STRONG-FIELD GRAVITY AND ORBITAL RESONANCE IN BLACK HOLES AND NEUTRON STARS — kHz QUASI-PERIODIC OSCILLATIONS (QPO)*

Włodek Kluźniak

Institute of Astronomy, Zielona Góra University
Lubuska 2, 65-265 Zielona Góra, Poland
Copernicus Astronomical Center
Bartycka 18, 00-716 Warszawa, Poland
Nordita
Blegdamsvej 17, DK-2100 Copenhagen, Denmark

and Marek Artur Abramowicz

Department of Astronomy and Astrophysics, Chalmers University
412-96 Göteborg, Sweden
Nordita
Blegdamsvej 17, DK-2100 Copenhagen, Denmark

(Received October 17, 2001)

We explain the origin of the puzzling high frequency peaks in the variability power spectra of accreting neutron stars and black holes as a non-linear 1:2 or 1:3 resonance between orbital and radial epicyclic motion. These resonances are present because the gravitational field deviates strongly from a Newtonian $1/r$ potential. Our theory agrees with the recently reported observations of two QPOs, at 300 Hz and 450 Hz, in the black hole candidate GRO J 1655-40.

PACS numbers: 97.60.Lf, 97.10.Gz, 97.10.Sj, 97.60.Jd

1. Introduction

There are about 10^2 low-mass X-ray binaries in our Galaxy, and each of them is a ten thousand, or more, times more powerful emitter of X-rays, than

* Presented at the XXV International School of Theoretical Physics "Particles and Astrophysics — Standard Models and Beyond", Ustroń, Poland, September 10–16, 2001.

our Sun is of visible radiation. Both this power and the characteristic X-ray spectrum speak of a very compact source, about 10 km accross as can easily be checked by using the black-body formula. These binaries are thought to contain neutron stars, or black holes in some cases, sucking matter out of the companion stars by virtue of their powerful gravitational attraction.

The X-ray photons coming from these sources are copious enough that a sufficiently large X-ray instrument in orbit about the Earth, such as the ones aboard the satellite called the Rossi X-ray Timing Explorer, can detect variations in the X-ray flux on timescales as short as milliseconds. Such variations have been observed, and at certain characteristic frequencies they are so prominent that they clearly reflect some kind of quasi-periodic phenomenon in the source. These prominent variations in the X-ray flux are called quasi-periodic oscillations (QPOs), and the kHz variety has attracted a great deal of attention, because they may reveal motion of matter around neutron stars and black holes in the tightest possible orbits, whose radius only slightly exceeds (by a factor of a few) the gravitational radius of the compact object. It is hoped that we may directly test the space-time metric in this strong-field regime. But, it is not sufficient to measure the orbital period—in Schwarzschild geometry the frequency of circular orbits as seen by an observer at infinity is given by the same formula as in Newton's theory, $\Omega = \sqrt{GM/r^3}$. However, in Einstein's theory of gravitation there are orbital effects which show qualitative departures from Newton's theory, and their signature may have been detected in black holes and neutron stars.

2. The marginally stable orbit and epicyclic frequency

All recent textbooks of general relativity contain a discussion of the innermost (marginally) stable circular orbit around black holes, where the effective potential for orbital motion has a "ledge-like" extremum (second derivative equal to zero), rather than the minimum familiar from Newtonian theory. A particle moving in stable circular motion can be thought of as resting in the minimum of the effective potential. If disturbed, it will execute oscillations about the minimum, which in Newtonian theory correspond to motion along an ellipse (a closed curve!), while in general relativity to motion along a "precessing ellipse" familiar from discussion of the perihelion motion of Mercury. In either case, the trajectory crosses the original circular orbit at characteristic time intervals, whose inverse is related to the frequency of oscillation in the well of the effective potential. Of course, as the well becomes wider, this epicyclic frequency increases, and in the limit of the well turning into a ledge the epicyclic frequency goes to zero. A test particle in the marginally stable orbit, if pushed towards the black hole, "falls off the ledge" and drops (spirals) into the black hole instead of suffering the usual restoring force characteristic of stable orbits. Thus, the epicyclic frequency goes to zero (no restoring force!) at the marginally stable orbit.

On the other hand, in stable orbits the restoring force preventing infall is related to angular momentum. "The centrifugal barrier" is present when a particle has too much angular momentum. More precisely, as long as the angular momentum L in neighboring circular orbits satisfies $dL/dr > 0$, with r the radius of the orbit, motion is stable. This is because a particle trying to move in has too much angular momentum to stay inside its original circular orbit, and a particle moving out has too little to stay outside its original circular orbit. It is easy enough to see that the opposite inequality, $dL/dr < 0$ corresponds to unstable orbits, and $dL/dr = 0$ corresponds to marginally stable orbits.

