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THE FIRST COMPACT OBJECTS IN THE MOND MODEL* **

S. Stachniewicz[a] and M. Kutschera[a,b]

[a]H. Niewodniczański Institute of Nuclear Physics
Radzikowskiego 152, 31-342 Kraków, Poland
[b]M. Smoluchowski Institute of Physics, Jagellonian University
Reymonta 4, 30-059 Kraków, Poland

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We trace the evolution of a spherically symmetric density perturbation in the MOdified Newtonian Dynamics (MOND) model. The background cosmological model is a Λ-dominated, low-Ω_b Friedmann model with no Cold Dark Matter. We include thermal processes and non-equilibrium chemical evolution of the collapsing gas. We find that the first density perturbations which collapse to form luminous objects have mass $\sim 10^5 M_\odot$. The time of the final collapse of these objects depends mainly on the value of the MOND acceleration a_0 and also on the baryon density Ω_b. For the "standard" value $a_0 = 1.2 \times 10^{-8}$ cm/s^2 the collapse starts at redshift $z \sim 160$ for $\Omega_b = 0.05$ and $z \sim 110$ for $\Omega_b = 0.02$.

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1. Introduction

Recent developments in cosmological observations have led to so-called cosmological concordance model with Ω_b about 0.03, Ω_m (dark+baryonic) about 0.3 and Ω_Λ about 0.7. However, as the ΛCDM models are dominated by hypothetical vacuum energy and non-baryonic dark matter contributions, some scientists look for different solutions. Perhaps the most interesting alternative model is the MOdified Newtonian Dynamics model (MOND)

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proposed by Milgrom [1]. It assumes that there is no non-baryonic dark matter (or it is negligible) and the lack of matter is only apparent due to modification of dynamics or gravity for small accelerations ($a \ll a_0$ where a_0 is some constant). This model seems to work very well for spiral galaxies and many other types of objects [2] but, however, it has some unresolved problems (*e.g.* lack of covariance).

Our aim here is to study what would be the implications of the MOND model for the formation of the very first objects in the Universe.

2. MOND *vs* the standard theory of linear perturbations

To apply the MOND model to structure formation calculations one encounters a number of difficulties. First of all, MOND is not a theory, it is rather a phenomenological model. In its present form MOND is inconsistent with the General Relativity. Up to now there were no successful attempts to find a generally covariant theory that could be a generalisation of the General Relativity and would give a MOND-like predictions in the low-gravity limit [2].

MOND is a model that modifies either dynamics or gravity (in this paper we assume this second possibility). It introduces a new fundamental scale, usually called a_0. Gravitational fields much stronger than a_0 are identical to their Newtonian limit $g_{\rm N}$ and very weak fields are $\sqrt{a_0 g_{\rm N}}$. According to Sanders and Verheijen [3] the value of the fundamental acceleration scale is $a_0 = 1.2 \times 10^{-8}{\rm cm/s}^2$. More precisely, the strength of the gravitational field may be written as

$$\mu\left(\frac{g}{a_0}\right)\vec{g} = \vec{g}_{\rm N}\,, \tag{1}$$

where $\mu(x)$ is some function that interpolates between these two extreme cases. This function is not specified in the model. We have decided to apply the function used by Sanders and Verheijen [3]:

$$\mu(x) \;=\; \frac{x}{\sqrt{1+x^2}} \tag{2}$$

and, finally,

$$\vec{g} \;=\; \vec{g}_{\rm N}\sqrt{\frac{1+\sqrt{1+\left(\frac{2}{x}\right)^2}}{2}}\,, \tag{3}$$

where $x = g_{\rm N}/a_0$.

If we consider the gravitational field of a point mass M, for distances $R > \sqrt{GM/a_0}$ the gravitational field would be in the MOND regime, where its strength would decrease as $1/R$ instead of $1/R^2$. It means that there is no escape velocity and all systems are gravitationally bound.

The consequences of the MOND for cosmology are not studied in details yet. Sanders [4] suggested that because in the early Universe the MOND radius is much lower than the radius of the horizon the evolution of the scale factor is described by the standard Friedmann equations. Here we follow this assumption and study the formation of the first objects in the Universe with modified dynamics.

