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NEUTRINO MASSES IN GUTs AND THE BARYON ASYMMETRY*

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We study the implications of large neutrino mixings for grand unified theories based on the seesaw mechanism. In SU(5) GUTs large mixings can be accommodated by means of $\mathrm{U}(1)_F$ flavour symmetries. In these models the heavy Majorana neutrinos are essentially decoupled from low energy neutrino physics. On the contrary in SO(10) GUTs large neutrino mixings severely constrain the mass spectrum of the heavy Majorana neutrinos. This leads to predictions for a variety of observables in neutrino physics as well as for the baryon asymmetry.

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1. Status of neutrino mixing

Recent results from the Sudbury Neutrino Observatory [1] and from the SuperKamiokande experiment [2] provide further evidence for neutrino oscillations as the solution of the solar neutrino problem. Neutrino oscillations can also account for the atmospheric neutrino anomaly [3,4]. It is remarkable that a consistent picture can be obtained with just three neutrinos, ν_e, ν_μ and ν_τ, undergoing 'nearest neighbour' oscillations, $\nu_e \leftrightarrow \nu_\mu$ and $\nu_\mu \leftrightarrow \nu_\tau$.

For massive neutrinos a mixing matrix U appears in the leptonic charged current,

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}} \sum_{\alpha,i} \overline{e}_\alpha \gamma^\mu (1-\gamma_5) U_{\alpha,i} \nu_i \; W_\mu^- \; + \dots \,, \tag{1}$$

where e_α and ν_i are mass eigenstates. In the case of three neutrinos, one for each generation, U is a unitary matrix.

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The experimental results on the ν_e deficit in the solar neutrino flux favour the LMA or LOW solutions [5] of the MSW conversion with large mixing angle. A large mixing also fits the atmospheric neutrino oscillations. As a result, the leptonic mixing matrix $U_{\alpha i}$ appears to be very different from the familiar CKM quark mixing matrix $V_{\alpha i}$. The emerging pattern is rather simple [6],

$$U = \begin{pmatrix} * & * & \diamond \\ * & * & * \\ * & * & * \end{pmatrix} . \tag{2}$$

Here the '$*$' denotes matrix elements whose value is consistent with the range $0.5 \ldots 0.8$, whereas for the matrix element '$\diamond$' only an upper bound exits, $|U_{e3}| < 0.16$. The neutrino masses may be hierarchical or quasi-degenerate. Note, however, that a possible hierarchy has to be much weaker than the known mass hierarchy of quarks and charged leptons.

Several interesting phenomenological schemes have been suggested, such as 'bi-maximal' or 'democratic' mixing, which describe the pattern (2) rather well [7]. Is is unclear, however, how these schemes are related to a more fundamental theory. We shall therefore focus on the question how large neutrino mixings can be obtained in a grand unified theory based on the gauge groups SU(5) or SO(10). In both cases we shall rely on the seesaw mechanism which naturally explains the smallness of light Majorana neutrino masses m_ν by the largeness of right-handed neutrino masses M [8],

$$m_\nu \simeq -m_{\mathrm{D}} \frac{1}{M} m_{\mathrm{D}}^T , \tag{3}$$

where m_{D} is the Dirac neutrino mass matrix. In unified theories m_{D} is related to the quark and charged lepton mass matrices. Since they have a large hierarchy, the almost non-hierarchical structure of the leptonic mixing matrix is very surprising and requires some explanation. In the following we shall discuss two qualitatively different examples based on the GUT groups SU(5) and SO(10), respectively, which illustrate present attempts to solve the puzzle of the large neutrino mixings.

2. Models with SU(5)

In the simplest GUT based on the gauge group SU(5) [9] quarks and leptons are grouped into the multiplets $\mathbf{10} = (q_{\mathrm{L}}, {u_{\mathrm{R}}}^c, {e_{\mathrm{R}}}^c)$, $\mathbf{5}^* = ({d_{\mathrm{R}}}^c, l_{\mathrm{L}})$ and $\mathbf{1} = \nu_{\mathrm{R}}$. Hence, unlike the gauge fields, quarks and leptons are not unified in a single irreducible representation. In particular, the right-handed neutrinos are gauge singlets and can therefore have Majorana masses not generated by spontaneous symmetry breaking. In addition one has three

Yukawa interactions, which couple the fermions to the Higgs fields $H_1(\mathbf{5})$ and $H_2(\mathbf{5}^*)$,

$$\mathcal{L} = h_{uij}\mathbf{10}_i\mathbf{10}_j H_1(\mathbf{5}) + h_{dij}\mathbf{5}^*{}_i\mathbf{10}_j H_2(\mathbf{5}^*) + h_{\nu ij}\mathbf{5}^*{}_i\mathbf{1}_j H_1(\mathbf{5}) + M_{ij}\mathbf{1}_i\mathbf{1}_j\,. \quad (4)$$

The mass matrices of up-quarks, down-quarks, charged leptons and the Dirac neutrino mass matrix are given by $m_u = h_u v_1$, $m_d = h_d v_2$, $m_e = m_d$ and $m_{\mathrm{D}} = h_\nu v_1$, respectively, with $v_1 = \langle H\rangle_1$ and $v_2 = \langle H\rangle_2$. The Majorana masses M are independent of the Higgs mechanism and can therefore be much larger than the electroweak scale v.

