# STANDARD COSMOLOGY IN THE DGP BRANE MODEL*

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Large extra dimensions provide interesting extensions of our parameter space for gravitational theories. There exist now brane models which can perfectly reproduce standard four-dimensional Friedmann cosmology. These models are not motivated by observations, but they can provide interesting new insights and approaches to the dimensionality problem in string theory. I describe the embedding of standard Friedmann cosmology in the DGP model, and in particular the realization of our current (dust + $\Lambda$)-dominated Universe.

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## 1. Introduction

In recent years large extra dimensions which can only be probed by gravitons and eventually non-standard matter have attracted a lot of attention. These models usually yield the correct Newtonian $(1/r)$-potential at large distances because the gravitational field is quenched on submillimeter transverse scales. This quenching appears either due to finite extension of the transverse dimensions [1] or due to submillimeter transverse curvature scales induced by negative cosmological constants[1] [2–5]. A common feature of both of these types of models and also of the old Kaluza–Klein type models is the prediction of deviations from four-dimensional Einstein gravity at short distances. If the transverse length scale is not too small this implies the possibility to generate bulk gravitons in accelerators [8] or stars [9–13].

In this regard the recent model of Dvali, Gabadadze and Porrati (DGP) [14, 15] is very different: It predicts that four-dimensional Einstein gravity is

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[1] Please consult *e.g.* [6, 7] for much more extensive lists of references.

a short-distance phenomenon with deviations showing up at large distances. The transition between four- and higher-dimensional gravitational potentials in the DGP model arises as a consequence of the presence of both brane and bulk Einstein terms in the action.

Furthermore, it was observed in [6] that the DGP model allows for an embedding of standard Friedmann cosmology in the sense that the cosmological evolution of the background metric on the brane can entirely be described by the standard Friedmann equation plus energy conservation on the brane. This was later generalized to arbitrary number of transverse dimensions in [19].

In Sec. 3 I review the standard embedding of Friedmann cosmology found in [6], and describe in particular the realization of a (dust + $\Lambda$)-dominated Universe in this framework.

## 2. The DGP model

The action of the DGP model reads

$$\begin{aligned} S &= \frac{m_4^3}{2}\int dt\int d^3\vec{x}\int dx^\perp\sqrt{-g}R \\ &\quad + \int dt\int d^3\vec{x}\left(\frac{m_3^2}{2}\sqrt{-g}R^{(d-1)} - m_4^3\sqrt{-g}\,\overline{K} + \mathcal{L}\right)\Big|_{x^\perp=0}, \end{aligned} \tag{1}$$

where Gaussian normal coordinates are employed:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + (dx^\perp)^2. \tag{2}$$

The transverse coordinate $|x^\perp|$ is a distance along orthogonal geodesics to the brane.

The $(3+1)$-dimensional submanifold $x^\perp = 0$ is usually denoted as a 3-brane, and $\mathcal{L}$ contains the matter degrees of freedom on this brane. Extrinsic curvature effects have been taken into account through a Gibbons–Hawking term [20–23] (which requires averaging over the two sides of the brane [6]), and $m_4$ and $m_3$ are reduced Planck masses in five and four dimensions, respectively.

The action (1) yields Einstein equations

$$\begin{aligned} &m_4^3\left(R_{MN} - \frac{1}{2}g_{MN}R\right) + m_3^2 g_M{}^\mu g_N{}^\nu\left(R_{\mu\nu}^{(d-1)} - \frac{1}{2}g_{\mu\nu}R^{(d-1)}\right)\delta(x^\perp) \\ &= g_M{}^\mu g_N{}^\nu T_{\mu\nu}\delta(x^\perp), \end{aligned} \tag{3}$$

corresponding to matching conditions

$$\lim_{\varepsilon\to+0}[K_{\mu\nu}]^{x^\perp=\varepsilon}_{x^\perp=-\varepsilon} = \frac{1}{m_4^3}\left(T_{\mu\nu}-\frac{1}{d-1}g_{\mu\nu}g^{\alpha\beta}T_{\alpha\beta}\right)\Bigg|_{x^\perp=0}$$
$$-\frac{m_3^2}{m_4^3}\left(R^{(d-1)}_{\mu\nu}-\frac{1}{2(d-1)}g_{\mu\nu}g^{\alpha\beta}R^{(d-1)}_{\alpha\beta}\right)\Bigg|_{x^\perp=0} \tag{4}$$

for the extrinsic curvature of the brane.