2.1. Collapse of a pressureless fluid in MOND

Let us consider a homogenous ball of density ϱ and some radius R, expanding uniformly in all directions with speed proportional to the distance from the centre. For a sphere of radius r the deceleration in the Newtonian limit is

$$g_{\mathrm{N}} = \frac{GM}{r^2} = \frac{4}{3}\pi G \varrho r \tag{4}$$

and, of course, expansion is scale-invariant because deceleration and velocity always are proportional to the radius r. However, we can find some radius r_0 where $g_{\mathrm{N}}(r) < a_0$ for $r < r_0$ — further we will call it a 'MOND radius'. For $r < r_0$ the gravity is in the MOND regime and the dynamics is changed. The evolution of the MOND radius r_0 in the early Universe is discussed in details by Sanders [4].

In the standard perturbation theory, if the mean density is comparable with the critical density of the Universe (it is true at least for large redshifts) the recollapse depends very strongly on the value of the overdensity. Regions with mean density less than the critical density will not recollapse at all and vice versa. Moreover, the time of the recollapse is very sensitive to the value of the density. In MOND it is different because there is no critical density. The overdensity does not play an essential role and the recollapse is similar for regions of different densities.

It is quite easy to derive the linear perturbation theory in the Newtonian limit (*e.g.* in Kolb and Turner [5]). In MOND it is much more difficult because there appear nonlinear terms connected with $\nabla\varphi$, where φ is the gravitational potential. However, it is known that the rare highest fluctuations in the primordial density field were nearly spherically symmetric [6], so as long as we are concerned with the very first bound objects in the Universe we may assume spherical symmetry.

Now let us consider a spherically symmetric overdensity with density profile

$$\varrho(r) = (1 + \delta(r))\bar{\varrho}\,. \tag{5}$$

Of course, the density changes with time because of expansion or recollapse, but the mass inside some shell i

$$M_i = \int_0^{r_i} 4\pi r^2 \varrho(r) dr \tag{6}$$

remains constant, so the deceleration for a shell of radius r_i will be equal to

$$\frac{d^2}{dt^2} r_i = -f(r_i)\,, \tag{7}$$

where $f(r_i)$ is the MOND gravitational force and it depends on GM_i/r_i and a_0. Let us drop the subscript i. If we multiply this equation by dr/dt we get

$$\frac{d}{dt}\left[\frac{1}{2}\left(\frac{dr}{dt}\right)^2\right] = -f(r)\frac{dr}{dt} \tag{8}$$

and after integrating over t we obtain

$$\frac{dr}{dt} = \sqrt{-2F(r) + C}\,, \tag{9}$$

where $F'(r) = f(r)$ and C is some constant which may be easily calculated if we know initial radius and velocity for a given function $F(r)$. This formula may be easily integrated, *e.g.* with the Runge-Kutta algorithm.

To show the difference between the MOND and the Newtonian gravity, we have performed a set of calculations. We have traced the evolution of a dust cloud (no pressure) expanding homogeneously in all directions. Initial radius r_i, velocity v_i, density ϱ_i and time t_i were taken from the $\Omega = 1$ Friedmann model with no radiation for $z + 1 = 500$ and $h = 0.65$. Initial radius was chosen to be the MOND radius at that time. We have performed eight runs:

- pure Newtonian gravity with the density equal to ϱ_i, $0.80\varrho_i$ and $1.25\varrho_i$,
- MOND with the density as above,
- MOND with the a_0 parameter ten times greater than the standard value and mean density equal to ϱ_i,
- MOND with the a_0 parameter ten times lower than the standard value and mean density equal to ϱ_i.

The results are displayed at Fig. 1. Solid curves show the trajectories in the MOND model and the long-, middle- and short-dashed ones show the trajectories for the Newtonian gravity, MOND with large a_0 and MOND with low a_0, respectively. For the "standard" MOND and the Newtonian gravity lower curves show the trajectories for runs with greater densities $(1.25\varrho_i)$ and upper curves show the trajectories for lower densities $(0.80\varrho_i)$.

