NONUNITARY NEUTRINO MIXING MATRIX AND *CP* VIOLATING NEUTRINO OSCILLATIONS* **

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In the standard approach to the neutrino oscillations a unitary relation among weak and mass eigenstates of light neutrinos is imposed. However, in many extensions of the SM left-handed, active neutrinos mix with additional heavy neutrino states. Consequences of this additional mixing, driven by experimental constraints, on the neutrino oscillations are considered.

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1. Introduction

At present, 3 light neutrinos, with masses at the eV or sub-eV scale [1] are known to exist. However, much heavier neutrino states ($m_N \geq \mathcal{O}(M_Z/2)$) are not excluded [2]. These, due to kinematical reasons do not contribute directly to the weak neutrino states which can undergo neutrino oscillations. They influence, however, neutrino oscillations since they modify the neutrino mixing matrix U. Let U_ν be the full neutrino mixing matrix, then the matrix

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U of dimension 3×3 constitutes the mixing submatrix of light neutrino states ($\nu_e,\nu_\mu,\nu_\tau \leftrightarrow \nu_1,\nu_2,\nu_3$ transitions)

$$U_\nu = \begin{pmatrix} U & V \\ V' & U' \end{pmatrix}. \tag{1}$$

The submatrix V (of dimension $(n_R\times 3)$) is responsible for the mixing of light neutrinos with n_R heavy states ($\nu_e,\nu_\mu,\nu_\tau \leftrightarrow \nu_4,\ldots,\nu_{n_R-3}$ transitions). The submatrix U' (of dimension $(n_R\times n_R)$) is responsible for mixing among heavy states. In the conventional see-saw mechanism $m_N \gg m_\nu$, where m_N and m_ν are masses of heavy and light states respectively, the elements of V are very small and U becomes unitary. This simply means that heavy neutrino states do not modify mixings among light neutrino states. From the theoretical point of view, V does not have to be negligible [3]. We will use the experimental data to constrain V [4], and more precisely the combination[1] $\left(VV^\dagger\right)_{\alpha\beta}$ $(\alpha,\beta=\{e,\mu,\tau\})$.

From the unitarity of U_ν we infer that

$$\left(UU^\dagger\right)_{\alpha\beta} = \delta_{\alpha\beta} - \left(VV^\dagger\right)_{\alpha\beta}\,. \tag{2}$$

The aim of this paper is to examine the effect of this modification of unitarity of V on neutrino oscillations. The subject is not new[2] [6]. Nevertheless, some issues, especially connected with CP violation effects have not yet been discussed. CP violation effects in the unitary neutrino oscillations case are known to be very fragile. If any element of the unitary U matrix (*e.g.* U_{e3}) is small then the effect of CP violation will be small either. And in fact, U_{e3} (see Eq. (6)) is known to be very small if not zero. Besides, the CP phase $\sin\delta$ must be substantial. Finally, the CP violating effects vanish with decreasing $\delta m^2_\odot$. For $\delta=\frac{\pi}{2}$, $\delta m^2_\odot$ given by LMA MSW solution and $U_{e3}>0$, the CP effects can be detectable [9], but even then it may happen that matter effects will mimic (or screen) the CP violation [10]. We show that the nonunitarity of U can be responsible for similar effects. If CP violation effects were detected with a strength larger than predicted by the unitary neutrino mixing approach, then heavy neutrino mixing could be held responsible for this effect. In the contrary case, some better bounds on the $\left(VV^\dagger\right)_{\alpha\beta}$ factors could be found.

In this paper we focus on neutrino oscillations in vacuum.

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[1] The elements of the V matrix can also be investigated, *e.g.* in heavy neutrino production processes [5].

[2] Recently, effects of a non-unitary mixing matrix U have been considered in [7] in a different context where new leptonic interactions have been included.

2. Neutrino oscillations in the presence of heavy neutrino states

In the standard neutrino oscillation theory of three flavours we start with neutrino weak eigenstates $\nu_\alpha = (\nu_e, \nu_\mu, \nu_\tau)$ as a combination of three mass eigenstates $\nu_i = (\nu_1, \nu_2, \nu_3)$

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i} \nu_i \,. \tag{3}$$

The form of the matrix U can be obtained using subsequent rotations around the axes spanned by massive neutrino states m_1, m_2, m_3

$$U = R_{23} R_{13} R_{12} \,. \tag{4}$$

R_{ij}'s represent rotations in the i–j plane by Θ_{ij} angle with additional phases, *e.g.* ($c_{12} \equiv \cos\Theta_{12}$, $s_{12} \equiv \sin\Theta_{12} \mathrm{e}^{i\delta_{12}}$)

$$R_{12} = \begin{pmatrix} c_{12} & s_{12}^* & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{5}$$