The use of Gaussian normal coordinates (2) implies that we can impose a harmonic gauge condition only on the longitudinal coordinates $x^\mu$:

$$\partial_\alpha h^\alpha{}_\mu + \partial_\perp h_{\perp\mu} = \tfrac{1}{2}\partial_\mu\left(h^\alpha{}_\alpha + h_{\perp\perp}\right), \tag{5}$$

but this is sufficient to get a decoupled equation for the gravitational potential of a static mass distribution:

The transverse equations in the gauge (5)

$$\begin{aligned} R_{\perp\perp} - R^\alpha{}_\alpha &= \tfrac{1}{2}\partial_\alpha\partial^\alpha\left(h^\beta{}_\beta - h_{\perp\perp}\right) + \partial_\perp\partial_\alpha h^\alpha{}_\perp = 0,\\ R_{\perp\mu} &= \tfrac{1}{2}\left(\partial_\mu\partial_\alpha h^\alpha{}_\perp - \partial_K\partial^K h_{\perp\mu}\right) + \tfrac{1}{4}\partial_\mu\partial_\perp\left(h_{\perp\perp} - h^\alpha{}_\alpha\right) = 0 \end{aligned}$$

can be solved by $h_{\perp\mu}=0$, $h_{\perp\perp}=h^\alpha{}_\alpha$, and the remaining equations take the form

$$\begin{aligned} &m_4^3(\partial_\alpha\partial^\alpha+\partial_\perp^2)h_{\mu\nu} + m_3^2\delta(x^\perp)\left(\partial_\alpha\partial^\alpha h_{\mu\nu} - \partial_\mu\partial_\nu h^\alpha{}_\alpha\right)\\ &= -2\delta(x^\perp)\left(T_{\mu\nu}-\frac{1}{d-1}\eta_{\mu\nu}\eta^{\alpha\beta}T_{\alpha\beta}\right). \end{aligned}$$

This yields the equation for the gravitational potential of a mass density $\varrho(\vec{r}) = M\delta(\vec{r})$ on $\mathcal{M}_{3,1}$:

$$m_4^3(\Delta+\partial_\perp^2)U(\vec{r},x^\perp) + m_3^2\delta(x^\perp)\Delta U(\vec{r},x^\perp) = \tfrac{2}{3}M\delta(\vec{r})\delta(x^\perp). \tag{6}$$

The resulting potential on the brane is [6, 14]

$$\begin{aligned} U(\vec{r}) &= -\frac{M}{6\pi m_3^2 r}\left[\cos\left(\frac{2m_4^3}{m_3^2}r\right) - \frac{2}{\pi}\cos\left(\frac{2m_4^3}{m_3^2}r\right)\mathrm{Si}\left(\frac{2m_4^3}{m_3^2}r\right)\right.\\ &\quad\left.+\frac{2}{\pi}\sin\left(\frac{2m_4^3}{m_3^2}r\right)\mathrm{ci}\left(\frac{2m_4^3}{m_3^2}r\right)\right], \end{aligned} \tag{7}$$

with the sine and cosine integrals

$$\begin{aligned} \mathrm{Si}(x) &= \int_0^x d\xi\,\frac{\sin\xi}{\xi},\\ \mathrm{ci}(x) &= -\int_x^\infty d\xi\,\frac{\cos\xi}{\xi}. \end{aligned}$$

