SOFTLY BROKEN LEPTON NUMBERS: AN APPROACH TO MAXIMAL NEUTRINO MIXING*

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We discuss models where the U(1) symmetries of lepton numbers are responsible for maximal neutrino mixing. We pay particular attention to an extension of the Standard Model (SM) with three right-handed neutrino singlets in which we require that the three lepton numbers L_e, L_μ, and L_τ be separately conserved in the Yukawa couplings, but assume that they are *softly* broken by the Majorana mass matrix $M_{\rm R}$ of the neutrino singlets. In this framework, where lepton-number breaking occurs at a scale much higher than the electroweak scale, deviations from family lepton number conservation are calculable, *i.e.*, finite, and lepton mixing stems exclusively from $M_{\rm R}$. We show that in this framework either maximal atmospheric neutrino mixing or maximal solar neutrino mixing or both can be imposed by invoking symmetries. In this way those maximal mixings are stable against radiative corrections. The model which achieves maximal (or nearly maximal) solar neutrino mixing assumes that there are two different scales in $M_{\rm R}$ and that the lepton number $\bar{L} = L_e - L_\mu - L_\tau$ is conserved in between them. We work out the difference between this model and the conventional scenario where (approximate) $\bar{L}$ invariance is imposed directly on the mass matrix of the *light* neutrinos.

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1. Neutrino masses, mixing and oscillations

The impressive results of the atmospheric [1] and solar [2] neutrino experiments have a natural explanation in terms of neutrino oscillations [3,4]. With respect to solar neutrinos, this point of view was further strengthened — and the possibility of an astrophysical solution [5] further weakened — by the first results of the SNO experiment [6]. In this context, the Large Mixing Angle (LMA) MSW solution of the solar neutrino deficit [7], with a solar mass-squared difference $\Delta m^2_\odot \sim 5 \times 10^{-5}$ eV^2, is emerging as the favoured scenario [8–13]; whereas the Small Mixing Angle (SMA) MSW solution seems to fade. The LOW solution, with $\Delta m^2_\odot \sim 10^{-7}$ eV^2, also gives a reasonable fit to the solar neutrino data. For the LMA MSW solution the solar mixing angle θ is large, but $\theta = 45^\circ$ is not allowed at 99% CL; for the LOW solution maximal mixing gives a somewhat better fit [12]. In the case of atmospheric neutrinos, the fit to the deficit of the muon-neutrino flux leads to the best-fit values $\Delta m^2_{\text{atm}} = 2.5 \times 10^{-3}$ eV^2 and $\psi = 45^\circ$ [14], with the atmospheric mixing angle ψ sitting squarely on maximal mixing.

In the following we shall consider exclusively neutrino mixing and oscillations among the known three active neutrinos. We shall not take into consideration the result of the LSND experiment [15] which — when interpreted in terms of neutrino oscillations — leads to a mass-squared difference $\Delta m^2_{\text{LSND}} \sim 1$ eV^2.

Neutrino mixing relates the left-handed neutrino flavour fields with the neutrino fields with definite mass m_j via

$$\nu_{\alpha L} = \sum_j U_{\alpha j} \nu_{jL} \quad (\alpha = e, \mu, \tau)\,, \tag{1}$$

where U is a unitary 3×3 matrix. Plugging Eq. (1) into the charged-current Lagrangian we obtain

$$\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}} W^-_\rho \sum_{\alpha=e,\mu,\tau} \bar{\ell}_\alpha \gamma^\rho \sum_j U_{\alpha j} \nu_{jL} + \text{h.c.} \tag{2}$$

Neutrino flavour eigenstates, which are coherent superpositions of the neutrino mass eigenstates, are produced and detected via charged-current interactions such that the neutrino flavour is defined via the charged lepton associated with the production or detection process. It is crucial in this context that $E^2 \gg m_j^2$, where E is the neutrino energy, holds in any realistic experiment. The general formula for the neutrino survival ($\alpha = \beta$) and transition ($\alpha \neq \beta$) probabilities is given by [3]

$$P_{\nu_\alpha \to \nu_\beta}\left(\frac{L}{E}\right) = \left| \sum_j U_{\beta j} U^*_{\alpha j} \exp\left(-i m_j^2 \frac{L}{2E}\right) \right|^2 , \tag{3}$$

where L denotes the distance in between the neutrino source and neutrino detection points. These probabilities depend on the ratio L/E and on the neutrino mass-squared differences[1].