An attractive framework to explain the observed mass hierarchies of quarks and charged leptons is the Froggatt–Nielsen mechanism [10] based on a spontaneously broken $\mathrm{U}(1)_F$ generation symmetry. The Yukawa couplings are assumed to arise from non-renormalizable interactions after a gauge singlet field Φ acquires a vacuum expectation value,

$$h_{ij} = g_{ij}\left(\frac{\langle\Phi\rangle}{\Lambda}\right)^{Q_i+Q_j}\,. \quad (5)$$

Here g_{ij} are couplings $\mathcal{O}(1)$ and Q_i are the U(1) charges of the various fermions, with $Q_\Phi = -1$. The interaction scale Λ is usually chosen to be very large, $\Lambda > \Lambda_{\mathrm{GUT}}$.

The symmetry group $\mathrm{SU}(5)\times\mathrm{U}(1)_F$ has been considered by a number of authors. Particularly interesting is the case with a 'lopsided' family structure where the chiral $\mathrm{U}(1)_F$ charges are different for the $\mathbf{5}^*$-plets and the $\mathbf{10}$-plets of the same family [11–13]. Note, that such lopsided charge assignments are not consistent with the embedding into a higher-dimensional gauge group, like $\mathrm{SO}(10)\times\mathrm{U}(1)_F$ or $E_6\times\mathrm{U}(1)_F$. An example of phenomenologically allowed lopsided charges Q_i is given in Table I.

TABLE I

Lopsided $\mathrm{U}(1)_F$ charges of SU(5) multiplets. From [14].

ψ_i	$\mathbf{10}_3$	$\mathbf{10}_2$	$\mathbf{10}_1$	$\mathbf{5}^*_3$	$\mathbf{5}^*_2$	$\mathbf{5}^*_1$	$\mathbf{1}_3$	$\mathbf{1}_2$	$\mathbf{1}_1$
Q_i	0	1	2	a	a	$a+1$	b	c	d

This charge assignment determines the structure of the Yukawa matrices, *e.g.*,

$$h_e = h_d \sim \begin{pmatrix} \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \end{pmatrix}, \quad (6)$$

where the parameter $\varepsilon = \langle \Phi \rangle / \Lambda$ controls the flavour mixing, and coefficients $\mathcal{O}(1)$ are unknown. The corresponding mass hierarchies for up-quarks, down-quarks and charged leptons are

$$m_t : m_c : m_u \simeq 1 : \varepsilon^2 : \varepsilon^4 , \tag{7}$$

$$m_b : m_s : m_d = m_\tau : m_\mu : m_e \simeq 1 : \varepsilon : \varepsilon^3 . \tag{8}$$

The differences between the observed down-quark mass hierarchy and the charged lepton mass hierarchy can be accounted for by introducing additional Higgs fields [15]. From a fit to the running quark and lepton masses at the GUT scale one obtains for the flavour mixing parameter $\varepsilon \simeq 0.06$.

The light neutrino mass matrix is obtained from the seesaw formula,

$$m_\nu = -m_{\mathrm{D}} \frac{1}{M} m_{\mathrm{D}}^T \sim \varepsilon^{2a} \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix} , \tag{9}$$

Note, that the structure of this matrix is determined by the $\mathrm{U}(1)_F$ charges of the $\mathbf{5}^*$-plets only. It is independent of the $\mathrm{U}(1)_F$ charges of the right-handed neutrinos.

Since all elements of the 2-3 submatrix of (9) are $\mathcal{O}(1)$, one naturally obtains a large ν_μ–ν_τ mixing angle Θ_{23} [11, 12]. At first sight one may expect that $\Theta_{12} = \mathcal{O}(\varepsilon)$, which would correspond to the SMA solution of the MSW conversion. However, one can also have a large mixing angle Θ_{12} if the determinent of the 2-3 submatrix of m_ν is $\mathcal{O}(\varepsilon)$ [16]. Choosing the coefficients $\mathcal{O}(1)$ randomly, in the spirit of 'flavour anarchy' [17], the SMA and the LMA solutions are about equally probable for $\varepsilon \simeq 0.1$ [18]. The corresponding neutrino masses are consistent with $m_2 \sim 5 \times 10^{-3}$ eV and $m_3 \sim 5 \times 10^{-2}$ eV. We conclude that the neutrino mass matrix (9) naturally yields a large angle Θ_{23}, with Θ_{12} large or small. In order to have maximal mixings the coefficients $\mathcal{O}(1)$ have to obey special relations.