The DGP model thus predicts a transition scale

$$\ell_{\rm DGP} = \frac{m_3^2}{2m_4^3} \tag{8}$$

between four-dimensional behavior and five-dimensional behavior of the gravitational potential:

$$\begin{aligned} r \ll \ell_{\rm DGP}: \; U(\vec{r}) = & -\frac{M}{6\pi m_3^2 r}\left[1+\left(\gamma-\frac{2}{\pi}\right)\frac{r}{\ell_{\rm DGP}}\right. \\ & \left. + \frac{r}{\ell_{\rm DGP}}\ln\left(\frac{r}{\ell_{\rm DGP}}\right)+\mathcal{O}\left(\frac{r^2}{\ell_{\rm DGP}^2}\right)\right], \\ r \gg \ell_{\rm DGP}: \; U(\vec{r}) = & -\frac{M}{6\pi^2 m_4^3 r^2}\left[1-2\frac{\ell_{\rm DGP}^2}{r^2}+\mathcal{O}\left(\frac{\ell_{\rm DGP}^4}{r^4}\right)\right]. \end{aligned}$$

$\gamma \simeq 0.577$ is Euler's constant.

If we would use the usual value of the reduced Planck mass for $m_3$, then the small $r$ potential would be stronger than the genuine four-dimensional potential by a factor $\frac{4}{3}$ because the coupling of the masses on the brane to the four-dimensional Ricci tensor is increased by this factor. This factor $\frac{4}{3}$ is in agreement with the tensorial structure of the graviton propagator reported in [14]. Therefore the four-dimensional reduced Planck mass is slightly larger in the DGP model than in ordinary Einstein gravity:

$$m_3 = (6\pi G_{N,3})^{-1/2} \simeq 2.8 \times 10^{18}\,\text{GeV}. \tag{9}$$

The potential is displayed in Fig. 1.

The current limit on deviations from Einstein gravity at *large distances* is still set by [16], see also [17,18].

The limit $\ell_{\rm DGP} > 10^{14}\,\text{m}$ would translate with (8),(9) into a bulk Planck mass $m_4 < 200\,\text{GeV}$. This may seem surprisingly low, but recall from (7) that the relevant graviton coupling scale at distances well below $\ell_{\rm DGP}$ is the large Planck mass $m_3$ on the brane, and lower $m_4$ means larger $\ell_{\rm DGP}$, making it even harder to detect any deviations from Einstein gravity.

It is certainly easy to constrain $\ell_{\rm DGP}$ to supergalactic scales, because the DGP model predicts a weakening of gravity at large distances, thus potentially increasing the need for dark matter.

However, interest in this model does not arise from the hope that one might detect any corresponding effects at galactic or not too large supergalactic scales: The interest in the model results from the observation that it provides a simple, yet surprising mechanism to accommodate four-dimensional gravity in a model with infinitely large extra dimensions.

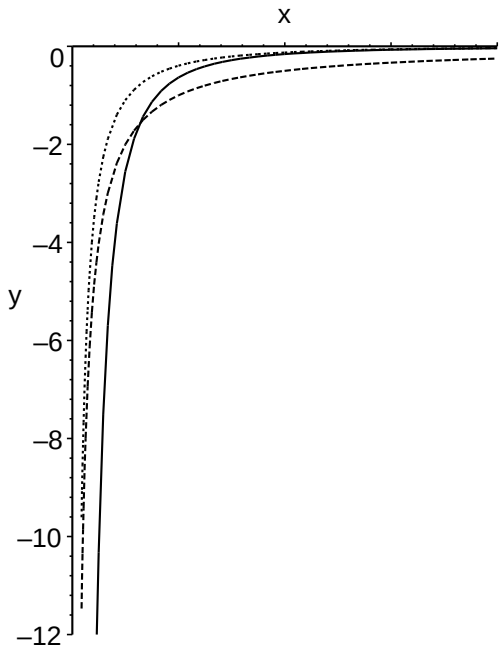


Fig. 1. The dotted line is the gravitational potential in the DGP model as a function of $x = r/\ell_{\rm DGP}$. The horizontal axis covers the region $0 \le x \le 4$. The vertical units correspond to $M/(12m_4^3)$. The dashed line is the ordinary three-dimensional Newton potential in these units, and the solid line is the corresponding potential in four spatial dimensions.