In the following we shall use the convention $m_1 < m_2$ for the ordering of the neutrino masses. Experimentally we know that $\Delta m^2_{\odot} \ll \Delta m^2_{\rm atm}$, hence we take $\Delta m^2_{\odot} = m^2_2 - m^2_1$ to be much smaller than $\Delta m^2_{\rm atm} = \left| m^2_3 - m^2_1 \right|$. We have to distinguish between the two mass spectra $m_3 > m_1 \simeq m_2$ and $m_3 < m_1 \simeq m_2$. The atmospheric neutrino mixing angle will be denoted by ψ and the solar mixing angle by θ.

It is an interesting fact that there are no indications in favour of electron–neutrino oscillations, neither in long baseline [16] nor in atmospheric neutrino experiments [1, 14]. This leads to the conclusion that U_{e3} is small. A three-neutrino fit [17] to the CHOOZ, solar, and atmospheric neutrino data (before SNO) leads to the upper bound $|U_{e3}|^2 \lesssim 0.04$. It is instructive to explore the limit $U_{e3} \to 0$, where decoupling of solar and atmospheric neutrino oscillations takes place [18]. In this limit, the mixing matrix factorizes:

$$U' = U_{\rm atm} U_{\odot} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (4)$$

or

$$U = \mathrm{diag}\,(1,-1,1)\, U' = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ \cos\psi\sin\theta & -\cos\psi\cos\theta & -\sin\psi \\ \sin\psi\sin\theta & -\sin\psi\cos\theta & \cos\psi \end{pmatrix}. \quad (5)$$

In Eqs. (4) and (5) all unphysical phases have been removed and a special phase convention, which will be used in the following, has been chosen in Eq. (5). In this picture one can show that the following transitions are realized:

$$U_{e3} = 0 \;\Rightarrow\; \left\{ \begin{array}{rl} \text{solar } \nu\text{s:} & |\nu_e\rangle \to \cos\psi\,|\nu_\mu\rangle - \sin\psi\,|\nu_\tau\rangle\,, \\ \text{atmospheric } \nu\text{s:} & |\nu_\mu\rangle \to |\nu_\tau\rangle \quad \text{and} \quad |\bar\nu_\mu\rangle \to |\bar\nu_\tau\rangle\,. \end{array} \right. \quad (6)$$

Let us summarize the picture which emerges from the comparison of three-neutrino oscillations with the solar and atmospheric neutrino data:

[1] For n neutrinos there are $n-1$ independent mass-squared differences.

1. $\Delta m^2_\odot / \Delta m^2_{\rm atm} \ll 1$;
2. $|U_{e3}|^2 \ll 1$;
3. θ large, but not 45°;
4. $\psi \simeq 45^\circ$.

In point 1, the ratio $\Delta m^2_\odot / \Delta m^2_{\rm atm}$ is of order 10^{-2} for the LMA MSW solution, 10^{-5}–10^{-4} for the LOW solution.

To these four points one could add the requirement that the neutrino masses are much smaller than the charged-lepton masses. One popular way of satisfying this requirement is given by the see-saw mechanism [19], which will be adopted in this paper. Concerning points 1–4, it seems difficult to identify plausible mechanisms to "explain" them, despite huge efforts in this direction. In particular, the feature that atmospheric mixing is maximal whereas solar mixing is large *but not maximal* represents a formidable task for model building [20].

In the present paper, which is based on Ref. [21], we discuss the effects of softly broken lepton numbers on the question of maximal or large neutrino mixing. We aim at obtaining those mixings from symmetries, in particular from the U(1) invariances associated with lepton numbers. In that way, when we are able to enforce (at least some of) the features enumerated above, our results will be stable against radiative corrections. We want to stress that this is not the case if "textures" or *Ansätze* are used for that end.

2. $L_e - L_\mu - L_\tau$ invariance

In this section we want to discuss the well-known [22] example of $\bar{L} \equiv L_e - L_\mu - L_\tau$ invariance of the *light-neutrino mass matrix* and its connection with maximal solar mixing. We do not consider heavy right-handed singlets $\nu_{\rm R}$ in this section.