To demonstrate that a marginally stable orbit exists around black holes, we will show that, necessarily, $dL/dr = 0$ at some radius. It can be shown that in the "optical" geometry of space near a static black hole *i.e.*, a geometry constructed from all distances measured by light signals received at infinity, the equation of motion in circular orbits (centered on the axis of symmetry) is quite reminiscent of Newtonian theory:

$$\frac{v^2}{R} = \frac{GM}{\tilde{r}^2}.$$

Here, R is the radius of curvature of the circular orbit and the circumference of the same orbit is $2\pi\tilde{r}$, with $L = v\tilde{r}$ being the specific (per unit mass) angular momentum. Hence, the angular momentum of a test particle in a circular orbit is given by $L^2 = GMR$, just as in Newtonian theory.

Now, it is well known that at a certain distance from the black hole photons can follow a closed circular orbit, this occurs at $r = 3M$ in Schwarzschild geometry. It should be also obvious that in the "optical" geometry all photons — by construction — follow geodesic lines *in space i.e.*, they are locally straight lines and, therefore, have $R = \infty$ everywhere, like straight lines and circles of infinite radius in a flat space. In particular, the circular photon orbit is also a locally straight line in the "optical" geometry and thus it has an infinite curvature radius. So, $R = \infty$, and, therefore, also $L = \infty$, at two different radii: at $r = \infty$, as well as in the (circular) photon orbit, at $r = 3M$ in the Schwarzschild metric. It follows that L must have a *minimum* in between, *e.g.*, somewhere in the interval $3M < r < \infty$ in the Schwarzschild metric. At this minimum of L, orbits of massive test particles are marginally stable (at $r = 6M$ in the Schwarzschild metric). Q.E.D.

After these introductory remarks, in the remainder of this contribution we present our explanation of the "kHz QPOs."

3. Millisecond variability in neutron stars and black holes

Observations of accreting neutron stars in low-mass X-ray binaries reveal two preferred frequencies, none of which is fixed. These frequencies, called kHz QPOs (quasi-periodic oscillations), show up as peaks in the observed power spectrum (the Fourier transform of the time variation) in the X-ray flux, [1]. Their properties are a major puzzle.

The peaks typically come in pairs, at frequencies ω_1 and ω_2 both on the order of a kHz ($\times 2\pi$), but both varying considerably in any given source (by several hundred Hertz in intervals of hours), with their difference $\omega_1 - \omega_2$ showing markedly less variation. That the difference is not constant (as would be expected if one of the two observed frequencies were a beat between the other frequency and the constant rotation rate of the star) is a clear indication that two fundamental frequencies are present, none of which is the stellar spin frequency.

Another puzzle has been that in black hole candidates only one high frequency QPO had been reported, with properties similar to one of the variable kHz QPOs in neutron stars. Observations of the black hole candidate GRO J 1655-40 demonstrate that in fact two high frequencies show up also in black-hole candidates.

4. Fixed frequencies in Newton's and Einstein's gravity

Newtonian gravity with the $1/r^2$ force law is scale-free, there is no preferred frequency (Fig. 1). If gravity were so described, only two fundamental frequencies would be expected for a quasi-spherical star with a thin accretion disk [2]: the stellar rotational frequency Ω_*, and the Keplerian orbital frequency at the surface of the star $\Omega_K(r_*) = (GM/r_*^3)^{1/2}$. The most important fact about these two frequencies is that they are fixed for a star of fixed angular momentum, fixed mass M, and fixed radius r_*.

It has long been recognized [3] that for black holes general relativity predicts instead two other preferred frequencies, also fixed for a given gravitating body: the orbital frequency in the innermost (marginally) stable orbit, $\Omega(r_{\rm ms})$, and the maximum epicyclic frequency $\omega_{\rm max} = \max(\omega_r) \equiv \omega_r(r_{\rm max})$, (Fig. 2). These reflect the presence of a characteristic scale, the gravitational radius $r_g = 2GM/c^2$. For example, in the Schwarzschild metric $r_{\rm ms} = 3r_g$, $r_{\rm max} = 4r_g$, $\Omega = \Omega_K$, and $\omega_{\rm max} = \Omega(4r_g)/2$. (The importance of the maximum in the epicyclic frequency was first stressed in a seminal paper by Kato and Fukue [4].) Thus, strong gravity presents us with two frequencies, but these are fixed for a given star and hence cannot be identified with the observed variable kHz QPOs in neutron stars.