The model can also explain the cosmological baryon asymmetry via leptogenesis [19] for an appropriate choice of the parameters in table 1 [14]. The mass of the heaviest Majorana neutrino is

$$M_3 \sim \varepsilon^{2(a+b)} \frac{v_1^2}{\overline{m}_\nu} \sim \varepsilon^{2(a+b)}\, 10^{15}\ \mathrm{GeV} , \tag{10}$$

where $\overline{m}_\nu = \sqrt{m_2 m_3} \sim 10^{-2}$ eV. The choice $a = b = 0$, $c = 1$, $d = 2$ yields the scenario of [20] where B–L is broken at the GUT scale.

For the CP asymmetry in the decays of the heavy neutrinos N_1,

$$\varepsilon_1 = \frac{\Gamma(N_1 \to l\, H_2) - \Gamma(N_1 \to l^c\, H_2^c)}{\Gamma(N_1 \to l\, H_2) + \Gamma(N_1 \to l^c\, H_2^c)} , \tag{11}$$

one has in the case $M_1 < M_{2,3}$,

$$\varepsilon_1 \simeq -\frac{3}{16\pi}\frac{M_1}{(h_\nu^\dagger h_\nu)_{11}}\mathrm{Im}\left(h_\nu^\dagger h_\nu \frac{1}{M} h_\nu^T h_\nu^*\right)_{11} \sim \frac{3}{16\pi}\varepsilon^{2(a+d)}\,. \qquad (12)$$

Successful baryogenesis requires $a+d=2$. With $\varepsilon \sim 0.1$ the corresponding CP asymmetry is $\varepsilon_1 \sim 10^{-6}$. The baryogenesis temperature is then $T_B \sim M_1 \sim \varepsilon^4 M_3 \sim 10^{10}$ GeV. The effective neutrino mass which controls the out-of-equilibrium condition of the decaying heavy Majorana neutrino is given by $\widetilde{m}_1 = (m_{\mathrm{D}}^\dagger m_{\mathrm{D}})_{11}/M_1 \sim 10^{-2}$ eV.

Thermal leptogenesis leads to the baryon asymmetry [21]

$$Y_B = \frac{n_B - n_{\overline{B}}}{s} = \kappa c_S \frac{\varepsilon_1}{g_*}\,, \qquad (13)$$

where n_B and s are baryon number and entropy densities, respectively; $g_* \sim 100$ is the number of degrees of freedom in the plasma of the early universe and $c_S = \mathcal{O}(1)$ is the conversion factor from lepton asymmetry to baryon asymmetry due to sphaleron processes. Washout processes are accounted for by $\kappa < 1$, which can be computed by solving the full Boltzmann equations [22, 23]. The resulting baryon asymmetry then reads

$$Y_B \sim \kappa\; 10^{-8}\,, \qquad (14)$$

With $\kappa \sim 0.1 \ldots 0.01$ this is indeed the correct order of magnitude in accord with observation, $Y_B \simeq (0.6-1)\times 10^{-10}$.

The magnitude for the generated baryon asymmetry depends crucially on the parameters ε_1, $\widetilde{m}_1$ and M_1. In the models with $\mathrm{SU}(5)\times\mathrm{U}(1)_F$ symmetry low energy neutrino physics is essentially decoupled from the heavy Majorana neutrinos and does not constrain the value of M_1. Hence, successful baryogenesis is consistent with the $\mathrm{SU}(5)\times\mathrm{U}(1)_F$ symmetry, but it cannot be considered a generic prediction. This is different in unified theories with larger gauge groups.

3. Models with SO(10)

The simplest grand unified theory which unifies one generation of quarks and leptons including the right-handed neutrino in a single irreducible representation is based on the gauge group SO(10) [24]. The quark and lepton mass matrices are obtained from the couplings of the fermion multiplet $\mathbf{16} = (q_{\mathrm{L}}, u_{\mathrm{R}}{}^c, e_{\mathrm{R}}{}^c, d_{\mathrm{R}}{}^c, l_{\mathrm{L}}, \nu_{\mathrm{R}})$ to the Higgs multiplets $H_1(\mathbf{10})$, $H_2(\mathbf{10})$ and $\Phi(\mathbf{126})$,

$$\mathcal{L} = h_{uij}\mathbf{16}_i\mathbf{16}_j H_1(\mathbf{10}) + h_{dij}\mathbf{16}_i\mathbf{16}_j H_2(\mathbf{10}) + h_{Nij}\mathbf{16}_i\mathbf{16}_j \Phi(\mathbf{126})\,. \qquad (15)$$