We need the mass terms for the neutrinos

$$\mathcal{L}^{(\nu)}_{\rm m} = \tfrac{1}{2} \nu^T_{\rm L} C^{-1} \mathcal{M}_\nu \nu_{\rm L} + \text{h.c.} \tag{7}$$

and for the charged leptons

$$\mathcal{L}^{(\ell)}_{\rm m} = -\bar{\ell}_{\rm R} M_\ell \ell_{\rm L} + \text{h.c.} \tag{8}$$

Invariance of the Lagrangian under $\bar{L}$ is an abbreviation for having a symmetry group U(1) in the following sense: the three left-handed lepton doublets

D_α and the right-handed singlets $\ell_{\alpha R}$ are multiplied by $e^{i\gamma}$, $e^{-i\gamma}$, $e^{-i\gamma}$, respectively, where γ takes on all real numbers. As an obvious consequence the mass matrices of Eqs. (7) and (8) have the forms

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & p & q \\ p & 0 & 0 \\ q & 0 & 0 \end{pmatrix}, \quad \text{and} \quad M_\ell = \begin{pmatrix} M_{ee} & 0 & 0 \\ 0 & M_{\mu\mu} & M_{\mu\tau} \\ 0 & M_{\tau\mu} & M_{\tau\tau} \end{pmatrix}, \tag{9}$$

respectively. The bi-diagonalization of the M_ℓ of Eq. (9) does not change the form of the $\mathcal{M}_\nu$ of Eq. (9). Therefore, without loss of generality we may assume that M_ℓ is diagonal and positive, and that p and q are real and positive.

The neutrino mass matrix of Eq. (9) has the eigenvalues

$$\begin{aligned} &\lambda_1 = m_1 = m_0\,, \quad \lambda_2 = -m_2 = -m_0\,, \quad \lambda_3 = m_3 = 0\,, \\ &\text{with} \quad m_0 = \sqrt{p^2+q^2}\,. \end{aligned} \tag{10}$$

Therefore, we obtain the mass-squared differences

$$\Delta m^2_{\text{atm}} = m_0^2 \quad \text{and} \quad \Delta m^2_\odot = 0\,. \tag{11}$$

Evidently, some small breaking of $\bar{L}$ invariance, whether soft or spontaneous, is necessary in order to achieve $\Delta m^2_\odot \neq 0$.

The neutrino mass matrix is diagonalized by

$$V^T \mathcal{M}_\nu V = \text{diag}\,(m_1, m_2, m_3)\,, \tag{12}$$

where the 3×3 unitary matrix V is determined as

$$V = U\,\text{diag}\,(1,-i,1) \quad \text{with} \quad U = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ \cos\psi/\sqrt{2} & -\cos\psi/\sqrt{2} & -\sin\psi \\ \sin\psi/\sqrt{2} & -\sin\psi/\sqrt{2} & \cos\psi \end{pmatrix}. \tag{13}$$

The main point of this mixing matrix is that it has maximal solar mixing, as a consequence of the assumption of $\bar{L}$ invariance. The atmospheric mixing angle, given by

$$\cos\psi = \frac{r}{\sqrt{1+r^2}}\,, \quad \sin\psi = \frac{1}{\sqrt{1+r^2}} \quad \text{with} \quad r = \frac{p}{q}\,, \tag{14}$$

will be large, if we do not allow for any fine tuning, but not necessarily close to 45°. Let us estimate the amount of *tuning* of r for bringing ψ into agreement with the range obtained from a fit to the atmospheric neutrino data [14]:

$$\sin^2 2\psi = \frac{4r^2}{(1+r^2)^2} > 0.89 \quad (90\%\ \text{CL}) \quad \Rightarrow \quad 0.7 \lesssim r \lesssim 1.4\,. \tag{15}$$

Thus, the present data allow a rather wide range for r. In addition, one has to take into account that the U of Eq. (13) was obtained at a stage where $\bar{L}$ was still unbroken. Breaking of the lepton number might move the solar angle θ a little away from 45°, but no significant corrections to θ and ψ are expected.