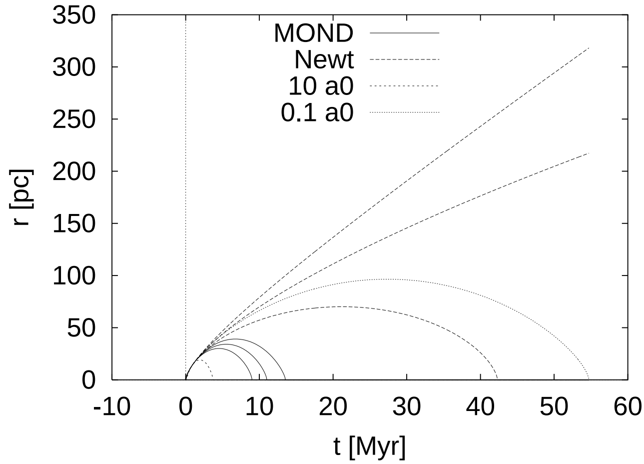

Fig. 1. Collapse of a pressureless fluid in MOND and in the Newtonian gravity.

As one could expect, raising or lowering the initial density by 25% does not have a big influence if we apply the MOND model while it plays a crucial role for the Newtonian gravity if the density is near the critical one (it is always true in the early Universe which is flat at least asymptotically at the beginning) — models with higher densities tend to recollapse while models with the lower one do not. The results are quite sensitive to the value of a_0 because it is the limit between the Newtonian and the MOND regimes, however, it is a quantitative effect only and the dust clouds in such models will always recollapse.

2.2. Collapse of a perfect gas

If we take a perfect gas instead of the pressureless fluid, the evolution will look different because effects of pressure will moderate the recollapse, especially for small systems. As we assume spherical symmetry, we use Lagrangian coordinates.

The dynamics is governed by the following equations:

$$\frac{dM}{dr} = 4\pi r^2 \varrho, \tag{10}$$

$$\frac{dr}{dt} = v\,, \tag{11}$$

$$\frac{dv}{dt} = -4\pi r^2 \frac{dp}{dM} - \frac{GM(r)}{r^2}\,, \tag{12}$$

$$\frac{du}{dt} = \frac{p}{\varrho^2}\frac{d\varrho}{dt} + \frac{\Lambda}{\varrho}\,, \tag{13}$$

where r is the radius of a sphere of mass M, u is the internal energy per unit mass, p is the pressure and ϱ is the mass density. Here Eq. (10) is the continuity equation, (11) and (12) give the acceleration and (13) accounts for the energy conservation. The last term in the Eq. (13) describes cooling/heating of the gas, with Λ being the energy absorption (emission) rate per unit volume, given in details in [7].

We use the equation of state of the perfect gas

$$p = (\gamma - 1)\varrho u\,, \tag{14}$$

where $\gamma = 5/3$, as the primordial baryonic matter after recombination is assumed to be composed mainly of monoatomic hydrogen and helium, with the fraction of molecular hydrogen H_2 always less than 10^{-3}.

In case of modified gravity, equation (12) will look a bit different:

$$\frac{dv}{dt} = -4\pi r^2 \frac{dp}{dM} - f\left(\frac{GM(r)}{r^2}\right), \tag{15}$$

where $f(x)$ is the function inverse to the function $\mu(x)$ mentioned before and it is asymptotically equal to x for $x \gg a_0$ and $\sqrt{a_0 x}$ for $x \ll a_0$.

3. Code used in the simulations

In the simulations we have used the code described in [7], based on the codes described by Thoul and Weinberg [8] and Haiman, Thoul and Loeb [9]. This is a standard, one-dimensional, second-order accurate Lagrangian finite-difference scheme. The only changes were modification of gravity and putting the dark matter fraction $\Omega_{\rm dm}$ equal to zero. However, it was necessary to make significant changes in initial conditions.

First of all, we start our calculations at the end of the radiation-dominated era. For $\Omega_b = \Omega_m = 0.02$, $z_{\rm eq} = 203$ and for $\Omega_b = 0.05$, $z_{\rm eq} = 507$ as given by the formula provided by Hu and Eiseinstein [10], $z_{\rm eq} = 2.50 \times 10^4 \Omega_0 h^2 \Theta_{2.7}^{-4}$, where $\Theta_{2.7} = T_\gamma/2.7$ K, assuming $h = 0.65$ and T_γ=2.7277 K. We have assumed that, as in the standard cosmology, initial overdensities may grow only in the matter-dominated era. We have developed and tested our own code to calculate the initial chemical composition and initial gas temperature. We have compared our results with the results by Galli and Palla [11] and they turned out to be very similar — the agreement is at a level of 10–20%. The difference was probably due to the fact that Galli and Palla have included more species (*e.g.* deuterium and lithium) and some reaction rates that they used were a bit different.

