LIMITATIONS ON QUANTUM INFORMATION FROM BLACK HOLE PHYSICS*

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After reviewing the relation of entropy to information, I derive the entropy bound as applied to bounded weakly gravitating systems, and review the bound's applications to cosmology as well as its extensions to higher dimensions. I then discuss why black holes behave as 1-D objects when emitting entropy, which suggests that a black hole swallows information at a rate restricted by the one-channel information capacity. I discuss fundamental limitations on the information borne by signal pulses in curved spacetime, from which I verify the mentioned bound on the rate of information disposal by a black hole.

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1. Introduction

Were one looking for a logo to symbolize the field of gravity physics in the last decade, none would be more apt than 't Hooft's holographic bound [1]: the entropy S (or information — see below) that can be contained in a physical system is bounded in terms of the area A of a surface enclosing it:

$$S \leq A(4\hbar)^{-1}\,. \tag{1}$$

(I assume units with $G = c = 1$.) Where does this comes from? According to Susskind [2], the holographic bound is required by the Generalized Second Law (GSL) [3] applied to the total collapse of a physical system into a black hole of its own making. Granted that many systems do not like to collapse spontaneously, so that this argument is not a general proof of the principle [4], a similar argument of wider applicability can be given for quiescent

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systems by considering either infall of the system into a large black hole, or a tiny auxiliary black hole which devours the system [5]. The holographic bound as above stated can be violated by rapidly evolving systems, but with Bousso's reinterpretation [6] of the meaning of A in Eq. (1), it works in these cases also.

Interest in the holographic bound, or the more encompassing holographic principle [1], is mostly for what it tells us about the structure of physical laws. However, it is clear that the holographic bound also serves as the final arbitrator of the promise of any futuristic information storage technology. Unfortunately, it is rather lenient in this respect: it merely requires that a device with dimension 1 cm hold no more than some 10^{66} bits of information. By contrast, all the books in the Library of Congress hold a paltry 10^{15} bits of information, and no state-of-the-art 1 cm size memory can hold all that. So I would like to ask if one can device a tighter bound on information storage than the holographic one? As I will show, the answer is positive in the form of the universal entropy bound, a spinoff of black hole physics. In light of the explosive development of fast communications, a further interesting question would be: what fundamental bounds can be set on the *flow* of information? I will show here that several new results in this well developed field can be had by considering black holes.

2. Information and entropy

I want to start by recapitulating how thermodynamic entropy of a quantum system and the information it can store are related. In quantum theory a system's *state*, whether pure or mixed, is described by an Hermitian operator ρ with unit trace. The entropy of a pure state is zero (because we know what state we are dealing with). The thermodynamic entropy of a mixed state is given by von Neumann's formula $S = -\operatorname{Tr}\,\rho\ln\rho$ (which will properly give zero if ρ corresponds to a pure state: $\operatorname{Tr}\,\rho^2 = 1$).

Suppose we find the eigenstates of ρ and their respective eigenvalues, which must obviously sum to unity because $\operatorname{Tr}\,\rho = 1$. According to quantum theory, each nonvanishing eigenvalue p_i stands for the probability with which the corresponding eigenstate turns up in state ρ. Working in the basis furnished by the eigenstates it is easy to see that $-\operatorname{Tr}\,\rho\ln\rho = -\sum_i p_i \ln p_i$. Now compare this result with Shannon's famous 1948 formula [7] for the peak information capacity — or information entropy — in bits of a system with distinguishable states which occur with *a priori* probabilities $\{p_i\}$:

$$I_{\max} = -\sum_i p_i \log_2 p_i \,. \tag{2}$$

We obviously have $I_{\max} = S \log_2 \mathrm{e}$: generically thermodynamic entropy sets an upper bound on the information storage capacity of the system, modulo the factor $\log_2 \mathrm{e}$ which converts from natural entropy units to bits.

The key word in the above identification is "distinguishable". The eigenstates of ρ are all orthogonal by virtue of its Hermitian character, so they are precisely distinguishable. Confusion can occur (and has often) if the distinguishability condition is compromised. Consider the four states

$$|\uparrow\rangle, |\downarrow\rangle, |\rightarrow\rangle, |\leftarrow\rangle$$

of a spin 1/2 particle corresponding to "up" and "down" spin with respect to spin components s_z and s_x, respectively. If we assign each probability $\frac{1}{4}$, Shannon's formula would predict maximum information capacity $I_{\max} = \log_2 4 = 2$. Obviously here

$$\rho = \frac{1}{4}\left(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| + |\rightarrow\rangle\langle\rightarrow| + |\leftarrow\rangle\langle\leftarrow|\right). \tag{3}$$