## 3. Standard cosmology in the DGP model

From the fact that the DGP model predicts deviations only at large distances one might hope that it could be ruled out from cosmological observations, but we will see that it can account for standard Friedmann cosmology at any distance scale on the brane:

Brane cosmology usually starts from the line element (with $x_i \equiv x^i$, $r^2 \equiv x_i x^i$)

$$\begin{aligned} ds^2 &= -n^2(x^\perp,t)dt^2 \\ &\quad +a^2(x^\perp,t)\left(\delta_{ij}+k\frac{x_i x_j}{1-kr^2}\right)dx^i dx^j + b^2(x^\perp,t){dx^\perp}^2 . \end{aligned} \tag{10}$$

This *ansatz* implies a brane cosmological principle in that it assumes that every hypersurface $x^\perp = \text{const.}$ is a Robertson–Walker spacetime with cosmological time $T|_{x^\perp} = \int |n(x^\perp,t)|dt$.

Building on the results of [25,26], the cosmological evolution equations of a 3-brane in a five-dimensional bulk following from (3) and (4) were presented in [27,28].

Here I will follow [6] and give the results for a brane of dimension $\nu+1$.

The Einstein tensors for the metric (10) in Gaussian normal coordinates ($b^2=1$) and in $d=\nu+1$ spatial dimensions are
on the hypersurfaces $x^\perp=$ const.:

$$G^{(\nu)}_{00} = \frac{1}{2}\nu(\nu-1)n^2\left(\frac{\dot a^2}{n^2a^2}+\frac{k}{a^2}\right) \tag{11}$$

$$G^{(\nu)}_{ij} = (\nu-1)\left(\frac{\dot n\dot a}{n^3a}-\frac{\ddot a}{n^2a}\right)g_{ij}-\frac{1}{2}(\nu-1)(\nu-2)\left(\frac{\dot a^2}{n^2a^2}+\frac{k}{a^2}\right)g_{ij}\,, \tag{12}$$

and in the bulk:

$$G_{00} = \frac{1}{2}\nu(\nu-1)n^2\left(\frac{\dot a^2}{n^2a^2}-\frac{a'^2}{a^2}+\frac{k}{a^2}\right)-\nu n^2\frac{a''}{a}\,, \tag{13}$$

$$G_{ij} = \frac{1}{2}(\nu-1)(\nu-2)\left(\frac{a'^2}{a^2}-\frac{\dot a^2}{n^2a^2}-\frac{k}{a^2}\right)g_{ij} + (\nu-1)\left(\frac{a''}{a}+\frac{n'a'}{na}-\frac{\ddot a}{n^2a}+\frac{\dot n\dot a}{n^3a}\right)g_{ij}+\frac{n''}{n}g_{ij}\,, \tag{14}$$

$$G_{0\perp} = \nu\left(\frac{n'}{n}\frac{\dot a}{a}-\frac{\dot a'}{a}\right), \tag{15}$$

$$G_{\perp\perp} = \frac{1}{2}\nu(\nu-1)\left(\frac{a'^2}{a^2}-\frac{\dot a^2}{n^2a^2}-\frac{k}{a^2}\right)+\nu\left(\frac{n'a'}{na}+\frac{\dot n\dot a}{n^3a}-\frac{\ddot a}{n^2a}\right). \tag{16}$$

The matching conditions (4) for an ideal fluid on the brane

$$T_{00}=\varrho n^2\,,\qquad T_{ij}=pg_{ij}$$

read

$$\lim_{\varepsilon\to+0}[\partial_\perp n]^{x^\perp=\varepsilon}_{x^\perp=-\varepsilon} = \frac{n}{\nu m^\nu_{\nu+1}}\left((\nu-1)\varrho+\nu p\right)\Big|_{x^\perp=0} + \frac{m^{\nu-1}_\nu}{m^\nu_{\nu+1}}(\nu-1)n\left(\frac{\ddot a}{n^2a}-\frac{\dot a^2}{2n^2a^2}-\frac{\dot n\dot a}{n^3a}-\frac{k}{2a^2}\right)\Big|_{x^\perp=0}, \tag{17}$$

$$\lim_{\varepsilon\to+0}[\partial_\perp a]^{x^\perp=\varepsilon}_{x^\perp=-\varepsilon} = \frac{m^{\nu-1}_\nu}{2m^\nu_{\nu+1}}(\nu-1)\left(\frac{\dot a^2}{n^2a}+\frac{k}{a}\right)\Big|_{x^\perp=0}-\frac{\varrho a}{\nu m^\nu_{\nu+1}}\Big|_{x^\perp=0}. \tag{18}$$