Let us, for instance, consider a specific example of soft $\bar{L}$ breaking, where $\mathcal{M}_\nu$ of Eq. (9) changes to

$$\mathcal{M}_\nu = \begin{pmatrix} a & p & q \\ p & 0 & 0 \\ q & 0 & 0 \end{pmatrix}. \tag{16}$$

(Models with this $\mathcal{M}_\nu$ have been described, for instance, in Ref. [23] and in the references therein.) It is easy to check that

$$\Delta m^2_{\text{atm}} \simeq m_0^2 \qquad \text{and} \qquad \Delta m^2_\odot \simeq 2 m_0 a \,. \tag{17}$$

Without loss of generality we have chosen $a > 0$. We can express a and the solar mixing angle by the mass-squared differences:

$$a \simeq \frac{\Delta m^2_\odot}{2\sqrt{\Delta m^2_{\text{atm}}}} \qquad \text{and} \qquad \sin^2 2\theta \simeq 1 - \frac{1}{16}\left(\frac{\Delta m^2_\odot}{\Delta m^2_{\text{atm}}}\right)^2 . \tag{18}$$

The first relation shows that $a \ll p$, q, whereas from the second relation we read off that, for all practical purposes, the solar mixing angle remains 45° even after $\bar{L}$ breaking of the type considered in Eq. (16).

It is a generic feature of models with approximate $\bar{L}$ invariance in the mass matrix of the light neutrinos that corrections to $\theta = 45^\circ$ are suppressed by $\left(\Delta m^2_\odot / \Delta m^2_{\text{atm}}\right)^2$ [24]. This is a drawback in view of the fit to present solar data, where a solar mixing angle very close to 45° is disfavoured. In addition, in the scheme under discussion atmospheric mixing comes out large but not necessarily close to 45°. There is no explanation for $\psi \simeq 45^\circ$ by using $\bar{L}$ invariance alone, as in this section.

3. A framework for imposing maximal atmospheric and/or maximal solar mixing

In this section we shall introduce a framework which will allow us to implement either maximal atmospheric neutrino mixing (Section 4) or (nearly) maximal solar neutrino mixing (Section 5) or both (Section 6). This framework is completely different from the one discussed in the previous section.

We begin with an extension of the Standard Model based on the following ingredients:

(i) The lepton sector of the Standard Model with three families, plus three right-handed singlets $\nu_{\rm R}$;

(ii) the see-saw mechanism;

(iii) $n_{\rm H}$ scalar doublets.

The see-saw mechanism provides us with a reason why the neutrino masses are much smaller than the charged-lepton masses. The number $n_{\rm H}$ of Higgs doublets is arbitrary in this section.

In order to fix the notation we recapitulate the see-saw mechanism. The scalar doublets and their Vacuum Expectation Values (VEVs) are denoted by

$$\phi_j = \begin{pmatrix} \varphi_j^+ \\ \varphi_j^0 \end{pmatrix} \quad \text{and} \quad \left\langle 0 \left| \varphi_j^0 \right| 0 \right\rangle = \frac{v_j}{\sqrt{2}} . \tag{19}$$

The right-handed neutrino singlets have a Majorana mass term

$$\mathcal{L}_{\rm M} = \frac{1}{2} \nu_{\rm R}^T C^{-1} M_{\rm R}^* \nu_{\rm R} + \text{h.c.} , \tag{20}$$

where $M_{\rm R}$ is symmetric. The Yukawa Lagrangian of the leptons is given by

$$\mathcal{L}_{\rm Y} = - \sum_{j=1}^{n_{\rm H}} \left[\bar{\ell}_{\rm R} \left(\varphi_j^- ,\ \varphi_j^{0*} \right) \varGamma_j + \bar{\nu}_{\rm R} \left(\varphi_j^0 ,\ -\varphi_j^+ \right) \varDelta_j \right] \begin{pmatrix} \nu_{\rm L} \\ \ell_{\rm L} \end{pmatrix} + \text{h.c.} \tag{21}$$

From this Lagrangian, we derive the charged-lepton mass matrix M_ℓ and the Dirac neutrino mass matrix $M_{\rm D}$ as

$$M_\ell = \frac{1}{\sqrt{2}} \sum_j v_j^* \varGamma_j \quad \text{and} \quad M_{\rm D} = \frac{1}{\sqrt{2}} \sum_j v_j \varDelta_j , \tag{22}$$

respectively. The Dirac and Majorana mass terms for the neutrinos are summarized in

$$\frac{1}{2} \left(\ \nu_{\rm L}^T ,\ \ \nu_{\rm L}^{\prime T} \ \right) C^{-1} \mathcal{M}_{\rm D+M} \begin{pmatrix} \nu_{\rm L} \\ \nu_{\rm L}^\prime \end{pmatrix} + \text{h.c.},$$

with

$$\mathcal{M}_{\rm D+M} = \begin{pmatrix} 0 & M_{\rm D}^T \\ M_{\rm D} & M_{\rm R} \end{pmatrix} , \tag{23}$$