Recalling that $|\rightarrow\rangle = 2^{-1/2}(|\uparrow\rangle + |\downarrow\rangle)$ while $|\rightarrow\rangle = 2^{-1/2}(|\uparrow\rangle - |\downarrow\rangle)$ and that $|\uparrow\rangle$ and $|\downarrow\rangle$ are orthonormal states, we easily work out that $\rho = \frac{1}{2}\left(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\right)$, which form tells us immediately that ρ's eigenvalues are $\{\frac{1}{2}, \frac{1}{2}\}$ and its eigenstates $\{|\uparrow\rangle, |\downarrow\rangle\}$. Calculating in the basis $\{|\uparrow\rangle, |\downarrow\rangle\}$ gives $S = \ln 2$. We thus find that $I_{\max} = 2S \log_2 \mathrm{e}$.

Does the above mean the spin system can store more information than allowed by the thermodynamic entropy? Not at all! The four states used are not independent, and so not mutually orthogonal. According to quantum theory nonorthogonal states are not fully distinguishable experimentally [8]. Thus when reading out information encoded in our four states, errors can occur, and this "noise" reduces the true information extractable below Shannon's formal $I_{\max}$. According to a fundamental theorem by Holevo [9], the information that can actually be read out of a system is bounded from above by $S \log_2 \mathrm{e}$, as claimed above.

Some confusion can still arise if we ignore the fact that von Neumann's S can have various values depending on the level of structure at which it is computed. The chemist, for instance, determines the entropy S of a piece of iron by methods that reach down to the atomic level; for him the states $\{i\}$ are atomic states. The communication engineer, by contrast, is interested in storing information in the magnetic domains of the iron in a magnetic tape. He groups a multitude of atomic states into domain states, and the S computed from the latter is definitely smaller than the chemist's S. There is no contradiction here; one must simply specify at which level S or the corresponding information capacity are calculated. Obviously, the deeper we go into the system's structure, the higher the entropy and information

capacity. In what follows I shall be interested in the entropy (information capacity) $S_{\rm X}$ calculated at level X, the deepest level of structure (the level of lepton, quark and gluon degrees of freedom). The corresponding $S_{\rm X}\log_2 {\rm e}$ bounds from above the information capacity of material media accessible with any achievable technology.

3. Poor man's road to the universal entropy bound

Consider the following *gedanken* experiment [10]. Drop a physical system $\mathcal{U}$ of unknown construction and constitution having mass-energy E and radius R into a Schwarzschild black hole of mass $M \gg E$ from a large distance $d \gg M$ away. This d is so chosen that the Hawking radiance carries away energy (as measured at infinity) equal to E while $\mathcal{U}$ is falling to the horizon where it is effectively assimilated by the black hole. At the end of the process the black hole is back at mass M and its entropy has not changed. Were the emission reversible, the radiated entropy would be $E/T_{\rm H}$ with $T_{\rm H} \equiv \hbar(8\pi M)^{-1}$. Irreversibility of radiation and spacetime curvature conspire to make the entropy emitted a factor ν larger; typical values, depending on particle species, are [11] $\nu = 1.35 \div 1.64$. Thus the overall change in world entropy is

$$\delta S_{\rm ext} = \delta S_{\rm rad} - S = \nu \frac{E}{T_{\rm H}} - S\,. \tag{4}$$

One can certainly choose M larger than R, say, by an order of magnitude so that the system will fall into the hole without being torn up: $M = \zeta R$ with $\zeta =$ a few. Thus by the *ordinary* second law we obtain the bound

$$S < 8\pi\nu\zeta R\frac{E}{\hbar}\,. \tag{5}$$

Our simple argument here leaves the factor $\nu\zeta$ somewhat fuzzy; but it is safe to say that $4\nu\zeta < 10^2$. Thus we have obtained a bound on the entropy of an arbitrary system $\mathcal{U}$ in terms of just its total energy E and radius R. This is the gist of the universal entropy bound [12].

Note that we could not derive (5) by using a heat reservoir in lieu of a black hole. A reservoir which has gained energy E upon $\mathcal{U}$'s assimilation, and has returned to its initial energy by radiating, does not necessarily return to its initial entropy, certainly not until $\mathcal{U}$ equilibrates with the rest of the reservoir. But a (nonrotating uncharged) black hole whose mass has not changed overall, retains its original entropy because that depends only on mass. In addition, for the black hole mass and radius are related in a simple way; this allowed us to replace $T_{\rm H}$ in terms of R. By contrast, for a generic reservoir, size is not simply related to temperature.

Note also that in our argument $\mathcal{U}$ is not allowed to be strongly gravitating (meaning $R \sim E$) because then M could not be large compared to E while ζ is of order a few, as I assumed. We thus have to assume that $R \gg E$ in addition: the universal entropy bound applies, *a priori*, only to weakly gravitating systems (but see below).