This corresponds to effective gravitational contributions to the pressure and energy density on the brane:

$$\begin{aligned}\varrho_G &= -\frac{1}{2}\nu(\nu-1)m_\nu^{\nu-1}\left(\frac{\dot a^2}{n^2a^2}+\frac{k}{a^2}\right),\\ p_G &= (\nu-1)m_\nu^{\nu-1}\left(\frac{\ddot a}{n^2a}-\frac{\dot n\dot a}{n^3a}\right)+\frac{1}{2}(\nu-1)(\nu-2)m_\nu^{\nu-1}\left(\frac{\dot a^2}{n^2a^2}+\frac{k}{a^2}\right).\end{aligned}$$

Energy conservation on the brane follows from the absence of transverse momentum, $T_{0\perp}=0$. With (15) this implies

$$\frac{n'}{n}=\frac{\dot a'}{\dot a} \tag{19}$$

and in particular

$$\lim_{\varepsilon\to+0}\left[\frac{n'}{n}\right]_{x^\perp=-\varepsilon}^{x^\perp=\varepsilon}=\lim_{\varepsilon\to+0}\left[\frac{\dot a'}{\dot a}\right]_{x^\perp=-\varepsilon}^{x^\perp=\varepsilon}.$$

Insertion of (17),(18) into this equation yields the sought for conservation equation

$$\dot\varrho a\Big|_{x^\perp=0}=-\nu(\varrho+p)\dot a\Big|_{x^\perp=0}. \tag{20}$$

Insertion of (19) into (13) and (16) for $x^\perp\neq 0$ yields a $\nu$-dimensional version of the integral of Binétruy *et al.* [26]:

$$\frac{2}{\nu n^2}a'a^\nu G_{00}=\frac{\partial}{\partial x^\perp}\left(\frac{\dot a^2}{n^2}a^{\nu-1}-a'^2a^{\nu-1}+ka^{\nu-1}\right)=0,$$

$$\frac{2}{\nu}\dot a a^\nu G_{\perp\perp}=-\frac{\partial}{\partial t}\left(\frac{\dot a^2}{n^2}a^{\nu-1}-a'^2a^{\nu-1}+ka^{\nu-1}\right)=0,$$

*i.e.*

$$I^+=\left(\frac{\dot a^2}{n^2}-a'^2+k\right)a^{\nu-1}\Big|_{x^\perp>0} \tag{21}$$

and

$$I^-=\left(\frac{\dot a^2}{n^2}-a'^2+k\right)a^{\nu-1}\Big|_{x^\perp<0} \tag{22}$$

are two constants, with $I^+=I^-$ if

$$\lim_{\varepsilon\to+0}a'\Big|_{x^\perp=\varepsilon}=\pm\lim_{\varepsilon\to+0}a'\Big|_{x^\perp=-\varepsilon}.$$

We have not yet taken into account $G_{ij} = 0$ in the bulk. However, Eq. (19) implies $\partial_\perp(n/\dot{a}) = 0$, and therefore

$$\frac{n''}{n} = \frac{\dot{a}''}{\dot{a}}\,.$$

This, the bulk equations $G_{00} = G_{\perp\perp} = 0$, and the constancy of $I^\pm$ imply that the bulk equation $G_{ij} = 0$ is already satisfied and does not provide any new information.

We can now simplify the previous equations by further restricting our Gaussian normal coordinates through the gauge

$$n(0,t) = 1 \tag{23}$$

by simply performing the transformation

$$t \;\Rightarrow\; t_{\rm FRW} = \int^t dt'\, n(0,t')$$

of the time coordinate. This gauge is convenient because it gives the usual cosmological time on the brane. Henceforth, this gauge will be adopted, but the index FRW will be omitted.