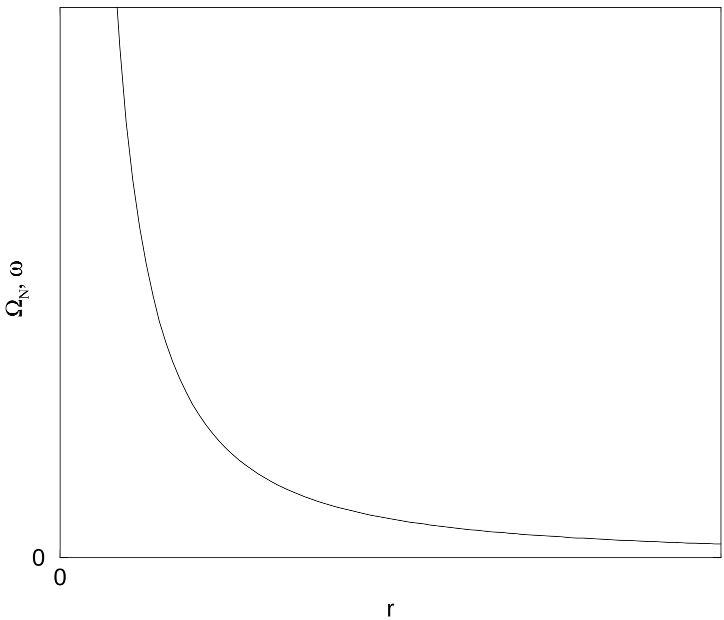

Fig. 1. The $1/r$ potential of a Newtonian point mass is scale-free. The orbital frequency in circular orbit and the frequency of epicyclic motions coincide (a small perturbation of a test particle originally in circular motion gives rise to periodic motions with the same period as in the original orbit).

However, we note that in contrast with Newtonian gravity of spherical bodies where the only frequency at a given radius is $\Omega_K(r)$, in general relativity turbulent noise may excite epicyclic motions at the different frequency ω_r, so inhomogeneities in flow at radius r contribute to the power spectrum at (angular) frequencies Ω, and ω_r, as well as at combination frequencies characteristic of coupled anharmonic oscillators (including rational fractions of eigenfrequencies), giving a rich structure to the power spectrum of X-ray variability, in agreement with observations.

5. Orbital resonance in Einstein's gravity

We point out that in general relativity, in addition to the fixed frequencies Ω_*, $\Omega(r_{\rm ms})$, $\omega_r(r_{\rm max})$, there are other preferred frequencies, those of 1:2, 1:3, *etc.*, resonances between orbital and radial epicyclic frequencies. These are possible because the ratio of orbital and radial epicyclic frequencies tends to large values near the marginally stable orbit: $\Omega(r)/\omega_r(r) \to \infty$, as $r \to r_{\rm ms}$ (Fig. 2). Frequencies in 1:2 or 1:3 ratio can be in resonance because epicyclic motion is anharmonic. As is usual for non-linear oscillators, the resonance occurs for a range of frequencies near the eigenfrequency of the oscillator, so the driving frequency (the orbital frequency here) need not be an exact multiple of the eigenfrequency of the epicyclic oscillator, nor need it be constant [5]. Thus, the resonant frequencies have just the properties which seemed puzzling in the power spectra of accreting neutron stars. We suggest that the kHz QPOs are caused by such resonances, and hence are manifestations of strong-field gravity.

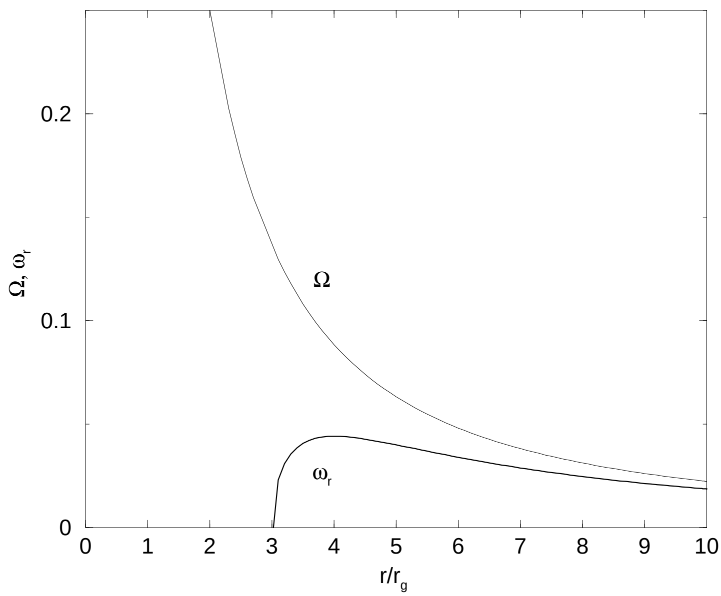

Fig. 2. In general relativity, the frequency of radial epicyclic motions, ω_r, goes to zero in the marginally stable orbit. The epicyclic frequency, ω_r, and the orbital frequency in circular orbits, Ω, both in units of c/r_g, are shown as a function of the circumferential radius in units of the gravitational radius $r_g = 2GM/c^2$ (after Kato *et al.*, 1998 [3]). The numerical values are for the Schwarzschild metric, where the marginally stable orbit is at $r = 3r_g$, but the qualitative features of the frequencies are general.