4. Results

We have performed eight runs, for various combinations of Ω_b (0.02 and 0.05), Ω_Λ ($1-\Omega_b$ and 0) and a_0 (1.2×10^{-8}cm/s^2 and 1.2×10^{-9}cm/s^2). Total mass of a cloud is about 3×10^5 M$_\odot$. Results are displayed on Figs. 2–7.

Fig. 2 shows trajectories of shells enclosing 7%, 17%, 27% ... 97% of the total mass for the models with the "standard" value of a_0.

Fig. 3 shows the same for the models with a_0 10 times lower than the "standard" value.

In figures 2–5 upper plots are for $\Omega_b = 0.02$, lower ones for $\Omega_b = 0.05$, plots at left for $\Omega_\Lambda = 1-\Omega_b$ and plots at right for $\Omega_\Lambda = 0$.

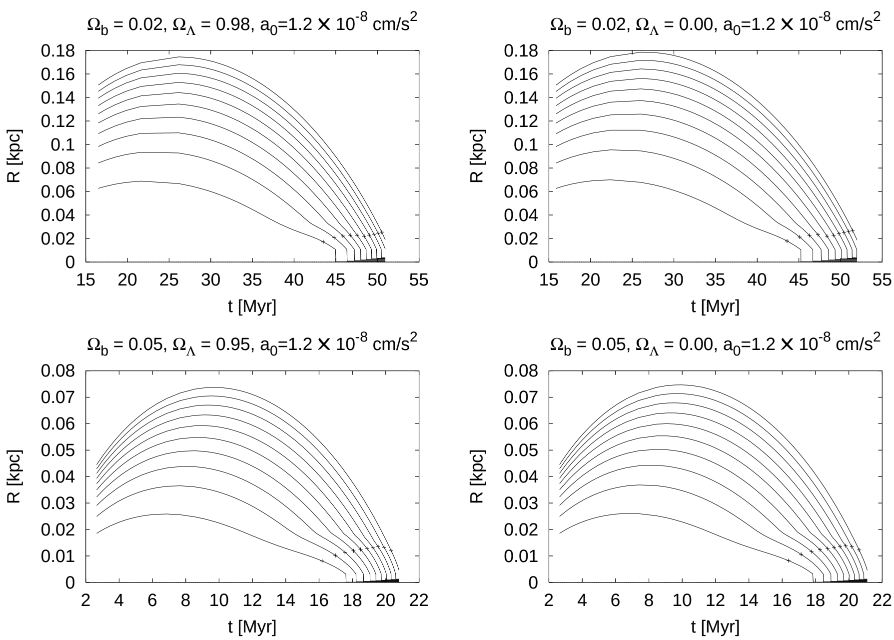

Fig. 2. Shell trajectories for the first four runs, for the "standard" value of a_0.