Eqs. (19),(23) imply that our basic dynamical variable is $a(x^\perp,t)$, with $n(x^\perp,t)$ given by

$$n(x^\perp,t) = \frac{\dot{a}(x^\perp,t)}{\dot{a}(0,t)}\,.$$

The basic set of cosmological equations in the present setting (without a cosmological constant in the bulk) are thus Eqs. (18),(20)–(22), which have to be amended with dispersion relations (or corresponding evolution equations) for the ideal fluid components on the brane:

$$\lim_{\varepsilon\to+0}\left[\partial_\perp a\right]_{x^\perp=-\varepsilon}^{x^\perp=\varepsilon}(t) = \frac{m_\nu^{\nu-1}}{2m_{\nu+1}^\nu}(\nu-1)\frac{\dot{a}^2(0,t)+k}{a(0,t)} - \frac{\varrho(t)a(0,t)}{\nu m_{\nu+1}^\nu},$$

$$I^+ = \left(\dot{a}^2(0,t) - a'^2(x^\perp,t) + k\right) a^{\nu-1}(x^\perp,t)\Big|_{x^\perp>0}\,,$$

$$I^- = \left(\dot{a}^2(0,t) - a'^2(x^\perp,t) + k\right) a^{\nu-1}(x^\perp,t)\Big|_{x^\perp<0}\,,$$

$$\dot{\varrho}(t)a(0,t) = -\nu(\varrho(t)+p(t))\dot{a}(0,t),$$

$$p(t) = p(\varrho(t))\,,$$

$$n(x^\perp,t) = \frac{\dot{a}(x^\perp,t)}{\dot{a}(0,t)}\,.$$

Our primary concern with regard to observational consequences is the evolution of the scale factor $a(0,t)$ on the brane, and we can use the integrals $I^{\pm}$ to eliminate the normal derivatives $a'(x^{\perp} \to \pm 0, t)$ from the brane analogue of the Friedmann equation:

$$\pm\sqrt{\dot{a}^2(0,t) + k - I^{+}a^{1-\nu}(0,t)} \mp \sqrt{\dot{a}^2(0,t) + k - I^{-}a^{1-\nu}(0,t)} = \frac{m_{\nu}^{\nu-1}}{2m_{\nu+1}^{\nu}}(\nu-1)\frac{\dot{a}^2(0,t)+k}{a(0,t)} - \frac{\varrho(t)a(0,t)}{\nu m_{\nu+1}^{\nu}}\,. \tag{24}$$

If this equation is solved for $a(0,t)$ by using the dispersion relation and energy conservation on the brane, then $a(x^{\perp},t)$ can be determined in the bulk from the constancy of $I^{\pm}$.

There must be at least one minus sign on the left hand side of (24) if the right hand side is negative, but the dynamics of the problem does not require symmetry across the brane. The constants $I^{\pm}$ must be considered as initial conditions, and if *e.g.* $I^{+} \neq I^{-}$, then there cannot be any symmetry across the brane.

If $m_{\nu} \neq 0$ and the normal derivatives on the brane have the same sign:

$$m_{\nu}a'(x^{\perp} \to +0, t)a'(x^{\perp} \to -0, t) > 0\,, \tag{25}$$

then the cosmology of our brane approximates ordinary Friedmann–Robertson–Walker cosmology during those epochs when

$$I^{\pm} \ll \left(\dot{a}^2(0,t) + k\right) a^{\nu-1}(0,t)\,.$$

In particular, this applies to late epochs in expanding open or flat branes ($k \neq 1$).