where $\nu'_{\rm L} \equiv (\nu_{\rm R})^c = C\bar{\nu}_{\rm R}^T$. The see-saw mechanism gives an effective mass term (7) for the light left-handed neutrinos with mass matrix [19]

$$\mathcal{M}_\nu = -M_{\rm D}^T M_{\rm R}^{-1} M_{\rm D}\,. \tag{24}$$

In general the mass matrix of the charged leptons will not be diagonal and, therefore, has to be bi-diagonalized by two unitary matrices $V_{\rm R}^\ell$ and $V_{\rm L}^\ell$:

$$V_{\rm R}^{\ell\,\dagger} M_\ell V_{\rm L}^\ell = {\rm diag}\,(m_e, m_\mu, m_\tau)\,. \tag{25}$$

Then we arrive at the neutrino mixing matrix U by

$$V_{\rm L}^{\ell\,\dagger} V = {\rm e}^{i\hat{\alpha}} U {\rm e}^{i\hat{\beta}}\,, \tag{26}$$

where V is given by Eq. (12), while ${\rm e}^{i\hat{\alpha}}$ and ${\rm e}^{i\hat{\beta}}$ are diagonal matrices of phase factors. The phases $\hat{\alpha}$ can be absorbed into the charged-lepton fields. The phases $\hat{\beta}$, usually called Majorana phases, are irrelevant for neutrino oscillations — although they play an important role in processes like neutrinoless double-beta decay.

Schematically, in the see-saw mechanism the mass of a light Majorana neutrino is obtained as $m_\nu \sim (m_\nu^{\rm D})^2/m_{\rm R}$, where $m_\nu^{\rm D}$ is a typical scale in the Dirac neutrino mass matrix $M_{\rm D}$ and $m_{\rm R}$ a typical order of magnitude of the eigenvalues of $\sqrt{M_{\rm R}^* M_{\rm R}}$. Identifying m_ν with $\sqrt{\Delta m^2_{\rm atm}} \simeq 0.05$ eV, and $m_\nu^{\rm D}$ with either m_μ or m_τ, one obtains the well-known order of magnitude for the right-handed scale $m_{\rm R} \sim 10^8$–10^{11} GeV.

Having discussed the conventional see-saw extension of the SM we come to the main point which defines our framework:

> **ASSUMPTION:**
> The lepton numbers L_e, L_μ, and L_τ are *conserved* in the $\mathcal{L}_{\rm Y}$ of Eq. (21), but they are *softly broken* in the $\mathcal{L}_{\rm M}$ of Eq. (20).

Note that the Majorana mass term of the right-handed neutrino singlets, $\mathcal{L}_{\rm M}$, has dimension 3; therefore, the breaking of the family lepton numbers is indeed soft. As a consequence of our assumption, the Yukawa coupling matrices Γ_j, Δ_j are diagonal. Therefore,

(i) $V_{\rm L}^\ell = \mathbf{1}$;

(ii) $M_{\rm D}$ is diagonal;

(iii) neutrino mixing stems exclusively from $M_{\rm R}$;

(iv) family lepton numbers $L_{e,\mu,\tau}$ and the total lepton number $L = L_e + L_\mu + L_\tau$ are broken at the heavy scale $m_{\rm R}$, which is much higher than the electroweak scale;

(v) deviations from lepton number conservation are calculable (finite).

Does the framework introduced by our assumption yield a viable theory, in view of $m_{\rm R}$ being much larger than the electroweak (ew) scale? First we remark that the renormalization-group evolution of the coupling matrices Γ_j, Δ_j does not induce flavour-changing elements, once we start with diagonal matrices; this is due to the soft nature of the breaking of the family lepton numbers. On the other hand, there are strong experimental constraints [25] on $\mu^- \to e^-\gamma$ and on $\mu^- \to e^-e^+e^-$. In our framework, the branching ratio of the first process is suppressed by $1/m_{\rm R}^4$, the latter one by eight powers of Yukawa couplings (the large scale enters only logarithmically in this case) [21]. Also the branching ratio for $Z \to e^-\mu^+ + e^+\mu^-$ is suppressed by $1/m_{\rm R}^4$ [26]. Therefore, breaking family lepton numbers *softly* at a very high scale does apparently lead to a sensible theory which is not in conflict with experimental results.

Within our framework we shall discuss in the forthcoming sections three models, realizing maximal atmospheric neutrino mixing, maximal solar mixing, and bimaximal mixing [27], respectively.

