromoelectric fields are approximately the same for different values of $\Delta\eta$. For longer times one may notice that the period of the field oscillations is slightly longer for larger values of $\Delta\eta$ — a faster initial decay of the field causes a faster back reaction of the induced currents, and the subsequent slow down of the decay.

6. Energy density and pressure of the plasma

The energy–momentum tensor of the quark–gluon plasma has a structure

$$T^{\mu\nu}_{\mathrm{matter}} = \left[\varepsilon\left(\tau\right) + P\left(\tau\right)\right] u^\mu u^\nu - P\left(\tau\right) g^{\mu\nu}, \tag{54}$$

where in our two-dimensional model

$$u^\mu = \frac{1}{\tau}\left(t, z\right) . \tag{55}$$

It is important to emphasize that in our case the standard form of the energy–momentum tensor, Eq. (54), does not follow from the assumption of the local thermodynamic equilibrium, but it is a direct consequence of the boost-invariance. Eq. (54) can be derived directly from the definition of the energy–momentum tensor with the help of the symmetry relations (37). The conservation law (43) together with Eq. (54) imply

$$\frac{d}{d\tau}\left[\varepsilon\left(\tau\right) + \frac{1}{2}\boldsymbol{\mathcal{E}}^2(\tau)\right] = -\frac{\varepsilon\left(\tau\right) + P\left(\tau\right)}{\tau} . \tag{56}$$

In the absence of the fields Eq. (56) is reduced to the Bjorken equation describing the evolution of the energy density in a boost-invariant hydrodynamic model [29]. On the other hand, neglecting the expansion effects, described by the term on the right-hand-side of Eq. (56), we obtain the simple conservation law for the total energy of the field and matter.

The non-equilibrium energy density ε and the non-equilibrium pressure P are

$$\varepsilon\left(\tau\right)=\frac{1}{u}\int d^2p_\perp\, dw\, v\left[N_f\sum_{i=1}^{3}\left(G_i^+ + G_i^-\right)+\sum_{i,j=1}^{3}\tilde{G}_{ij}\right] \tag{57}$$

and

$$P\left(\tau\right)=\frac{1}{u}\int d^2p_\perp\, dw\, \frac{w^2}{v}\left[N_f\sum_{i=1}^{3}\left(G_i^+ + G_i^-\right)+\sum_{i,j=1}^{3}\tilde{G}_{ij}\right]. \tag{58}$$

The time dependence of the energy density $\varepsilon\left(\tau\right)$, following from Eq. (57), is shown in Figs 4 and 5. In Fig. 4 we fix the maximal quasirapidity interval $\Delta\eta = 6$ and show the results for different strengths of the initial field, $k=2$, 3 and 5. We observe that the energy density grows very rapidly and reaches maximum at a fraction of a fermi. Later the energy density decreases, which is an effect connected with the longitudinal expansion of the system, imposed by the boost invariance. The maximal values of the energy density are very close to those found in Ref. [2]. In Fig. 5 we show the results for the fixed initial value of the field, $k=3$, and for different quasirapidity intervals, $\Delta\eta=4$, 6 and 8. It is interesting to observe that with increasing values of $\Delta\eta$ the maximal energy density gets smaller. This behavior is connected with the time dependence of the chromoelectric field. For larger values of

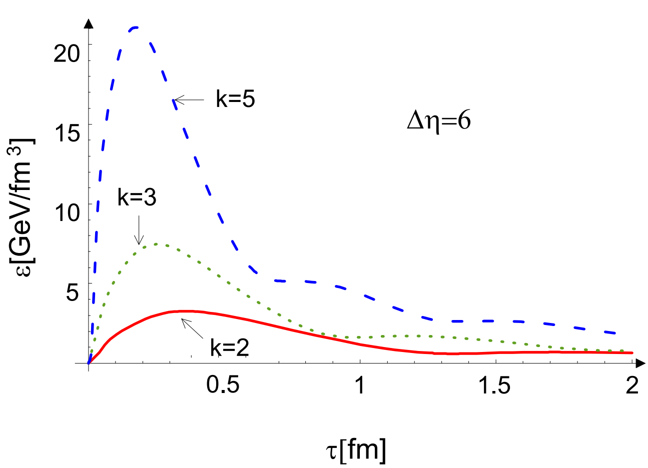

Fig. 4. Time dependence of the energy density of the plasma, Eq. (57), for different values of k and $\Delta\eta$ =6.

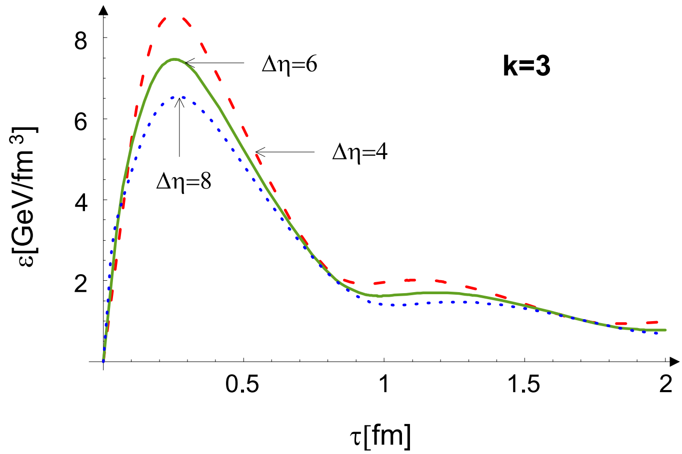

Fig. 5. Time dependence of the energy density of the plasma for different values of $\Delta\eta$ and $k = 3$.