Taking $\delta_{12} = \delta_{23} = 0$ (two of the three complex phases do not influence the oscillation probability [11]) we obtain the classical parametrization of the U matrix [12] ($\delta_{13} \equiv \delta$)

$$\begin{aligned} U &= \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}\mathrm{e}^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}\mathrm{e}^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - s_{13}c_{23}c_{12}\mathrm{e}^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}\mathrm{e}^{i\delta} & c_{23}c_{13} \end{pmatrix} . \end{aligned} \tag{6}$$

Let us now include effects of the matrix V to the matrix U (Eqs. (1), (2)). We will do it by introducing three new parameters ε_i, $i = 1, 2, 3$ which are connected directly to the elements of the matrix V in the case of the 4×4 matrix Eq. (1).

The general 4×4 matrix Eq. (1) can be parametrized by 6 rotation angles (and 6 phases) in the following way

$$U_\nu = R_{34} R_{24} R_{14} R_{23} R_{13} R_{12} , \tag{7}$$

where the rotations take place in the 4 dimensional space spanned by four massive neutrino states, *e.g.* [13, 14] ($s_{12} \equiv \sin\Theta_{12} \mathrm{e}^{i\delta_{12}}$)

$$R_{12} = \begin{pmatrix} c_{12} & s_{12}^* & 0 & 0 \\ -s_{12} & c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \tag{8}$$

Let us note that Eq. (7) differs from Eq. (4) by three additional rotations $R_{34}R_{24}R_{14}$ in the plane to which the additional fourth neutrino state belongs. When the fourth state is much heavier than the light states, the rotation angles are small. Let us take $|s_{14}| \equiv |\varepsilon_1| \ll 1$, $|s_{24}| \equiv |\varepsilon_2| \ll 1$ and $|s_{34}| \equiv |\varepsilon_3| \ll 1$. Then we can expand Eq. (7) to get

$$U_\nu = \begin{pmatrix} \begin{pmatrix} U(\varepsilon_i) \end{pmatrix} & \begin{matrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{matrix} \\ g(\varepsilon_i) & 1 - \frac{1}{2}(|\varepsilon_1|^2 + |\varepsilon_2|^2 + |\varepsilon_3|^2) \end{pmatrix} . \tag{9}$$

In the limit $\varepsilon_i \to 0$ $U(\varepsilon_i) \to U$ (Eq. (6)) and $g(\varepsilon_i) \to 0$. $U(\varepsilon_i)$ is the desired matrix which we will use in the neutrino oscillation formula instead of the U matrix in Eq. (3). We will not show the explicit form of $U(\varepsilon_i)$ as it is straightforward but space consuming. Our parametrization through (complex) ε factors holds in the general case of n heavy states and can be easily connected to the quantities which are usually constrained by experimental data, *e.g.*

$$\left| \left(VV^\dagger \right)_{ee} \right| = \sum_{i=\mathrm{heavy}} |V_{ei}|^2 \equiv |\varepsilon_1|^2 \leq 0.0054 \,, \tag{10}$$

and similarly,

$$\left| \left(VV^\dagger \right)_{e\mu} \right| = |\varepsilon_1 \varepsilon_2| \leq 10^{-4} \,, \tag{11}$$

$$\left| \left(VV^\dagger \right)_{\mu\tau} \right| = |\varepsilon_2 \varepsilon_3| \leq 10^{-2} \,. \tag{12}$$

The very strict constraint, Eq. (11) comes from the lack of the $\mu \to e\gamma$ decay [4, 6, 15]. Constraints, Eqs. (10), (12) are consequences of global fits to experimental data [4, 6] (*e.g.* lepton universality, invisible Z decay, CKM unitarity). There is also a constraint on $\sum_{i=\mathrm{heavy}} |V_{ei}|^2/M_i$ coming from the neutrinoless double beta decay [8]. In our approach we do not have to use any information on the heavy neutrino mass spectrum. We just assume that the masses are above 100 GeV. The constraint from $(\beta\beta)_{0\nu}$ is then fulfilled. With the parametrization Eq. (9) it is straightforward to write the modified neutrino oscillation probability