But what about Hawking radiation pressure. Is it important? Could it blow $\mathcal{U}$ outwards? If we approximate the radiance as black-body radiance of temperature $T_{\rm H}$ coming from a sphere of radius $2M$, the energy flux at Schwarzschild coordinate r from the hole is

$$F(r) = \frac{\mathcal{N}\hbar}{61,440(\pi M r)^2}\,, \tag{6}$$

where $\mathcal{N}$ stands for the effective number of massless species radiated (photons contribute 1 to $\mathcal{N}$ and each neutrino species 7/16). This estimate is known to be off by a factor of only a few [13]. This energy (and momentum) flux results in a radiation pressure force $f_{\rm rad}(r) = \pi R^2 F(r)$ on $\mathcal{U}$. More precisely, species which reflect well off $\mathcal{U}$ are approximately twice as effective at exerting force as just stated, while those (neutrinos and gravitons) which go right through $\mathcal{U}$ contribute very little; the $\mathcal{N}$ must thus be reinterpreted accordingly. I have ignored relativistic corrections so that the result, as qualified, is correct mostly for $r \gg M$.

Writing the gravitational force on $\mathcal{U}$ in the Newtonian approximation, $f_{\rm grav}(r) = ME/r^2$, one sees that

$$\frac{f_{\rm rad}(r)}{f_{\rm grav}(r)} = \frac{\mathcal{N}_{\rm eff}\,\hbar R^2}{61,440\pi^2 M^3 E}\,. \tag{7}$$

I have written $\mathcal{N}_{\rm eff}$ here because, as mentioned, some species just pass through $\mathcal{U}$ without exerting force on it. In addition, only those species actually represented in the radiation flowing out during $\mathcal{U}$'s infall have a chance to exert forces. Now an Hawking quantum bears an energy of order $T_{\rm H}$, so the number of quanta radiated together with energy E is approximately $8\pi ME/\hbar$. However, for any bound system $\hbar/E < R$ (system larger than its own Compton length), so by our stipulation that $M = \zeta R > R$, the number of species can be large compared to unity. Since a species can exert pressure only if it is represented by at least one quantum, one obviously has $\mathcal{N}_{\rm eff} < 8\pi ME/\hbar$. Therefore,

$$\frac{f_{\rm rad}(r)}{f_{\rm grav}(r)} < \frac{R^2}{7680\pi M^2} \ll 1\,. \tag{8}$$

Radiation pressure is thus negligible, and $\mathcal{U}$'s fall is very nearly on a geodesic, at least until $\mathcal{U}$ approaches to within a few Schwarzschild radii. It is intuitively clear that if $d \gg M$, the last (relativistic) stage cannot make any difference, and $\mathcal{U}$ must plunge to the horizon.

Whether $d \gg M$ as assumed must be checked. I have taken d such that the infall time equals the time t for the hole to radiate energy E. Newtonially $d \approx 2(t^2M/\pi^2)^{1/3}$, while Eq. (6) gives the estimate $t \approx 5 \times 10^4 EM^2\hbar^{-1}\mathcal{N}^{-1}$ with $\mathcal{N}$ now the full species number. From these equations and $M = \zeta R$ we get that $d \approx 1.2 \times 10^3(\zeta ER/\mathcal{N}\hbar)^{2/3}M$. Thus for $\mathcal{N} < 10^2$ (conservative estimate of *our* world's massless particle content), we have $d > 57M \gg M$ for all systems $\mathcal{U}$ satisfying our assumption $R > \hbar/E$.

4. Ramifications of the universal entropy bound

In asymptotically flat four-dimensional spacetime ($D = n + 1 = 4$) the holographic bound restricts the entropy of a finite system $\mathcal{U}$ with energy E and circumscribing radius R (E and R measured in proper frame) by Eq. (1). For weakly gravitating systems ($R \ll E$) the universal entropy bound in its original version [12] is

$$S \leq 2\pi \frac{ER}{\hbar}\,. \tag{9}$$

Typically for laboratory sized systems $E < 10^{-23}R$, while for astronomical systems — barring neutron stars and black holes — $E < 10^{-5}R$. Thus, with few exceptions, the entropy bound is many orders of magnitude tighter than the holographic one. For instance, it limits the information capacity of a 1 cm device made of ordinary matter to be less than 10^{37} bits, which limit no longer looks unreachable.