$\Delta\eta$ the decay of the field is slower (except for the very beginning of the decay process) and the growth of the energy density is weaker.

We have checked that our numerically evaluated functions $\varepsilon(\tau)$ and $P(\tau)$ obey Eq. (56). The time-dependence of the non-equilibrium pressure $P(\tau)$ is depicted in Figs 6 and 7. One can notice that the minima of $P(\tau)$ correspond to the extremes of the chromoelectric field. At these points the longitudinal momenta of the particles practically vanish and the field is the strongest.

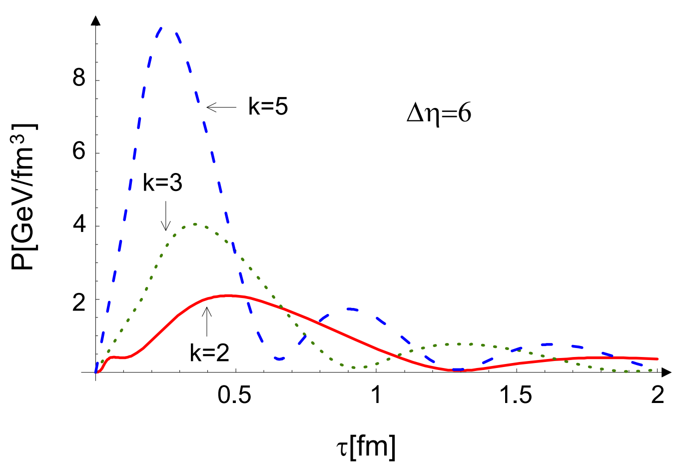

Fig. 6. Time dependence of the pressure of the plasma, as defined by Eq. (58), $k = 2, 3, 5$ and $\Delta\eta = 6$.

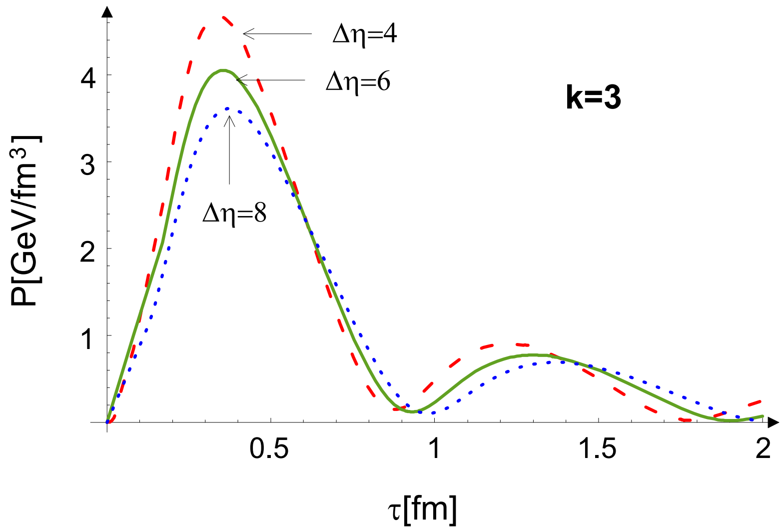

Fig. 7. Time dependence of the pressure of the plasma, $k = 3$ and $\Delta\eta = 4, 6, 8$.

This behavior reminds oscillations of a simple string. Another interesting feature of the process discussed here is that the ratio $P(\tau)/\varepsilon(\tau)$ oscillates around the mean value $P/\varepsilon \sim 1/3$, which corresponds to the equilibrium limit. Imposing condition $P/\varepsilon = 1/3$ in Eq. (56) we may treat this expression as an equation determining ε (the time dependent chromoelectric field enters here as the only input). We have found that the solutions of this equation are very good approximations of the exact solutions shown in Figs 4 and 5.

7. Conclusions

In this paper, the boost invariant tunneling has been incorporated into the framework of the kinetic theory describing production of the quark–gluon plasma in strong color fields. Our description of the tunneling process includes the effect of a finite space-like distance formed between the particles. Such a distance appears, since the energy and momentum conservation laws should be satisfied locally during the tunneling. In our approach, the particles emerging from the vacuum have finite longitudinal momenta, which is a direct consequence of the tunneling along the trajectories of constant invariant time.

We restrict the possibility of creation of pairs at very large distances (in order to retain the boost-invariance of the system, this condition is formulated in the quasirapidity space). In this way we mimic the constraints imposed by the fact that total energies of the realistic systems are finite. We find that the finite-size corrections are crucial to have non-trivial solutions of the kinetic equations. For the realistic finite-size corrections (as suggested by the accessible rapidity range in the present experiments with the ultra-relativistic heavy-ions) we find the solutions of the kinetic equations which are very close (qualitatively and quantitatively) to the results of the previous investigations: the chromoelectric fields oscillate, and large densities of quarks and gluons are produced in a very short time. The increase of the region allowed for tunneling does not lead to the increase of the produced maximal energy density of quarks and gluons. This feature of the model can be used to interpret small differences in many characteristics of the heavy-ion collisions at the SPS and RHIC energies.

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