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta} &= N_\alpha^2 N_\beta^2 \Big\{ \Big(\delta_{\alpha\beta} - \Big| \Big(VV^\dagger \Big)_{\alpha\beta} \Big| \Big)^2 \\ &\quad -4 \sum_{a>b} \tilde{R}^{ab}_{\alpha\beta} \sin^2 \Delta_{ab} - 8 \tilde{I}^{12}_{\alpha\beta} \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \\ &\quad -2 \left[A^{(1)}_{\alpha\beta} \sin 2\Delta_{31} + A^{(2)}_{\alpha\beta} \sin 2\Delta_{32} \right] \Big\}, \end{aligned} \tag{13}$$

where

$$\Delta_{ab} = 1.27 \delta m^2_{ab} [\mathrm{eV}] \frac{L[\mathrm{km}]}{E[\mathrm{GeV}]}, \qquad \delta m^2_{ab} = m_a^2 - m_b^2 . \tag{14}$$

$\tilde{R}^{ab}_{\alpha\beta}$ are modified definitions taken from the standard, unitary approach

$$\tilde{R}^{ab}_{\alpha\beta} = \mathrm{Re} \left[W^{ab}_{\alpha\beta} \right], \tag{15}$$

$$\tilde{I}^{ab}_{\alpha\beta} = \mathrm{Im} \left[W^{ab}_{\alpha\beta} \right], \tag{16}$$

$$W^{ab}_{\alpha\beta}(\varepsilon_i) = U_{\alpha a} U_{\beta b} U^*_{\alpha b} U^*_{\beta a} . \tag{17}$$

$N_{\alpha(\beta)}$ are factors which normalize the three light neutrino states to 1

$$N_\alpha^2 = \frac{1}{1 - (VV^\dagger)_{\alpha\alpha}} . \tag{18}$$

The last row in Eq. (13) with

$$A^{(i)}_{\alpha\beta}(\varepsilon_i) = \mathrm{Im} \left[U^*_{\alpha i} U_{\beta i} \left(VV^\dagger \right)_{\alpha\beta} \right] \tag{19}$$

deserves an extra comment. Its appearance is a consequence of the modification of the Jarlskog factors, which for unitary U fulfill the following relations

$$I^{ab}_{\alpha\beta} = -I^{ba}_{\alpha\beta} = -I^{ab}_{\beta\alpha} = I^{ba}_{\beta\alpha} . \tag{20}$$

When U is not unitary, Eq. (2) leads to

$$\tilde{I}^{12}_{\alpha\beta} = -\tilde{I}^{32}_{\alpha\beta} - \mathrm{Im} \left(U^*_{\alpha 2} U_{\beta 2} \left(VV^\dagger \right)_{\alpha\beta} \right), \tag{21}$$

$$\tilde{I}^{21}_{\alpha\beta} = -\tilde{I}^{31}_{\alpha\beta} - \mathrm{Im} \left(U^*_{\alpha 1} U_{\beta 1} \left(VV^\dagger \right)_{\alpha\beta} \right), \tag{22}$$

$$\tilde{I}^{23}_{\alpha\beta} = -\tilde{I}^{13}_{\alpha\beta} - \mathrm{Im} \left(U^*_{\alpha 3} U_{\beta 3} \left(VV^\dagger \right)_{\alpha\beta} \right) \tag{23}$$

and, therefore, to the last term in Eq. (13).

As discussed in [6], the effects of the normalization factors will be difficult to observe in experiments. Here we will focus on the influence of the additional neutrino mixing of light neutrinos represented by the ε_i's on the CP violating effects. The novelty here is the appearance of the third line in Eq. (13). This term is not very sensitive to Δ_{21} when it is small. Therefore CP violation can occur even if $\delta m_{12}^2 = 0$. However, CP violation is now possible with two neutrino oscillations. In addition, the CP effect with three neutrino flavours, contrary to unitary oscillations, can be substantial even if one of the elements of the mixing matrix is very small.

3. *CP* violating effects in neutrino oscillations

CP violating effects can be seen in neutrino appearance experiments. Let us consider the following standard quantities

$$\begin{aligned} A_{CP}(\alpha;\beta) &= \frac{P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)}{P(\nu_\alpha \to \nu_\beta) + P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)}\,, \\ A_T(\alpha;\beta) &= \frac{P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)}{P(\nu_\alpha \to \nu_\beta) + P(\nu_\beta \to \nu_\alpha)}\,. \end{aligned}$$

In vacuum $A_{CP} = A_T$. The same is true in the case of the new Eq. (13) when a nonunitary matrix U is present.