In the original derivation of bound (9) I imagined that $\mathcal{U}$ is lowered slowly from far away to the horizon of a stationary black hole, while all the freed potential energy is allowed to do work on a distant agent (a Geroch process [14]). I then applied the GSL to infer the bound. This derivation was criticized [15] for not taking into account the buoyancy of $\mathcal{U}$ in the Unruh radiation surrounding it by virtue of its acceleration. However, I have shown [16] that correction for buoyancy — itself an intricate calculation — merely increases the 2π coefficient in Eq. (9) by a tiny amount provided only that one assumes that $R \geq \hbar/E$, as done in Sec. 3.

The entropy in bounds (5) or (9) refers to the $S_{\rm X}$ defined in Sec. 2. This is because gravitation plays a crucial role in many generic ways of deriving the entropy bound [12, 16]. And gravitation is unique among the interactions in that it is aware of all degrees of freedom in its sources (according to the equivalence principle all energy gravitates). It would thus be odd if the entropy bound took into account only entropy corresponding to intermediate degrees of freedom, and so ignored energy carrying states at the deeper levels?

Of late it has been realized that the entropy bound also applies in higher dimensions. For instance, Bousso [17] has shown, via the Geroch process

argument, that bound (9) applies *verbatim* in asymptotically flat spacetime with any $D = n + 1$ dimensions. Bousso rewrites the bound in terms of $\mathcal{U}$'s gravitational radius $r_{\rm g}$ as inferred from the D-dimensional Schwarzschild solution,

$$ {r_{\rm g}}^{n-2} = \frac{8\Gamma(n/2)E}{(n-1)\pi^{n/2-1}}\,, \tag{10} $$

whereby it takes the form

$$ S \leq \frac{(n-1)\pi^{n/2}\, r_{\rm g}^{n-2} R}{4\Gamma(n/2)\,\hbar^{(n-1)/2}}\,. \tag{11} $$

According to Bousso this form of the bound applies to all systems in D-dimensional de Sitter spacetime which occupy a small part of the space inside the cosmological horizon whose radius is denoted by R.

As mentioned in Sec. 3, when we come to strongly gravitating systems ($E \sim R$), we cannot derive the bound (9) or even the weaker version (5) by the methods just expounded. However, it so happens that the bound (9) is actually obeyed — and saturated at that — by all $D = 4$ Kerr–Newman black holes provided one interprets E as the black hole's mass and R as the Boyer–Lindquist coordinate of the horizon, r_+ (see references in [19]). Further, spherical *black holes* in $D > 4$ spacetime (for which the horizon has a $(n-1)$ dimensional "area") also obey (9) in asymptotically flat spacetimes, and the bound (11) in asymptotically de Sitter spacetimes. However, black holes in higher dimensions no longer saturate these bounds as for $D = 4$ [17].

The sway of the entropy bound also extends to cosmology. For example, Verlinde [18] has shown that the entropy S of a complete closed Robertson–Walker Universe in D spacetime dimensions whose contents are described by a Conformal Field Theory (CFT) — the deeper description of a number of massless fields possibly in interaction — with large central charge (essentially many particle species), is subject to the generic bound

$$ S \leq \frac{2\pi R}{n\hbar}[E_{\rm C}(2E - E_{\rm C})]^{1/2}, \tag{12} $$

where R is the radius of the S^n space, E the total energy in the fields and $E_{\rm C}$ the Casimir (vacuum) energy (which shows up because the cosmological space is compact). Verlinde points out that for fixed E the maximum of his bound is $2\pi RE/(n\hbar)$, which never exceeds the original entropy bound (9); indeed Verlinde adopts $S \leq 2\pi RE/(n\hbar)$ as the fiducial form of that bound. A number of recent papers (see [19] for references) have substantiated Verlinde's bound; they culminate years of efforts by many to make meaningful statements about the entropy (and by implication the maximum information) that can be contained in a whole Universe.

For *strongly* gravitating systems in asymptotically flat spacetime with $D = 4$, the holographic bound and the formal entropy bound make very similar predictions, but for $D > 4$ the holographic bound is the tighter of the two. Unless D is very large, the entropy bound is the tighter bound for weakly gravitating systems, such as those we meet in everyday life.

5. Black holes as information pipes

If the holographic bound (1) can be construed as telling us that a generic physical system in 4-D spacetime is fundamentally two-dimensional in space, then a black hole in 4-D spacetime when viewed as an information absorber or entropy emitter, is fundamentally one-dimensional in space [20]. I proceed to explain.

In the discipline treating information flow — communication theory — the notion of a channel is central. In flat spacetime a channel is a complete set of unidirectionally propagating modes of some field, with the modes enumerated by a single parameter. For example, all electromagnetic modes in free space with fixed wave vector *direction* and particular linear polarization constitute a channel; the modes are parametrized solely by frequency. An example would be a straight infinitely long coaxial cable (which is well known to transmit all frequencies) capped at its entrance by the analog of a polaroid filter. Acoustic and neutrino channels can also be defined. Note that a channel is intrinsically one-dimensional.