Here we have assumed that the two Higgs doublets of the standard model are contained in the two[1] ten-plets H_1 and H_1, respectively. This yields the quark mass matrices $m_u = h_u v_1$, $m_d = h_d v_2$, with $v_1 = \langle H \rangle_1$ and $v_2 = \langle H \rangle_2$, and the lepton mass matrices

$$m_{\mathrm{D}} = m_u \,, \qquad m_e = m_d \,. \tag{16}$$

Contrary to SU(5) GUTs, the Dirac neutrino and the up-quark mass matrices are now related. Note, that all matrices are symmetric. The Majorana mass matrix $M = h_N \langle \Phi \rangle$ is also generated by spontaneous symmetry breaking and a priori independent of m_u and m_d.

With $m_{\mathrm{D}} = m_u$ the seesaw mass relation becomes

$$m_\nu \simeq -m_u \frac{1}{M} m_u^T \,. \tag{17}$$

The large neutrino mixings now appear very puzzling, since the quark mass matrices are hierarchical and the quark mixings are small. It turns out, however, that because of the known properties of the up-quark mass matrix this puzzle can be resolved provided the heavy neutrino masses also obey a specific hierarchy. This then leads to predictions for a number of observables in neutrino physics including the cosmological baryon asymmetry. In the following we shall describe these implications of large neutrino mixings in SO(10) GUTs following Ref. [27]. The role of the heavy neutrino mass hierarchy for the light neutrino mixings has previously been discussed in different contexts [25].

From the phenomenology of weak decays we know that the quark matrices have approximately the form [28, 29],

$$m_{u,d} \propto \begin{pmatrix} 0 & \varepsilon^3 \mathrm{e}^{i\phi} & 0 \\ \varepsilon^3 \mathrm{e}^{i\phi} & \rho\varepsilon^2 & \eta\varepsilon^2 \\ 0 & \eta\varepsilon^2 & \mathrm{e}^{i\psi} \end{pmatrix} . \tag{18}$$

Here $\varepsilon \ll 1$ is the parameter which determines the flavour mixing, and

$$\rho = |\rho| \mathrm{e}^{i\alpha} \,, \quad \eta = |\eta| \mathrm{e}^{i\beta} \,, \tag{19}$$

are complex parameters $\mathcal{O}(1)$. We have chosen a 'hierarchical' basis, where off-diagonal matrix elements are small compared to the product of the corresponding eigenvalues, $|m_{ij}|^2 \leq \mathcal{O}(|m_i m_j|)$. In contrast to the usual assumption of Hermitian mass matrices [28, 29], SO(10) invariance dictates the matrices to be symmetric. All parameters may take different values for

[1] Note, that this is unavoidable in models with SO(10) breaking by orbifold compactification [26].

up- and down-quarks. Typical choices for ε are $\varepsilon_u = 0.07$, $\varepsilon_d = 0.21$ [29]. The agreement with data can be improved by adding in the 1–3 element a term $\mathcal{O}(\varepsilon^4)$ [30, 31] which, however, is not important for the following analysis. Data also imply one product of phases to be 'maximal', *i.e.*, $\Delta = \phi_u - \alpha_u - \phi_d + \alpha_d \simeq \pi/2$.

We do not know the structure of the Majorana mass matrix $M = h_N \langle \Phi \rangle$. However, in models with family symmetries it should be similar to the quark mass matrices, *i.e.*, the structure should be independent of the Higgs field. In this case, one expects

$$M = \left(\begin{array}{ccc} 0 & M_{12} & 0 \\ M_{12} & M_{22} & M_{23} \\ 0 & M_{23} & M_{33} \end{array} \right) , \tag{20}$$

with $M_{12} \ll M_{22} \sim M_{23} \ll M_{33}$. M is diagonalized by a unitary matrix, $U^{(N)\dagger} M U^{(N)*} = \mathrm{diag}(M_1, M_2, M_3)$. Using the seesaw formula one can now evaluate the light neutrino mass matrix. Since the choice of the Majorana matrix m_N fixes a basis for the right-handed neutrinos the allowed phase redefinitions of the Dirac mass matrix m_{D} are restricted. In Eq. (18) the phases of all matrix elements have therefore been kept.