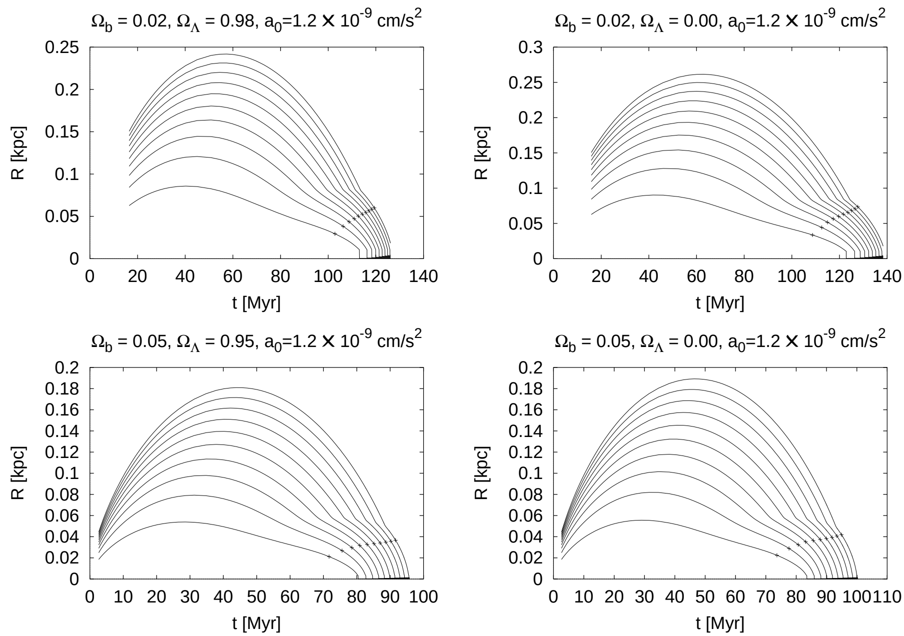

Fig. 3. Shell trajectories for the last four runs, for a_0 10 times lower than the "standard" value.

These results show that:

- if we set the cosmological constant to zero, it affects the results very slightly – collapse is a bit slower (it is due to the fact that non-zero cosmological constant affects behaviour of the scale factor and, thus, the initial velocities);
- time of collapse depends very strongly on a_0;
- results are more sensitive to the value of Ω_b for higher a_0;
- mass $\sim 10^5$ $M_\odot$ is the boundary between the regime when the gas effects dominate and the cloud virializes without further collapse to a compact object (or at least this collapse is much slower), and the collapse regime when the gas effects may be neglected. The boundary mass is a bit higher for lower a_0.

The last point needs some comments. Scientists who explore the origin of the Large Scale Structure often neglect gas terms, *i.e.* set pressure and internal energy equal to zero. Our paper is not the first one that deals with the structure formation in MOND — this problem was also explored by Sanders ([4] and [12]) and Nusser [13] but they were interested mainly in the large structure formation and they did not include gas effects. It is a good approximation for large scales but gas effects play a crucial role in small scale structure formation.

If we look *e.g.* at Fig. 2, for the outer shells the collapse is almost "symmetric" to the expansion, the slow-down by gas pressure is very tiny. In contrast, the most innermost shells show a big asymmetry — expansion, collapse and then shock waves make the matter to virialize (the kinetic energy of a collapsing shell is turned into the internal energy of the gas) so the collapse is stopped. Only pressure of the outer shells and the cooling processes (especially the cooling by molecular hydrogen H_2) force them to collapse.

Figs. 4 and 5 show the same as Figs. 2 and 3, but as a function of redshift instead of the cosmic time. They show that the collapse of the first compact objects in MOND is really very fast — for high Ω_b and "standard" a_0 it happens at $z \sim 160$, for lower Ω_b it happens at $z \sim 110$ and for lower a_0 at $z \sim 50$–60. It is a very strong prediction and hopefully it may be tested in the future by the Next Generation Space Telescope (NGST) or (perhaps) some of its successors — if they discover luminous objects or ionised gas at $z > 20$–30 it will strongly support the MOND model.

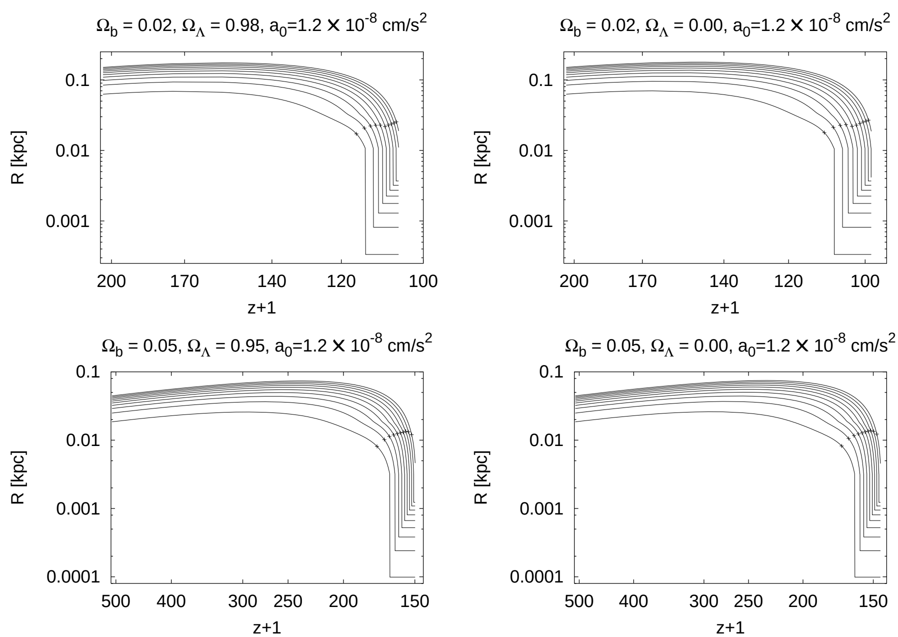

Fig. 4. Shell trajectories as a function of redshift, for the first four runs or the "standard" value of a_0.