We assume the following values of the standard parameters

$$\begin{aligned} \delta m_{21}^2 &= 5 \times 10^{-5}\ \mathrm{eV}^2\,, \\ \delta m_{32}^2 &= 3 \times 10^{-3}\ \mathrm{eV}^2\,, \\ \Theta_{12} &\simeq 35^\circ\,,\ \Theta_{23} \simeq 40^\circ\,,\ \Theta_{13} \simeq 5^\circ\,, \\ \delta &\simeq \pm 90^\circ\,. \end{aligned} \tag{24}$$

These values are consistent with CHOOZ [16], the LMA MSW solution of the solar neutrino problem [17] and the SuperKamiokande data [18]. For the nonstandard parameters we take

$$|\varepsilon_1| = 0.001\,, \qquad |\varepsilon_2| = 0.1\,, \qquad |\varepsilon_3| = 0.1\,. \tag{25}$$

which are consistent with Eqs. (10)–(12).

Figs. 1, 2 show the results for two cases of $A_{CP}(e;\mu)$ and $A_{CP}(\mu;\tau)$, and long-baseline ($L = 732$ km, *e.g* MINOS) or short-baseline ($L = 250$ km, *e.g.* K2K) experiments. The neutrino energies are chosen to be between 2 GeV and 30 GeV. We can see that the effects of the nonstandard heavy neutrino mixings can be quite large, even much bigger than in the unitary approach when $\varepsilon_i = 0$.

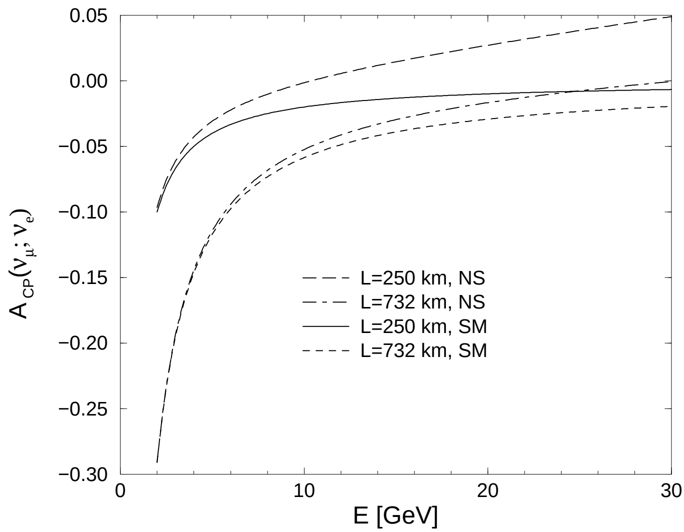

Fig. 1. The $A_{CP}(\mu;e)$ asymmetry as function of neutrino energy. The label 'NS' means that Eqs. (24), (25) are taken into account. The label 'SM' means that the values of neutrino parameters as given in Eq. (24) and $\varepsilon_i = 0$ have been taken.

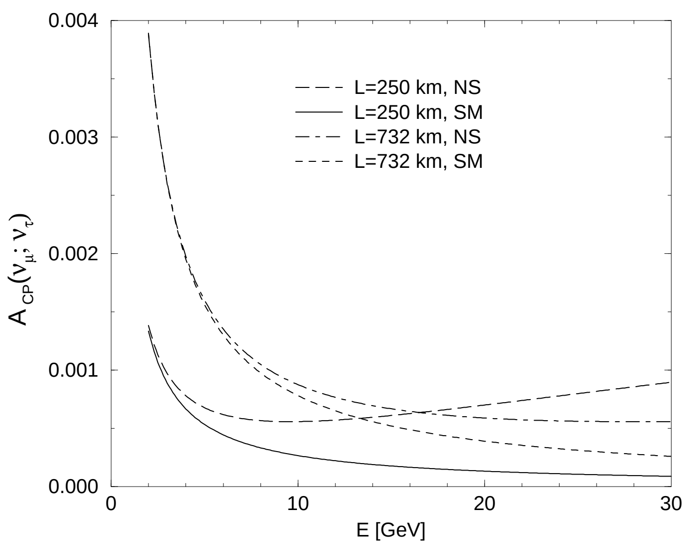

Fig. 2. The $A_{CP}(\mu;\tau)$ asymmetry as function of neutrino energy. The label 'NS' means that Eqs. (24), (25) are taken into account. The label 'SM' means that the values of neutrino parameters as given in Eq. (24) and $\varepsilon_i = 0$ have been taken.

In Figs. 3, 4 the results are given for genuine CP effects of NS sector when some of the ε_i's are chosen to be complex and $\delta = 0$.