The ν_μ–ν_τ mixing angle is known to be large. This leads us to require $m_{\nu_{i,j}} = \mathcal{O}(1)$ for $i,j = 2,3$. It is remarkable that this determines the hierarchy of the heavy Majorana mass matrix to be[2]

$$M_{12} : M_{22} : M_{33} = \varepsilon^5 : \varepsilon^4 : 1 \,. \tag{21}$$

With $M_{33} \simeq M_3$, $M_{22} = \sigma\varepsilon^4 M_3$, $M_{23} = \zeta\varepsilon^4 M_3 \sim M_{22}$ and $M_{12} = \varepsilon^5 M_3$, one obtains for masses and mixings to order $\mathcal{O}(\varepsilon^4)$

$$M_1 \simeq -\frac{\varepsilon^6}{\sigma} M_3 \,, \quad M_2 \simeq \sigma\varepsilon^4 M_3 \,, \tag{22}$$

$$U^{(N)}_{12} = -U^{(N)}_{21} = \frac{\varepsilon}{\sigma} \,, \quad U^{(N)}_{23} = \mathcal{O}(\varepsilon^4) \,, \quad U^{(N)}_{13} = 0 \,. \tag{23}$$

Note, that σ can always be chosen real whereas ζ is in general complex. This yields for the light neutrino mass matrix

$$m_\nu = -\left(\begin{array}{ccc} 0 & \varepsilon \mathrm{e}^{2i\phi} & 0 \\ \varepsilon \mathrm{e}^{2i\phi} & -\sigma \mathrm{e}^{2i\phi} + 2\rho \mathrm{e}^{i\phi} & \eta \mathrm{e}^{i\phi} \\ 0 & \eta \mathrm{e}^{i\phi} & \mathrm{e}^{2i\psi} \end{array} \right) \frac{v_1^2}{M_3} \,. \tag{24}$$

[2] We also note that this result is independent of the zeroes in the mass matrix (18) if its 1–3 element is smaller than ε^3, as required by data.

The complex parameter ζ does not enter because of the hierarchy. The matrix (24) has the same structure as the mass matrix (9) in the $\mathrm{SU}(5)\times\mathrm{U}(1)_F$ model, except for additional texture zeroes. Since, as required, all elements of the 2–3 submatrix are $\mathcal{O}(1)$, the mixing angle Θ_{23} is naturally large. A large mixing angle Θ_{12} can again occur in case of a small determinant of the 2–3 submatrix,

$$(-\sigma + 2\rho \mathrm{e}^{-i\phi})\mathrm{e}^{2i\psi} - \eta^2 \equiv \delta \mathrm{e}^{2i\gamma} = \mathcal{O}(\varepsilon)\,. \tag{25}$$

Such a condition can be fulfilled without fine tuning if $\sigma, \rho, \eta = \mathcal{O}(1)$. It implies relations between the moduli as well as the phases of ρ and η. In the special case of a somewhat smaller mass of the second heavy neutrino, *i.e.*, $|\sigma| < |\rho|$, the condition (25) becomes

$$\psi - \beta \simeq \tfrac{1}{2}(\phi - \alpha)\,, \qquad |\eta|^2 \simeq 2|\rho|\,, \tag{26}$$

The mass matrix m_ν can again be diagonalized by a unitary matrix, $U^{(\nu)\dagger} m_\nu U^{(\nu)*} = \mathrm{diag}(m_1, m_2, m_3)$. A straightforward calculation yields ($s_{ij} = \sin\Theta_{ij}$, $c_{ij} = \cos\Theta_{ij}$, $\xi = \varepsilon/(1+|\eta|^2)$),

$$U^{(\nu)} = \begin{pmatrix} c_{12}\mathrm{e}^{i(\phi-\beta+\psi-\gamma)} & s_{12}\mathrm{e}^{i(\phi-\beta+\psi-\gamma)} & \xi s_{23}\mathrm{e}^{i(\phi-\beta+\psi)} \\ -c_{23}s_{12}\mathrm{e}^{i(\phi+\beta-\psi+\gamma)} & c_{23}c_{12}\mathrm{e}^{i(\phi+\beta-\psi+\gamma)} & s_{23}\mathrm{e}^{i(\phi+\beta-\psi)} \\ s_{23}s_{12}\mathrm{e}^{i(\gamma+\psi)} & -s_{23}c_{12}\mathrm{e}^{i(\gamma+\psi)} & c_{23}\mathrm{e}^{i\psi} \end{pmatrix}, \tag{27}$$

with the mixing angles,

$$\tan 2\Theta_{23} \simeq \frac{2|\eta|}{1-|\eta|^2}\,, \qquad \tan 2\Theta_{12} \simeq 2\sqrt{1+|\eta|^2}\frac{\varepsilon}{\delta}\,. \tag{28}$$