What is the maximum *rate*, in quantum theory, at which information may be transmitted through a channel for prescribed power P? The answer has been known since the 1960's; however, let me work with the particularly lucid version given by Pendry [21]. Pendry thinks of a possible signal state as corresponding to a particular set of occupation numbers for the various propagating modes. He assumes the channel is uniform in the direction of propagation, which allows him to label the modes by momentum p. But he allows for dispersion, so that a quantum with momentum p has some energy $\varepsilon(p)$. Then the propagation velocity of the quanta is the group velocity $v(p) = d\varepsilon(p)/dp$. Up to a factor $\log_2 \mathrm{e}$ the information rate capacity must equal the maximal one-way entropy current for given P, which obviously occurs for the thermal state, if one discards from the latter the modes moving opposite the direction of interest.

Now the entropy $s(p)$ of any boson mode of momentum p in a thermal state (temperature T) is [22]

$$s(p) = \frac{\frac{\varepsilon(p)}{T}}{\mathrm{e}^{\varepsilon(p)/T} - 1} - \ln\left(1 - \mathrm{e}^{-\varepsilon(p)/T}\right) , \tag{13}$$

so the entropy current in one direction is

$$\dot{S} = \int_0^\infty s(p)\, v(p)\, \frac{dp}{2\pi\hbar} = \int_0^\infty s(p)\, \frac{d\varepsilon}{dp}\, \frac{dp}{2\pi\hbar}\,, \tag{14}$$

where $dp/2\pi\hbar$ is the number of modes per unit length in the interval dp which propagate in one direction. This factor, when multiplied by the group velocity, gives the one-way current of modes.

Suppose $\varepsilon(p)$ is monotonic and extends over the range $[0, \infty)$; we may then cancel dp and integrate over ε. Then after substitution of Eq. (13) and integration by parts we have

$$\dot{S} = \frac{2}{T} \int_0^\infty \frac{\varepsilon}{\mathrm{e}^{\varepsilon/T} - 1}\, \frac{d\varepsilon}{2\pi\hbar} = \frac{2}{T} \int_0^\infty \frac{\varepsilon(p)}{\mathrm{e}^{\varepsilon(p)/T} - 1}\, v(p)\, \frac{dp}{2\pi\hbar}\,. \tag{15}$$

The first factor in each integrand is the mean energy per mode, so that the integral represents the one-way power P in the channel. Thus

$$\dot{S} = \frac{2P}{T}\,. \tag{16}$$

The integral for P in the first form of Eq. (15) can easily be done:

$$P = \frac{\pi (T)^2}{12\hbar}\,. \tag{17}$$

Eliminating T between the last two expressions gives Pendry's limit

$$\dot{S} = \left(\frac{\pi P}{3\hbar}\right)^{1/2} \qquad \text{or} \qquad \dot{I}_{\max} = \left(\frac{\pi P}{3\hbar}\right)^{1/2} \log_2 \mathrm{e}\,. \tag{18}$$

For a fermion channel P in Eq. (17) is a factor 2 smaller, and consequently $\dot{S}$ in Eq. (18) is reduced by a factor $\sqrt{2}$.

The function $\dot{S}(P)$ in Eq. (18) is the so called *capacity of a noiseless quantum channel*. Surprisingly, it is independent, not only of the form of the mode velocity $v(p)$, but also of its scale. Thus the phonon channel capacity is as large as the photon channel capacity despite the difference in speeds. Why? Although phonons convey information at lower speed, the energy of a phonon is proportionately smaller than that of a photon in the equivalent mode. Thus when the capacities of channels harnessing various carriers are expressed in terms of power, they turn out to involve the same constants. Formula (18) neatly characterizes what we mean by one-dimensional transmission of entropy or information. It refers to transmission

by use of a single species of quantum and a specific polarization; different species and alternative polarizations engender separate channels. Although framed in a flat spacetime context, its lack of sensitivity to the dispersion relation of the transmitting *milieu* should make Pendry's limit relevant to curved spacetime also. This because electrodynamics in curved spacetime is equivalent to flat spacetime electrodynamics in a suitable dielectric and paramagnetic medium [23]. We shall see in Sec. 7 that this hunch is justified.