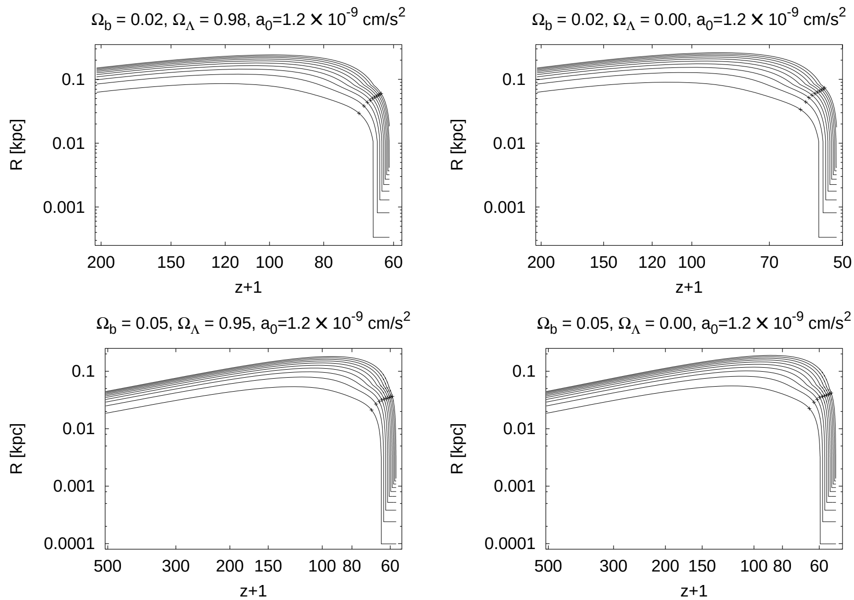

Fig. 5. Shell trajectories as a function of redshift, for the last four runs or a_0 10 times lower than the "standard" value.

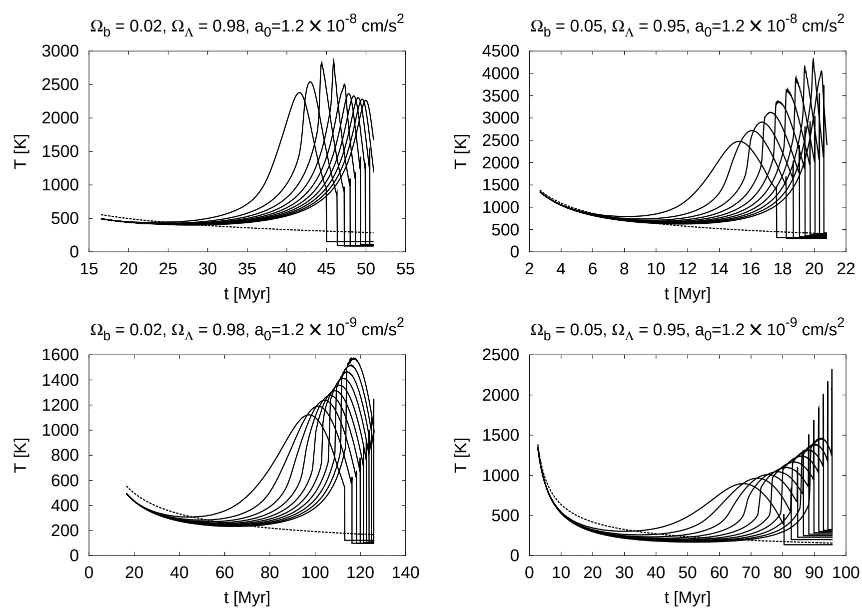

Fig. 6. Shell temperatures for the runs with non-zero cosmological constant as a function of cosmic time.