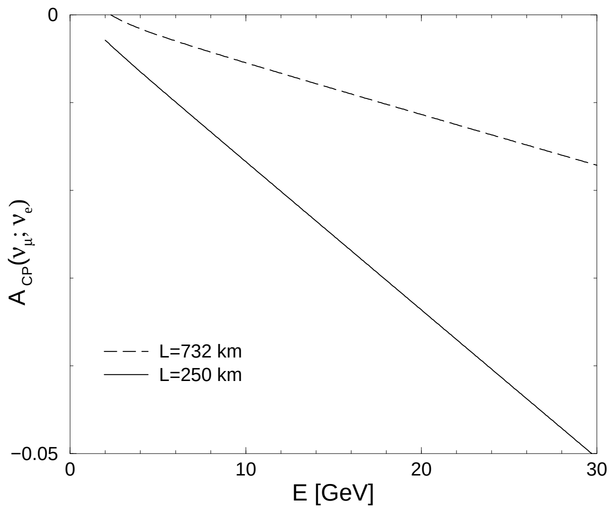

Fig. 3. The $A_{CP}(\mu; e)$ asymmetry generated by the NS sector. The results are for the parameters Eq. (24) but with $\delta = 0$. $\varepsilon_1 = 0.001$, $\varepsilon_2 = 0.1\,i$, $\varepsilon_3 = 0.1$. This choice is consistent with Eqs. (10)–(12).

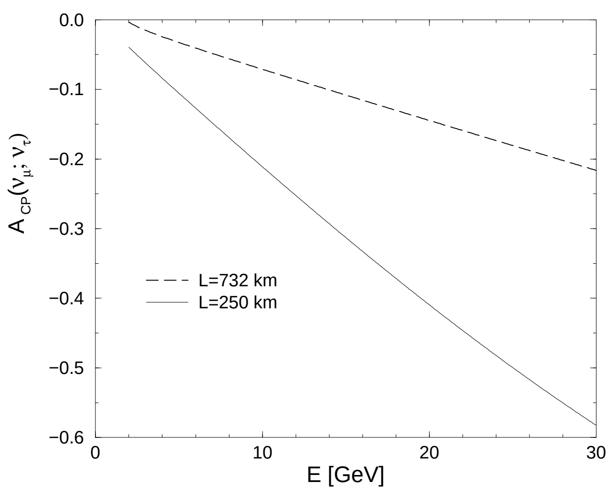

Fig. 4. The $A_{CP}(\mu; \tau)$ asymmetry generated by the NS sector. The results are for the parameters Eq. (24) but with $\delta = 0$. $\varepsilon_2 = 0.1$, $\varepsilon_3 = 0.1\,i$, $\varepsilon_1 = 10^{-4}$. This choice is consistent with Eqs. (10)–(12).

We would like to finish with a somehow academic example of what more do 'nonorthogonal' neutrino states mean. When a unitary U is used in the description of neutrino oscillations, the following relation holds

$$\sum_{\alpha} P_{\alpha\beta} = 1\,, \tag{26}$$

e.g.:

$$P_{ee} + P_{e\mu} + P_{e\tau} = 1\,.$$

It simply means that the number of emitted neutrinos of the given flavour will be the same as the number of final neutrinos of any type. However, for a nonunitary U this relation is not fulfill. Let us see it in a simple case of two flavours, when U is defined as ($\Theta_2 = \Theta_1 + \varepsilon$)

$$U = \begin{pmatrix} \cos\Theta_1 & \sin\Theta_1 \\ -\sin\Theta_2 & \cos\Theta_2 \end{pmatrix}. \tag{27}$$

In this case we get

$$\begin{aligned} \sum_{\alpha=e,\mu} P_{e\alpha} &= P_{ee} + P_{e\mu} = 1 + 4\varepsilon \sin^2 \varDelta_{21} \sin\Theta_1 \cos\Theta_1 \cos 2\Theta_1 + \mathcal{O}(\varepsilon^2)\,, \\ \sum_{\alpha=e,\mu} P_{\mu\alpha} &= P_{\mu e} + P_{\mu\mu} = 1 - 4\varepsilon \sin^2 \varDelta_{21} \sin\Theta_1 \cos\Theta_1 \cos 2\Theta_1 + \mathcal{O}(\varepsilon^2)\,. \end{aligned} \tag{28}$$

We can see that the sum can be either larger or smaller than 1. A similar result holds for a 3 dimensional U.

4. Conclusions

Three lessons can be learned from the results. First of all short baseline experiments are sensitive to the NS sector. Some improvements of the constraints Eqs. (10)–(12) are possible in this case when no signal for A_{CP} is found. Second, the NS effects connected to the complexity of ε_i can mimic SM effects of δ. Third, cancellations between the SM and NS effects can appear.

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