It is instructive to contrast the results just obtained with the power and entropy emission rate in a single boson polarization of a closed black body surface with temperature T and area A in flat 4-D spacetime. By the Stefan–Boltzmann law this is

$$P = \frac{\pi^2 T^4 A}{120\hbar^3} \qquad \dot{S} = \frac{4}{3}\frac{P}{T}\,, \tag{19}$$

whereby

$$\dot{S} = \frac{2}{3}\left(\frac{2\pi^2 A P^3}{15\hbar^3}\right)^{1/4}, \tag{20}$$

[for fermions P carries an extra factor $7/8$ and formula (20) an extra factor $(8/7)^{1/4}$]. Our manifestly 3-D transmission system deviates from the sleek formula (18) in the exponent of P and in the appearance of the measure A of the system. In emission from a closed curve of length L in two-dimensional space, the factor $(LP^2)^{1/3}$ would replace $(AP^3)^{1/4}$. We may thus gather the dimensionality of the transmission system from the exponent of P in the expression $\dot{S}(P)$ [it is $n/(n+1)$ for $D = n+1$ spacetime dimensions], as well as from the value of the coefficient of P/T in expressions for $\dot{S}$ like (16) or (19) [it is $(n+1)/n$].

Radiation from a Schwarzschild black hole in 4-D spacetime is also given by Eqs. (19) (or their fermion version) with $A = 4\pi(2M)^2$ and $T = T_{\rm H}$, except we must correct the expression for P by a species dependent factor $\bar{\Gamma}$ of order unity [13], and replace the $4/3$ in the expression for $\dot{S}$ by the species dependent factor ν already mentioned in Sec. 4. Eliminating M between the equations we obtain, in lieu of Eq. (20),

$$\dot{S} = \left(\frac{\nu^2 \bar{\Gamma} \pi P}{480\hbar}\right)^{1/2}. \tag{21}$$

(For fermions there is an extra factor $7/8$ inside the radical.) This looks completely different from the law (20) for the hot closed surface because, unlike for a hot body, a black hole's temperature is related to its mass.

However, (21) *is* of the same form as Pendry's limit (18) for one-channel transmission. From Page [11,13] we get $\bar{\Gamma} = 1.6267$ and $\nu = 1.5003$ for one

photon polarization, so the numerical coefficient of (21) is 15.1 times that in (18). Repeating the above exercise for one species of neutrinos we again find formulae like (21) and (18), this time with $\bar{\Gamma} = 18.045$ and $\nu = 1.6391$; the numerical coefficient of (21) is 48.1 times that of the fermion version of (18).

Thus when judged by its entropy emission properties, a black hole in 4-D spacetime is more like a 1-D channel than like a surface in 3-D space. Why is this? A formal answer is that, because of the way $T_{\rm H}$ is related to the black hole's radius $2M$, Hawking emission prefers to emerge in the lowest angular momentum mode possible. To exit with impact parameter $< 2M$ and angular momentum $j\hbar$, a quantum must have energy (momentum) $\hbar\omega > j\hbar/2M$. But in the Hawking thermal distribution the dominant $\hbar\omega$ is of order $T_{\rm H} = \hbar(8\pi M)^{-1}$. Thus the emerging j's tend to be small. For example, 97.9% of the photon energy emerges in the $j = 1$ modes ($j = 0$ is forbidden for photons), and 96.3% of the neutrino power is in the $j = \frac{1}{2}$ modes [13]. Thus the black hole emits as close to radially as possible. This means that, crudely speaking, it does so through just one channel.

If a black hole emits entropy like a one-dimensional system, we might guess it should absorb information like a one dimensional system. This hunch will be verified in Sec. 7. As a first step I extend to curved spacetime some of the insights regarding information flow.

6. Information pulses in curved spacetime

The discussion in Sec. 5 tacitly assumed steady state streaming of information and energy. But what if information is delivered in pulses? Can one state a bound generalizing (18)? Can one include effects of gravitation on the information transfer rate? To answer these questions let us extend the notion of channel to curved spacetime, at least to stationary curved spacetime. Again, a channel will be a complete set of unidirectional modes of some field that can be enumerated with a single parameter. Each channel is characterized by species of quanta, polarization (helicity), trajectory, *etc.* In Sec. 5 I characterized the *signal* in a particular channel by power. For a pulse it seems a better idea to use both the signal's duration τ and its energy E. Since in curved spacetime a channel is not generally uniform, I choose to measure these parameters in a local Lorentz frame (I shall show presently that it does not matter which one). With this precaution sections of the channel may be treated as in flat spacetime.