Note, that the 1–3 element of the mixing matrix is small, $U_{13}^{(\nu)} = \mathcal{O}(\varepsilon)$. The masses of the light neutrinos are

$$\begin{aligned} m_1 &\simeq -\tan^2\Theta_{12}\ m_2\,, \\ m_2 &\simeq \frac{\varepsilon}{(1+|\eta|^2)^{3/2}}\cot\Theta_{12}\ m_3\,, \\ m_3 &\simeq (1+|\eta|^2)\ \frac{v_1^2}{M_3}\,. \end{aligned} \tag{29}$$

This corresponds to the weak hierarchy,

$$m_1 : m_2 : m_3 = \varepsilon : \varepsilon : 1\,, \tag{30}$$

with $m_2^2 \sim m_1^2 \sim \Delta m_{21}^2 = m_2^2 - m_1^2 \sim \varepsilon^2$. Since $\varepsilon \sim 0.1$, this pattern is consistent with the LMA solution of the solar neutrino problem, but not with the LOW solution.

The large ν_μ–ν_τ mixing has been obtained as consequence of the required very large mass hierarchy (22) of the heavy Majorana neutrinos. The large ν_e–ν_μ mixing follows from the particular values of parameters $\mathcal{O}(1)$. Hence, one expects two large mixing angles, but single maximal or bi-maximal mixing would require fine tuning. On the other hand, one definite prediction is the occurrence of exactly one small matrix element, $U_{13}^{(\nu)} = \mathcal{O}(\varepsilon)$. Note, that the obtained pattern of neutrino mixings is independent of the off-diagonal elements of the mass matrix M. For instance, replacing the texture (20) by a diagonal matrix, $M = \mathrm{diag}(M_1, M_2, M_3)$, leads to the same pattern of neutrino mixings.

In order to calculate various observables in neutrino physics we need the leptonic mixing matrix

$$U = U^{(e)\dagger} U^{(\nu)}\,, \tag{31}$$

where $U^{(e)}$ is the charged lepton mixing matrix. In our framework we expect $U^{(e)} \simeq V^{(d)}$, and also $V = V^{(u)\dagger} V^{(d)} \simeq V^{(d)}$ for the CKM matrix since $\varepsilon_u < \varepsilon_d$. This yields for the leptonic mixing matrix

$$U \simeq V^\dagger U^{(\nu)}\,. \tag{32}$$

To leading order in the Cabibbo angle $\lambda \simeq 0.2$ we only need the off-diagonal elements $V_{12}^{(d)} = \overline{\lambda} = -V_{21}^{(d)*}$. Since the matrix m_d is complex, the Cabibbo angle is modified by phases, $\overline{\lambda} = \lambda \exp\{i(\phi_d - \alpha_d)\}$. The resulting leptonic mixing matrix is indeed of the wanted form (2) with all matrix elements $\mathcal{O}(1)$, except U_{13},

$$U_{13} = \xi s_{23} \mathrm{e}^{i(\phi-\beta+\psi)} - \overline{\lambda} s_{23} \mathrm{e}^{i(\phi+\beta-\psi)} = \mathcal{O}(\lambda, \varepsilon) \sim 0.1\,, \tag{33}$$

which is close to the experimental limit.

Let us now consider the CP violation in neutrino oscillations. Observable effects are controlled by the Jarlskog parameter J_l [32] ($\widetilde{\varepsilon}_{ij} = \sum_{k=1}^3 \varepsilon_{ijk}$)

$$\mathrm{Im}\{U_{\alpha i} U_{\beta j} U_{\alpha j}^* U_{\beta i}^*\} = \widetilde{\varepsilon}_{\alpha\beta} \widetilde{\varepsilon}_{ij} J_l\,, \tag{34}$$

for which one finds

$$J_l \simeq \lambda s_{12} c_{12} c_{23} s_{23}^2 \sin\left(2(\beta - \psi + \gamma) + \phi_d - \alpha_d\right). \tag{35}$$

In the case of a small mass difference Δm_{12}^2 the CP asymmetry $P(\nu_\mu \to \nu_e) - P(\overline{\nu}_\mu \to \overline{\nu}_e)$ is proportional to δ (*cf.* (25)). Hence, the dependence of J_l on the angle γ is not surprising.

For large mixing, $c_{ij} \simeq s_{ij} \simeq 1/\sqrt{2}$, and in the special case (26) one obtains from the SO(10) phase relation $\phi - \alpha = \phi_u - \alpha_u$ and $\phi_u - \alpha_u - \phi_d + \alpha_d = \mathit{\Delta} \simeq \pi/2$,

$$J_l \simeq \frac{\lambda}{4\sqrt{2}} \sin\left(-\frac{\pi}{2} + 2\gamma\right). \tag{36}$$

For small γ this corresponds to maximal CP violation, but without a deeper understanding of the fermion mass matrices this case is not singled out. Due to the large neutrino mixing angles, J_l is much bigger than the Jarlskog parameter in the quark sector, $J_q = \mathcal{O}(\lambda^6) \sim 10^{-5}$, which may lead to observable effects at future neutrino factories [33].