4. Maximal atmospheric neutrino mixing

It turns out that to enforce maximal atmospheric mixing we need (at least) three scalar doublets ($n_{\rm H} = 3$), and we have to introduce two Z_2 symmetries:

$$Z_2 : \ \nu_{\mu{\rm R}} \leftrightarrow \nu_{\tau{\rm R}} \,, \ D_\mu \leftrightarrow D_\tau \,, \ \mu_{\rm R} \leftrightarrow \tau_{\rm R} \,, \ \phi_3 \to -\phi_3 \,, \tag{27}$$

the left-handed lepton doublets are denoted by D_α,

$$Z_2' : \ \mu_{\rm R} \to -\mu_{\rm R} \,, \ \tau_{\rm R} \to -\tau_{\rm R} \,, \ \phi_2 \to -\phi_2 \,, \ \phi_3 \to -\phi_3 \,. \tag{28}$$

Fields not appearing in these equations transform trivially. The motivation for Z_2 is quite straightforward. In $M_{\rm R}$ this symmetry leads to

$$(M_{\rm R})_{e\mu} = (M_{\rm R})_{e\tau} \quad \text{and} \quad (M_{\rm R})_{\mu\mu} = (M_{\rm R})_{\tau\tau} \,. \tag{29}$$

Because of Z_2', the second and third scalar doublet have no Yukawa couplings to the neutrino fields $\nu_{\rm R}$. Therefore, the Dirac neutrino mass matrix has the form

$$M_{\rm D} = {\rm diag}\,(a, b, b) \,. \tag{30}$$

As a consequence, the light-neutrino Majorana mass matrix has the same structure as M_{R}:

$$\mathcal{M}_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}. \tag{31}$$

Maximal atmospheric neutrino mixing and $U_{e3} = 0$ immediately follow from this structure of $\mathcal{M}_\nu$. This structure of $\mathcal{M}_\nu$ in the basis where the charged-lepton mass matrix is diagonal has previously been suggested by several authors (*e.g.* Ref. [28]). We stress that in our case this structure results from a symmetry, *i.e.*, we have a *model* and not just a *texture* for $\mathcal{M}_\nu$. In the phase convention used for U of Eq. (5), we obtain the neutrino mixing matrix

$$U = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta/\sqrt{2} & -\cos\theta/\sqrt{2} & -1/\sqrt{2} \\ \sin\theta/\sqrt{2} & -\cos\theta/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}. \tag{32}$$

One can check that the solar mixing angle θ is expressed by the parameters of $\mathcal{M}_\nu$ of Eq. (31) as

$$\tan 2\theta = 2\sqrt{2}\,\frac{|x^* y + y^*(z+w)|}{|z+w|^2 - |x|^2}. \tag{33}$$

By virtue of the symmetry Z_2', the scalar doublets $\phi_{2,3}$ couple only to ℓ_{R}. The non-zero VEV of ϕ_3 breaks Z_2 invariance in the charged-lepton sector already at tree level and allows for $m_\mu \neq m_\tau$; in the neutrino sector, where only ϕ_1 couples, and since ϕ_1 transforms trivially under Z_2, this symmetry is valid at the tree level. For details see Ref. [21].

It is instructive to recall the symmetries invoked. The three $\mathrm{U}(1)_{L_\alpha}$ ($\alpha = e, \mu, \tau$) are broken softly at scale m_{R} by $\mathcal{L}_{\mathrm{M}}$; the symmetry $Z_2 \times Z_2'$ is broken spontaneously, together with the SM gauge group, by the VEVs of the three Higgs doublets.

In summary, the model of this section fulfills the following:

(i) The atmospheric mixing angle ψ is exactly 45°;

(ii) U_{e3} is exactly zero;

(iii) the solar mixing angle θ is free, but large if one avoids fine tunings;

(iv) $\Delta m^2_\odot / \Delta m^2_{\mathrm{atm}}$ is free; it must be made small by fine tuning;

(v) the see-saw mechanism is responsible for the smallness of the neutrino masses $m_{1,2,3}$.

For the first and second items the rationale is given by the Z_2 symmetry together with the softly broken family lepton numbers.