How is the true $I_{\rm max}$ of a pulse related to its E and τ? Since information is dimensionless, $I_{\rm max}$ must be a function of dimensionless combinations of E, τ, channel parameters and the fundamental constants $c, \hbar$ and G:

$$I_{\rm max} = \Im\left(\frac{E\tau}{\hbar}, GEc^{-5}\tau^{-1}\right) . \tag{22}$$

Here $\Im(\xi, \varpi)$ is some nonnegative valued function of the dimensionless parameters ξ and ϖ characteristic of the channel. This is called the Characteristic Information Function (CIF) [24, 25]. The shape of $\Im$ depend on things like the polarization and nature of the transmitting medium. I shall assume this medium, if any, is nondissipative and nondispersive. Thus it is characterized by a single signal velocity c_s; the dimensionless parameter c_s/c is one of the determinants of the shape of $\Im$. I shall exclude channels which transmit massive quanta, *e.g.* electrons, because rest mass is energy in a form not useful for communication, so that the strictest limits on information flow should emerge for massless signal carriers. Hence masses do not enter into the shape of $\Im$. The variable $\varpi \equiv GEc^{-5}\tau^{-1}$ is of order of E divided by the signal's self-potential energy, and very large for ordinary signals. So I first work with the limiting formula as $\varpi \to \infty$.

Let us check what happens in flat spacetime for steady state signaling. This implies we deal with a long stream of information and that the signal can be characterized in a suitable frame as statistically stationary. The peak information transfer *rate* and power can then be inferred from a finite section of the signal of duration τ bearing information $I_{\max}$ and energy E. It should matter little how long a stretch in τ is used so long as it is not too short, and $\dot{I}_{\max} \equiv E\tau^{-1}$ should come out fully determined by the power $P \equiv E\tau^{-1}$. But this is consistent with Eq. (22) only if $\Im(\xi, \infty) \propto \sqrt{\xi}$, for only then does τ cancel out. With this form we recover Pendry's formula $\dot{I}_{\max} \propto (P/\hbar)^{1/2}$, which we know to be the correct answer for steady state flow in flat spacetime.

The dividing line between steady state signaling and signaling by means of very long pulses is fuzzy. This suggests that long pulse signals must also obey a Pendry type formula, albeit approximately, *cf.* [26]. The law $\dot{I}_{\max} \propto (P/\hbar)^{1/2}$ is evidently inapplicable to *brief* information pulses. For such it may be replaced by a linear upper bound [27] which may even transcend some of the limitations I imposed to define $\Im(\xi, \varpi)$. Consider the information I to be encoded in some material structure $\mathcal{V}$ of radius R and rest energy E which maintains its integrity and dimensions as it travels from emitter to receiver. From Eq. (9) we have the strict inequality $I < 2\pi ER\hbar^{-1}\log_2 \mathrm{e}$. The rate at which the information is assimilated by the receiver is obviously restricted by the local time τ it takes for $\mathcal{V}$ to sweep by it. From special relativity $\tau > 2R\gamma^{-1}$ with Lorentz's γ accounting for the Fitzgerald contraction of $\mathcal{V}$ in the frame of the receiver. Thus the peak information reception rate is $I/\tau < \pi\gamma E\hbar^{-1}\log_2 \mathrm{e}$, or

$$\dot{I}_{\mathrm{rec}} < \pi E_{\mathrm{rec}}\hbar^{-1}\log_2 \mathrm{e}\,, \tag{23}$$

where $E_{\rm rec} \equiv \gamma E$ is $\mathcal{V}$'s energy as measured in the receiver's frame. Bound (23) replaces the information version of Eq. (18) when it comes to pulses. Since $\xi \equiv E_{\rm rec}\tau\hbar^{-1}$ we have the strict linear bound $\Im(\xi,\infty) < (\pi \log_2 {\rm e})\xi$, a bound which is supported by much evidence [24,25] even when the signal has no rest frame. I must stress that the linear bound applies only for small ξ; for $\xi \gg 1$ one may use Pendry's formula.

Detailed calculations [19,25] show that $E\tau$ is unchanged in the passage between Lorentz frames, regardless of whether transmission is through a medium or vacuum. Thus the law $I_{\rm max} = \Im(E\tau/\hbar,\infty)$ is Lorentz invariant not only in vacuum where this is required by relativity, but also in the presence of a preferred frame established by the medium. We can thus use $I_{\rm max} = \Im(E\tau/\hbar,\infty)$ both in the medium's and in the signal emitter's (receiver's) Lorentz frame, provided we do so at a fixed point.

But how is the information transmission rate related at two point along the channel? In flat spacetime, and in the absence of dispersion, E and τ are evidently conserved with propagation. And in the absence of dissipation so is the information, so that $I_{\rm max} = \Im(E\tau/\hbar,\infty)$ is valid at every point along the channel. Once we are in stationary *curved* spacetime, E and τ are subject to redshift and dilation effects, respectively. However, the two effects act in opposite senses so that $E\tau$ is again conserved throughout the signal's flight. Therefore, the formula is meaningful throughout the channel. In fact one can use global values (as measured at infinity) of E and τ in the formula. In conclusion, one and the same formula limits information transmission, propagation and reception rates.