Specifically, we suggest that one of the observed high QPO frequencies could be one of the variable orbital frequencies driving the 1:2, or 1:3 epicyclic resonance. All other things being equal, the most prominent peaks are expected where most of the luminosity is generated, and that is between the marginally stable orbit, at $r = r_{\rm ms}$, where the disk terminates and the X-ray flux vanishes, and the radius $r_{\rm max}$ where ω_r peaks. For the Schwarzschild metric the position (radius) of the 1:2 resonance, r_2, coincides with the epicyclic maximum, $r_2 = r_{\rm max}$, and for rotating bodies $r_2 > r_{\rm max}$, typically. For realistic metrics of rapidly rotating neutron stars, the radius of the 1:3 resonance, r_3, is close to $r_{\rm max}$. We expect the prominent resonance in the power spectrum to be the one closest to $r_{\max}$ on the side of the star, so for rapidly rotating neutron stars the most prominent resonance should be 1:3, *i.e.*, the one with eigenfrequency $\omega_r(r_3) = \Omega(r_3)/3$.

For maximally rotating Kerr metric, r_3 and $r_{\max}$ nearly coincide, but in view of the relatively low accretion rate (per mass) of most Galactic black-hole candidates, such as 1655-40, it would not be surprising if their metrics were not maximally rotating Kerr. For accretion in the Schwarzschild metric not much luminosity is released at $r_3 = (9/8)r_{\rm ms}$. Thus, there should be much less power in the black hole QPO, than is the case for neutron stars, as indeed is observed.

We might remark that observations reveal similar electromagnetic spectra of the X-ray emissions of black hole candidates and of neutron stars, at least in some states. This strongly suggests that accretion disks in neutron stars are similar to those in black holes, as they would be if $r_* < r_{\rm ms}$, as preferred by our model. The fact that only one high frequency QPO is observed in black hole candidates has been puzzling. Recently,‡ the discovery of a second high frequency peak in the black-hole candidate 1655-40 has been reported [6]. If the two frequencies are the orbital frequency and its beat with the epicyclic frequency (Ω and $\Omega - \omega_r$) in 1:3 resonance ($\omega_r : \Omega = 1 : 3$), as suggested by us, they should be (approximately) in 3:2 ratio. The reported frequencies in 1655-40 are about 450 Hz and 300 Hz. If this frequency ratio does indeed reflect the presence of a resonance between orbital and epicyclic motions, these values of the frequencies, taken together with the known mass of the compact source, tightly constrain the angular momentum of the black hole [7].

In summary, strong-field effects of general relativity, and in particular metric properties of space-time around rapidly rotating neutron stars, make natural the excitation of a 1:3 or 1:2 anharmonic epicyclic resonance, driven by orbital motion whose variable (orbital) frequency may be imprinted on the X-ray flux as a fairly prominent kHz QPO. A second QPO may be present at a frequency differing from the first by the epicyclic frequency of the same resonance, with the difference frequency varying to a lesser degree than the QPO frequency. The same high frequency QPOs may appear in black hole systems, and indeed they now have been reported in the black hole candidate GRO J 1655-40 [6].

The authors appreciate the hospitality and generous support of Nordita, where most of this work has been done in November 2000. The Authors wish to say that the main body of this manuscript — available electronically at `http://xxx.lanl.gov/astro-ph/0105057` — was submitted to *Phys. Rev. Lett.* in December 2000, before the observational results of Strohmayer were announced.

REFERENCES

[1] M. van der Klis, *Annu. Rev. Astron. Astrophys.* **38**, 717 (2000).

[2] It is thought that fluid in accretion disks of small vertical extent ("thin disks") approximately follows circular geodesic trajectories. Observations of water masers in some thin disks in active galactic nuclei (AGNs) do indeed reveal a Keplerian distribution of velocity for the water masers, $v \equiv \Omega r = (GM/r)^{1/2}$: M. Miyoshi, *et al.*, *Nature* **373**, 127 (1995).

[3] S. Kato, S. Mineshige, J. Fukue, *Black Hole Accretion Disks*, Kyoto University Press, Kyoto 1998.

[4] S. Kato, J. Fukue, *Proc. Astron. Soc. Japan*, **32**, 377 (1980).

[5] L.D. Landau, E.M. Lifschitz, *Classical Mechanics*, Pergamon, 1973.

[6] T.E. Strohmayer, *Astrophys. J.* **552**, L49 (2001).

[7] M.A. Abramowicz, W. Kluźniak, *Astron. Astrophys.* **374**, L19 (2001).