5. Nearly maximal solar neutrino mixing

In this section we dispense with the discrete symmetries of the previous section, and a single Higgs doublet is sufficient. We take up again the idea of approximate $\bar{L} = L_e - L_\mu - L_\tau$ invariance, this time implemented in the $\mathcal{L}_{\rm M}$ of Eq. (20): we assume that there are two scales $m_{\rm R} \ll \bar{m}_{\rm R}$ in $M_{\rm R}$ such that $\bar{L}$ is conserved in between[2]. Thus we have the following picture:

$$\begin{array}{ccccccc} \text{ew scale} & \ll & m_{\rm R} & \ll & \bar{m}_{\rm R} & \ll & \begin{array}{c}\text{Planck (?)}\\ \text{scale}\end{array} \\ & & \uparrow & & \uparrow & & \uparrow \\ & & \bar{L}\text{ broken} & & \begin{array}{c}L_{e,\mu,\tau}\text{ broken,}\\ \bar{L}\text{ conserved}\end{array} & & \begin{array}{c}L_{e,\mu,\tau}\\ \text{conserved}\end{array} \end{array}$$

The individual lepton numbers are softly broken at $\bar{m}_{\rm R}$, but the linear combination $\bar{L}$ of the individual lepton numbers survives down to $m_{\rm R}$, where it is also softly broken. Defining $\varepsilon = m_{\rm R}/\bar{m}_{\rm R} \ll 1$, with our implementation of approximate $\bar{L}$ invariance the matrix $M_{\rm R}$ has the form

$$M_{\rm R} = \begin{pmatrix} u & p/\varepsilon & q/\varepsilon \\ p/\varepsilon & r & t \\ q/\varepsilon & t & s \end{pmatrix}, \tag{34}$$

where u, p, q, r, s, and t are assumed to be all of the order of magnitude $m_{\rm R}$. Note that since $M_{\rm D}$ is diagonal, $\bar{L}$ is not violated in the Dirac neutrino mass matrix. After applying the see-saw formula (24), with some lengthy algebra we obtain the mass spectrum of the light neutrinos [21]

$$\begin{array}{rcl} m_{1,2} & = & \varepsilon\, m' \pm \varepsilon^2 m'' \\ m_3 & = & m_0 + \varepsilon m_0' \end{array} \qquad \text{with} \quad m',\ m'',\ m_0,\ m_0' \sim (m_\nu^{\rm D})^2/m_{\rm R}\,. \tag{35}$$

Note that this mass spectrum is very different from the one obtained in Section 2, with approximate $\bar{L}$ invariance in $\mathcal{M}_\nu$ itself, instead of in $M_{\rm R}$. The two spectra are compared in figure 1. It is interesting that the spectrum (35) does not fit into either of the usual categories "hierarchy" and "inverted hierarchy", since it has the properties

$$m_2 - m_1 \ll \tfrac{1}{2}\,(m_1 + m_2) \ll m_3\,. \tag{36}$$

[2] An earlier variant of the idea of soft $\bar{L}$ breaking by $\mathcal{L}_{\rm M}$ is found in Ref. [29].

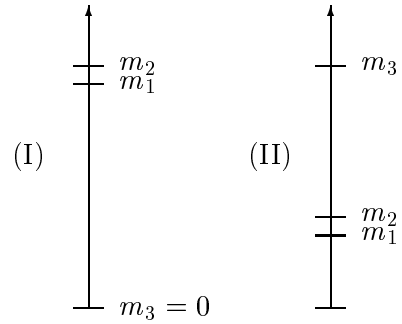

Fig. 1. Neutrino spectrum (I) shows the inverted hierarchy obtained in the scenario discussed in Section 2. For comparison, spectrum (II) of the model of Section 5 is also depicted. The latter spectrum, characterized by Eq. (36), is neither hierarchical nor of inverse hierarchy.

In summary, the characteristic features of this model (for details see Ref. [21]) are given by

$$\Delta m^2_{\rm atm} \simeq m_0^2\,, \qquad \frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} \sim \varepsilon^3\,, \qquad U_{e3} \sim \varepsilon\,, \qquad 1 - \sin^2 2\theta \sim \varepsilon^2\,. \quad (37)$$