When self-gravity of the signal pulse is not negligible, ϖ reappears as a possible argument of $\Im$. However, it is clear that E/τ is not a Lorentz scalar, so inclusion of ϖ would spoil the local Lorentz invariance of Eq. (22) and violate special relativity for signals propagating in vacuum in a flat background. In a curved background there are further arguments against inclusion of ϖ in $\Im$. In vacuum we can use the requirement of local Lorentz invariance to bar ϖ's appearance, for a sufficiently brief signal should admit being encompassed in its entirety by local Lorentz frames. Further, ϖ evidently decreases as the signal propagates *outward* in the gravitational potential. Thus, $\Im(E\tau/\hbar,\varpi)$ would decrease either outwardly (if $\Im$ increases with ϖ) or inwardly (if it decreases as ϖ increases). If a signal's information saturates the bound $\Im(E\tau/\hbar,\varpi)$ at some point in the potential, then by conservation of information it will exceed the bound once it has propagated somewhat in the direction in which $\Im$ decreases. This leads to a contradiction. One could try to resolve the problem by defining $I_{\rm max}$ only in terms of the *minimum* value of ϖ in the channel. But it seems strange that, at least for brief signals, one cannot state $I_{\rm max}$ in terms of local quantities.

Thus for signals propagating in vacuum in flat or curved spacetime, ϖ cannot appear in $\Im$. It is unclear whether this conclusion extends to signal propagation in a medium. For one thing in curved spacetime a medium is never homogeneous, which means, among other things, that c_s varies. This in itself puts in doubt our argument for simplicity of the formula (22).

7. Dumping information into a black hole

Suppose we are granted a certain power $\mathcal{P}$ to accomplish the task of getting rid of a stream of possibly compromising information by dumping it into a black hole. What is the maximum information dumping rate?

To answer this I first argue that if the signal comes from a distance, it is transmitted down the hole through a single channel — more or less — per field species and polarization. Let us recall the rule for field mode counting. In one space dimension a length L contains $(2\pi)^{-1}L\Delta k$ modes in the wave vector interval Δk. In 3-D we have $(2\pi)^{-3}L_xL_yL_z\Delta k_x\Delta k_y\Delta k_z$ modes. From this we may conclude that if a flat 2-surface of area $\mathcal{A}$ radiates into a narrow solid angle $\Delta\Omega$ about its normal, the number of modes out to a distance L from it whose wave vector magnitudes lie between k and $k+\Delta k$ is $(2\pi)^{-3}\mathcal{A}Lk^2\Delta\Omega\Delta k$. The factor $(2\pi)^{-1}L\Delta k$ is obviously the number of modes emitted sequentially in each direction and distinguished by their values of k. One can thus think of $\mathcal{W}=(2\pi)^{-2}\mathcal{A}k^2\Delta\Omega$ as the number of active *channels*.

Now let a transmitter with effective area $\mathcal{A}$ send an information bearing signal towards a Schwarzschild black hole of mass M surrounded by vacuum and situated at distance $d\gg 2M$. Let $\mathcal{A}$ be oriented with its normal towards the black hole; evidently $\mathcal{A}<4\pi d^2$. As viewed from the transmitter the black hole subtends solid angle $\Delta\Omega=\pi(2M)^2/d^2$. What should we take for k in the formula for $\mathcal{W}$? Being interested in the highest information for given energy (other things being equal), we certainly want to use the smallest k (smallest $\hbar\omega$) possible. But signals composed of too small k's will just be scattered by the black hole. The borderline is $k=2\pi/\lambda\approx 2\pi/(2M)$. With this we find $\mathcal{W}<4\pi^2$, which means that, optimally, information is transmitted down a black hole through just a few channels per field species and polarization. This is independent of the scales M and d of the problem.

In light of this we employ the one-channel formula (22); according to our argument in Sec. 6, we drop the argument ϖ. Further, since $E\tau$ is conserved in Schwarzschild (stationary) spacetime, and closely equals $\mathcal{E}t$, the values being measured at infinity, we have $I_{\max}=\Im(\mathcal{E}t/\hbar)$. This for a pulse of duration t as seen from infinity. If we are dealing with a steady state stream of energy and information ($t\to\infty$ and $\mathcal{E}\to\infty$ with $\mathcal{P}\equiv\lim(\mathcal{E}/t)$ finite), we have, by the logic of the paragraph following Eq. (22), that the maximum

information disposal rate into the black hole is $\dot{I}_{\max} \sim (\mathcal{P}/\hbar)^{1/2}$, as hinted at the end of Sec. 5. We thus discover that the power required to dispose of information into a black hole grows *quadratically* with the information dumping rate.

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