According to the seesaw mechanism neutrinos are Majorana fermions. This can be directly tested in neutrinoless double β-decay. The decay amplitude is proportional to the complex mass

$$\begin{aligned}\langle m\rangle &= \sum_i U_{ei}^2 m_i = -(UU^{(\nu)\dagger}m_\nu U^{(\nu)*}U^T)_{ee} \simeq -(V^{(d)\dagger}m_\nu V^{(d)*})_{ee} \\ &= -\frac{1}{1+|\eta|^2}\left(\lambda^2|\eta|^2 \mathrm{e}^{2i(\phi_d-\alpha_d+\beta+\phi-\psi)} - 2\lambda\varepsilon \mathrm{e}^{i(\phi_d-\alpha_d+2\phi)}\right) m_3\,. \quad (37)\end{aligned}$$

With $m_3 \simeq \sqrt{\Delta m^2_{\mathrm{atm}}} \simeq 5\times 10^{-2}$ eV this yields $\langle m\rangle \sim 10^{-3}$ eV, more than two orders of magnitude below the present experimental upper bound [34].

Finally, consider again the baryon asymmetry which should eventually be related to the CP violation in neutrino oscillations and quark mixing. This possibility has recently been discussed also in other contexts [35, 36]. In the special case[3] (26) one obtains for the CP asymmetry,

$$\varepsilon_1 \simeq \frac{3}{16\pi}\varepsilon^6 \frac{|\eta|^2}{\sigma}\frac{(1+|\rho|)^2}{|\eta|^2+|\rho|^2}\sin(\phi_u-\alpha_u)\,. \quad (38)$$

As expected ε_1 depends only on phases of the up-quark matrix and not on the combination of up- and down-quark phases Δ which appears in the CKM matrix. In addition, the parameter σ enters. Hence, the baryon asymmetry is not completely determined by properties of the quark matrices and the CP violation in the neutrino sector.

Numerically, with $\varepsilon \sim 0.1$ one has $\varepsilon_1 \sim 10^{-7}$, $|M_1| \simeq (\varepsilon^6/|\sigma|)(1+|\eta|^2)v_1^2/m_3 \sim 10^9$ GeV and $\widetilde{m}_1 \sim (|\eta|^2+|\rho|^2)/(\sigma(1+|\eta|^2))m_3 \sim 10^{-2}$ eV. The baryon asymmetry is then given by

$$Y_B \sim -\kappa\,\mathrm{sign}(\sigma)\ \sin(\phi_u-\alpha_u)\times 10^{-9}\,. \quad (39)$$

The parameters ε_1, M_1 and $\widetilde{m}_1$ are rather similar to those considered in the previous section. Hence, a solution of the Boltzmann equations can be expected to yield again a baryon asymmetry in accord with the observation.

[3] For the discussion of the general case, see Ref. [27].

4. Conclusions

Large neutrino mixings, together with the known small quark mixings, have important implications for the structure of GUTs. In SU(5) models this difference between the lepton and quark sectors can be explained by lopsided $U(1)_F$ family symmetries. In these models the heavy Majorana neutrino masses are not constrained by low energy physics, *i.e.*, light neutrino masses and mixings. Successful leptogenesis then depends on the choice of the heavy neutrino masses and is not a generic prediction of the theory.

In SO(10) models the implications of large neutrino mixings are much more stringent because of the connection between Dirac neutrino and up-quark mass matrices. It is remarkable that the requirement of large neutrino mixings determines the relative magnitude of the heavy Majorana neutrino masses in terms of the known quark mass hierarchy. This leads to predictions for neutrino mixings and masses, CP violation in neutrino oscillations and neutrinoless double β-decay. The predicted order of magnitude for the baryon asymmetry is in accord with observation. It would be very interesting to relate directly the CP violation in the quark sector and in neutrino oscillations to the baryon asymmetry. This, however, will require a deeper understanding of the quark and lepton mass matrices.

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REFERENCES

[1] SNO Collaboration, Q.R. Ahmad *et al.*, *Phys. Rev. Lett.* **87**, 071301 (2001).

[2] SuperKamiokande Collaboration, S. Fukuda *et al.*, *Phys. Rev. Lett.* **86**, 5651 (2001).

[3] SuperKamiokande Collaboration, S. Fukuda *et al.*, *Phys. Rev. Lett.* **81**, 1562 (1998).

[4] K2K Collaboration, S.H. Ahn *et al.*, *Phys. Lett.* **B511**, 178 (2001).