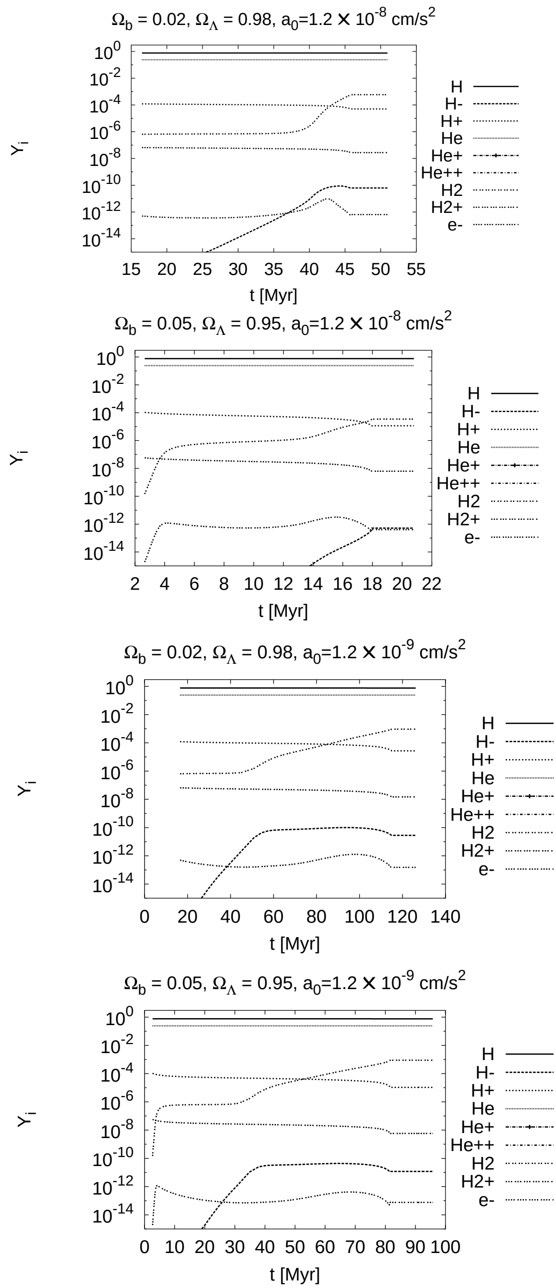

Fig. 7. The evolution of chemical composition for a shell enclosing 12% total mass, for the runs with non-zero cosmological constant.

The last two figures show the evolution of shell temperatures (Fig. 6) and chemical composition of a shell enclosing 12% of the total mass (Fig. 7) for the runs with non-zero cosmological constant. Upper plots are for the "standard" a_0, lower ones for $a_0/10$, plots at left for $\Omega_b = 0.02$ and plots at right for $\Omega_b = 0.05$. Shell temperatures behave in a very similar way: they fall (due to the Hubble expansion), then rapidly grow (collapse) and again fall (due to the cooling processes). For the "standard" a_0 maximal temperatures are significantly higher (up to 4500 K for the high-Ω_b model) so the cooling is much more efficient and the final collapse is faster. It is the effect of higher gravity and, thus, higher outer pressure. The evolution of chemical composition is qualitatively similar for all runs and all shells. It behaves similarly to the predictions of background cosmological models up to the time of collapse. The greatest differences are for H_2^+ and especially H_2 which eventually reaches the final abundance of the order of 10^{-3}. One should stress that at the time of collapse chemical reactions are much faster because their rates are proportional to ϱ^2.

It is worth to note that in order to avoid arbitrarily small time-steps, we have adopted a numerical trick described by Thoul and Weinberg [8]: if a shell falls below some radius r_c it is set to some constant value and chemical reactions and cooling are frozen. It means that the "flat" parts of the curves at the end of each plot sometimes preceeded by a strange behaviour of the shell temperature (very narrow "peaks" for the low-a_0 models) are artificial.

5. Conclusions

If the assumptions of the MOND were correct, the "Dark Ages" end very early — about $z \sim 110$–160 for the "standard" a_0 and about $z \sim 60$ for a_0 10 times lower. It may be a good test of MOND in future because in the standard (Λ)CDM scenario the first luminous objects may appear only for $z \sim 10$–20. However, much more work is necessary in order to understand MOND and, in particular, the MOND cosmology properly.

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