While for the LOW solution of the solar neutrino deficit we estimate, from the value of $\Delta m^2_\odot/\Delta m^2_{\rm atm}$, $\varepsilon \sim 1/30$, for the LMA MSW solution we have $\varepsilon \sim 1/4$. Thus, in the LMA MSW case the two scales $m_{\rm R}$ and $\bar{m}_{\rm R}$ are rather close. Concentrating on the LMA MSW solution and using the order of magnitude estimate for θ in Eq. (37) despite of the not very small ε, we obtain $\theta \sim 38^\circ$. This is not so far from the best-fit value for the solar mixing angle[3] of Ref. [12], $\theta \simeq 32^\circ$. Furthermore, $|U_{e3}|^2 \sim \varepsilon^2$ might be on the border of becoming discernible in the data. We want to stress, however, that we have presented here only a crude picture of the situation, since ratios of the undetermined constants in $M_{\rm R}$ of Eq. (34), which are assumed to be all of similar order of magnitude, enter into the precise version of Eq. (37) (see Ref. [21]). Concerning the atmospheric mixing angle ψ, we can only say that it must be large but we have no rationale for setting it very close to 45°.

As we have seen, in the scheme of soft $\bar{L}$ breaking developed here, the solar mixing angle is not necessarily close to 45°, and U_{e3} can deviate appreciably from zero. Thus, not only in the neutrino mass spectra but also in neutrino mixing, we have striking differences between the scenario of this section and the one presented in Section 2.

[3] This is the value of the fit with a free ^{8}B flux; the value of the fit where the ^{8}B flux is restricted by the Solar Standard Model is slightly lower [12].

6. Bimaximal neutrino mixing

Bimaximal neutrino mixing can be obtained by combining the models of Sections 4 and 5. Then, the number of Higgs doublets must again be three, and we must again have the $Z_2 \times Z_2'$ symmetry in the Lagrangian prior to spontaneous symmetry breaking with, moreover, intermediate $\bar{L}$ invariance in $M_{\rm R}$. In this scenario we preserve the good relations $1 - \sin^2 2\theta \sim \varepsilon^2$ and $\Delta m^2_{\odot}/\Delta m^2_{\rm atm} \sim \varepsilon^3$, and are additionally rewarded with $\psi = 45^\circ$ and $U_{e3} = 0$ at tree level.

7. Conclusions

In this conference report we have first discussed the scenario with approximate $\bar{L} = L_e - L_\mu - L_\tau$ invariance in the mass matrix of the light neutrinos [22]. We have then considered a novel framework for imposing maximal neutrino mixing, either for atmospheric neutrinos or for solar neutrinos or for both, which was introduced in Ref. [21]. This framework consists of the SM with three families and three right-handed neutrino singlets with a heavy Majorana mass term with mass matrix $M_{\rm R}$, such that the three individual lepton numbers $L_{e,\mu,\tau}$ are conserved in the Yukawa couplings but *softly broken* by the Majorana mass term of the right-handed singlets. This is a very interesting scenario where deviations from $L_{e,\mu,\tau}$ and from $L = L_e + L_\mu + L_\tau$ conservation are calculable and controlled by the mass matrix $M_{\rm R}$. The mass matrix of the charged leptons is automatically diagonal.

Within this framework we have imposed different symmetries in order to obtain three different models of neutrino mixing:

1. $Z_2 \times Z_2'$ symmetry: In this case, where three Higgs doublets are necessary, we have obtained an atmospheric mixing angle of 45°, $U_{e3} = 0$, and a large but otherwise free solar mixing angle. We want to stress again that *symmetries* are responsible for these results, not "textures" or *Ansätze*.

2. Intermediate $\bar{L}$ conservation in $M_{\rm R}$: Here we need only one Higgs doublet, but we have two scales in $M_{\rm R}$ such that at the higher scale the individual family lepton numbers are broken down softly to the linear combination $\bar{L}$, which is then broken, also softly, at the lower scale. In this case the atmospheric mixing angle is free but large in general, whereas the solar mixing angle should be in the range around 44°, with the LOW solution of the solar neutrino deficit, or around 38°, with the LMA MSW solution. Also U_{e3} can differ considerably from zero.

3. Combination of the two symmetries leads to bimaximal mixing with U_{e3} again vanishing.

We have compared the scenario where $\bar{L}$ invariance is imposed directly on the light-neutrino mass matrix (Section 2) with the new scheme of item 2 where $\bar{L}$ invariance is rather imposed on $M_{\rm R}$ (Section 5).

Our new mechanisms for enforcing maximal neutrino mixings are accommodated in a rather simple extension of the SM. In particular, neither supersymmetry nor Grand Unified Theories (GUT) are invoked. Quite on the contrary, it seems difficult to incorporate our framework into a GUT.

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