### *3.1. The embedding of standard Friedmann cosmology*

Standard cosmology may be realized in the DGP model in an even more direct way:

If (25) holds and $I^{+} = I^{-}$, then (24) reduces *entirely* to the ordinary Friedmann equation for a $(\nu+1)$-dimensional spacetime [6]. This embedding of standard Friedmann cosmology is then given by the following set of cosmological evolution equations:

$$\frac{\dot{a}^2(0,t)+k}{a^2(0,t)} = \frac{2\varrho(t)}{\nu(\nu-1)m_\nu^{\nu-1}}\,,$$

$$I = \left(\dot{a}^2(0,t) - a'^2(x^\perp,t) + k\right) a^{\nu-1}(x^\perp,t)\,,$$

$$\dot{\varrho}(t)a(0,t) = -\nu(\varrho(t)+p(t))\dot{a}(0,t)\,,$$

$$p(t) = p(\varrho(t)),$$

$$n(x^\perp,t) = \frac{\dot{a}(x^\perp,t)}{\dot{a}(0,t)}\,.$$

The evolution of the background geometry of the observable Universe according to the Friedmann equation can thus be embedded in the DGP model, with the behavior of $a(x^\perp,t)$ off the brane determined solely by the integral $I$ and the boundary condition $a(0,t)$ from the Friedmann equation.

This embedding will be asymmetric in all realistic cases, because the requirement that the Friedmann equation holds on the brane is equivalent to the smoothness condition

$$\lim_{\varepsilon\to+0}\partial_\perp a(\varepsilon,t) = \lim_{\varepsilon\to+0}\partial_\perp a(-\varepsilon,t)\,.$$

This could yield a symmetric embedding only for $a'(0,t)=0$, but this is incompatible with the time-independence of the integral $I$ (apart from the exotic case $k=-1$, $\dot{a}^2=1$). And *vice versa*: The previously often employed assumption that embeddings would have to be symmetric implied a cusp at the brane and a corresponding violation of the Friedmann equation on the brane. Observation of the Hubble flow thus might have had observable consequences on brane cosmology, see [29, 30] for a discussion of this, but in the present embedding scenarios the Hubble flow cannot rule out brane scenarios.

In the sequel we will choose the sign of $x^\perp$ in the direction of increasing scale factor:

$$a'(0,t) > 0. \tag{26}$$

The possibility of a direct embedding of Friedmann cosmology is a consequence of the fact that the evolution of the background geometry (10) and the source terms $\varrho$, $p$ are supposed to depend only on $t$ and $x^\perp$. This implies the possibility to decouple the brane and the bulk contributions in the Einstein equation for the background metric, and in this case deviations from Friedmann–Robertson–Walker cosmology would only show up in specific $\vec{x}$-dependent effects like the evolution of cosmological perturbations and structure formation.

In the relevant case $\nu = 3$ we find from the second equation

$$I = \left(\dot{a}^2(0,t) - a'^2(x^\perp,t) + k\right) a^2(x^\perp,t),$$

and from the equation for $n(x^\perp,t)$, the solutions for the metric components off the brane in terms of the metric on the brane (with the sign convention from (26)):

$$a^2(x^\perp,t) = a^2(0,t) + \left(\dot{a}^2(0,t)+k\right) x^{\perp 2} + 2\sqrt{(\dot{a}^2(0,t)+k)\, a^2(0,t) - I}\, x^\perp, \tag{27}$$

$$n(x^\perp,t) = \left[a(0,t) + \ddot{a}(0,t) x^{\perp 2} + a(0,t) x^\perp \frac{a(0,t)\ddot{a}(0,t) + \dot{a}^2(0,t) + k}{\sqrt{(\dot{a}^2(0,t)+k)\, a^2(0,t) - I}}\right]$$
$$\times \left[a^2(0,t) + \left(\dot{a}^2(0,t)+k\right) x^{\perp 2} + 2\sqrt{(\dot{a}^2(0,t)+k)\, a^2(0,t) - I}\, x^\perp\right]^{-1/2}. \tag{28}$$

This embedding of Friedmann cosmology on the brane becomes particularly simple for $I = 0$:

$$a(x^\perp,t) = a(0,t) + \sqrt{\dot{a}^2(0,t)+k}\, x^\perp, \tag{29}$$

$$n(x^\perp,t) = 1 + \frac{\ddot{a}(0,t)}{\sqrt{\dot{a}^2(0,t)+k}} x^\perp. \tag{30}$$

### *3.2. Radiation dominated spatially flat Universe in the DGP model*

For early Universe cosmology spatial curvature and the cosmological constant are negligible compared to the radiation dominated matter density.