[5] M.C. Gonzalez-Garcia, P.C. de Holanda, C. Peña-Garay, J.W.F. Valle, *Nucl. Phys.* **B573**, 3 (2000); J.N. Bahcall, M.C. Gonzalez-Garcia, C. Peña-Garay, *J. High Energy Phys.* **0108**, 014 (2001); G.L. Fogli, E. Lisi, D. Montanino, A. Palazzo, *Phys. Rev.* **D64**, 093007 (2001).

[6] M. Fukugita, M. Tanimoto, *Phys. Lett.* **B515**, 30 (2001).

[7] For a review and references, see G. Altarelli, F. Feruglio, *Phys. Rep.* **320C**, 295 (1999); H. Fritzsch, Z. Xing, *Prog. Part. Nucl. Phys.* **45**, 1 (2000).

[8] T. Yanagida, in *Workshop on Unified Theories*, KEK report 79-18 (1979) p. 95; M. Gell-Mann, P. Ramond, R. Slansky, in *Supergravity*, North Holland, Amsterdam, 1979, eds. P. van Nieuwenhuizen, D. Freedman, p. 315.

[9] H. Georgi, S.L. Glashow, *Phys. Rev. Lett.* **32**, 438 (1974).

[10] C.D. Froggatt, H.B. Nielsen, *Nucl. Phys.* **B147**, 277 (1979).

[11] J. Sato, T. Yanagida, *Phys. Lett.* **B430**, 127 (1998).

[12] N. Irges, S. Lavignac, P. Ramond, *Phys. Rev.* **D58**, 035003 (1998).

[13] J. Bijnens, C. Wetterich, *Nucl. Phys.* **B292**, 443 (1987).

[14] W. Buchmüller, T. Yanagida, *Phys. Lett.* **B445**, 399 (1999).

[15] H. Georgi, C. Jarlskog, *Phys. Lett.* **B86**, 297 (1979).

[16] F. Vissani, *J. High Energy Phys.* **9811**, 25 (1998).

[17] L. Hall, H. Murayama, N. Weiner, *Phys. Rev. Lett.* **1984**, 2572 (2000).

[18] J. Sato, T. Yanagida, *Phys. Lett.* **B493**, 356 (2000).

[19] M. Fukugita, T. Yanagida, *Phys. Lett.* **B174**, 45 (1986).

[20] W. Buchmüller, M. Plümacher, *Phys. Lett.* **B389**, 73 (1996).

[21] For a review and references, see W. Buchmüller, M. Plümacher, *Int. J. Mod. Phys.* **A15**, 5086 (2000); `hep-ph/0007176`.

[22] M.A. Luty, *Phys. Rev.* **D45**, 455 (1992).

[23] M. Plümacher, *Z. Phys.* **C74**, 549 (1997); *Nucl. Phys.* **B530**, 207 (1998).

[24] H. Georgi, in *Particles and Fields*, ed. C.E. Carlson, AIP, NY, 1975, p. 575; H. Fritzsch, P. Minkowski, *Ann. Phys. (NY)* **93**, 193 (1975).

[25] A. Yu. Smirnov, *Phys. Rev.* **D48**, 3264 (1993); G. Altarelli, F. Feruglio, I. Masina, *Phys. Lett.* **B472**, 382 (2000); D. Falcone, *Phys. Lett.* **B479**, 1 (2000); C. Albright, S. Barr, *Phys. Rev.* **D64**, 073010 (2001).

[26] T. Asaka, W. Buchmüller, L. Covi, `hep-ph/0108021`; L. Hall, Y. Nomura, T. Okui, D. Smith, `hep-ph/0108071`, to appear in *Phys. Rev.* **D**.

[27] W. Buchmüller, D. Wyler, `hep-ph/0108216`.

[28] H. Fritzsch, Z. Xing, Ref. [7].

[29] R. Rosenfeld, J.L. Rosner, *Phys. Lett.* **B516**, 408 (2001).

[30] G. Branco, D. Emmanuel-Costa, R. Gonzalez Felipe, *Phys. Lett.* **B483**, 87 (2000).

[31] R.G. Roberts, A. Romanino, G.G. Ross, L. Velasco-Sevilla, *Nucl. Phys.* **B615**, 358 (2001).

[32] C. Jarlskog, *Phys. Rev. Lett.* **55**, 1039 (1985).

[33] A. Blondel *et al.*, *Nucl. Instrum. Methods* **A451**, 102 (2000); C. Albright *et. al.*, `hep-ex/0008064`.

[34] Heidelberg–Moscow Collaboration, L. Baudis *et al.*, *Phys. Rev. Lett.* **83**, 41 (1999).

[35] A.S. Joshipura, E.A. Paschos, W. Rodejohann, *J. High Energy Phys.* **0108**, 029 (2001).

[36] G.C. Branco, T. Morozumi, B.M. Nobre, M.N. Rebelo, `hep-ph/0107164`.