The energy density and scale factor on the brane evolve in the standard way

$$\varrho(t) = \frac{3m_3^2}{4t^2},$$
$$a(0,t) = C\sqrt{t}, \tag{31}$$

and the metric off the brane is given by [6]

$$a^2(x^\perp,t) = \frac{C^2}{4t} x^{\perp 2} + \sqrt{C^4 - 4I}\, x^\perp + C^2 t$$

and

$$n^2(x^\perp,t) = \frac{C^2}{4t^2} \frac{\left(4t^2 - x^{\perp 2}\right)^2}{C^2 x^{\perp 2} + 4\sqrt{C^4 - 4I}\, x^\perp t + 4C^2 t^2}.$$

This yields in particular for $I = 0$:

$$a(x^\perp, t) = C\left(\sqrt{t} + \frac{x^\perp}{2\sqrt{t}}\right),$$

$$n(x^\perp, t) = 1 - \frac{x^\perp}{2t}.$$

There appear coordinate singularities on the spacelike hypercone $x^\perp = \pm 2t$. This is presumably a consequence of the fact that the orthogonal geodesics emerging from the brane (which we used to set up our Gaussian normal system) do not cover the full five-dimensional spacetime.

### *3.3. (Dust + $\Lambda$)-dominated Universe in the DGP model*

The evolution of the background metric in a (dust + $\Lambda$)-dominated Universe is readily inferred from energy conservation and the Friedmann equation. The energy density in the dust evolves according to

$$\varrho_{\text{dust}} = \frac{\Lambda}{\sinh^2\left(\frac{\sqrt{3\Lambda}}{2m_3}t\right)}, \tag{32}$$

and the scale factor on the brane is

$$a(0,t) = \left(\frac{\sinh(\sqrt{3\Lambda}t/2m_3)}{\sinh(\sqrt{3\Lambda}t_0/2m_3)}\right)^{\frac{2}{3}}. \tag{33}$$

From subsection 3.1 it is clear that minimal embeddings correspond to $I = 0$, and Eqs. (29),(30) yield

$$a(x^\perp, t) = \left(\frac{\sinh(\sqrt{3\Lambda}t/2m_3)}{\sinh(\sqrt{3\Lambda}t_0/2m_3)}\right)^{\frac{2}{3}} \left[1 + \sqrt{\frac{\Lambda}{3}}\coth\left(\frac{\sqrt{3\Lambda}}{2m_3}t\right)\frac{x^\perp}{m_3}\right], \tag{34}$$

$$\begin{aligned} n(x^\perp, t) &= 1 + \frac{\ddot{a}(0,t)}{\dot{a}(0,t)}x^\perp \\ &= 1 + \frac{\sqrt{3\Lambda}}{2m_3}\left[\tanh\left(\frac{\sqrt{3\Lambda}}{2m_3}t\right) - \frac{2}{3}\coth\left(\frac{\sqrt{3\Lambda}}{2m_3}t\right)\right]x^\perp. \end{aligned} \tag{35}$$

## 4. Conclusions

Brane models provide a somewhat exotic, yet interesting extension of our parameter space for gravitational theories. We have seen that even standard Friedmann cosmology on the brane can be accommodated by brane models with Einstein terms both on the brane and in the bulk, thus implying that cosmological tests of these models should come from structure formation, where the DGP model should have problems due to the weakening of gravity at large distances.

Another matter of concern for brane models in general are gauge fields. While it may be mathematically possible to restrict non-gravitational terms in the action *a priori* to brane contributions, logically this may not seem entirely satisfactory. It is relatively easy to limit the penetration depth for scalars and fermions, but I am not aware of any satisfactory mechanism to constrain the penetration depth of time-dependent gauge fields (static sources are no problem, see [31]).

Still, it is of interest to study the properties of these models: Brane models like the DGP model provide a framework for extensions of the brane models of string theory to infinitely large transverse dimensions, thus potentially shedding new light on the dimensionality problem in string theory.

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