

THE γ^*p TOTAL CROSS SECTION AT LOW x^* **

D. SCHILDKNECHT

Fakultät für Physik, Universität Bielefeld
 Universitätsstrasse 25, 33615 Bielefeld, Germany

(Received October 17, 2001)

The scaling in $\sigma_{\gamma^*p}(W^2, Q^2)$ cross sections (for $Q^2/W^2 \ll 1$) in terms of the scaling variable $\eta = (Q^2 + m_0^2)/\Lambda^2(W^2)$ is interpreted in the Generalized Vector Dominance/Color-Dipole Picture (GVD/CDP). The quantity $\Lambda^2(W^2)$ is identified as the average gluon transverse momentum absorbed by the $q\bar{q}$ state, $\langle \vec{l}_\perp^2 \rangle = (1/6)\Lambda^2(W^2)$. At any Q^2 , for $W^2 \rightarrow \infty$, the cross sections for virtual and real photons become universal, $\sigma_{\gamma^*p}(W^2, Q^2)/\sigma_{\gamma p}(W^2) \rightarrow 1$. The gluon density corresponding to the color-dipole cross section in the appropriate limit is found to be consistent with the results from QCD fits.

PACS numbers: 12.38.-t, 12.40.-y, 13.60.-r

Two important observations [1] were made on Deep Inelastic Scattering (DIS) at low values of the Björken scaling variable $x_{\text{Bj}} \cong Q^2/W^2 \ll 1$, since HERA started running in 1993:

- (i) The diffractive production of high-mass states (of masses $M_X \lesssim 30$ GeV) at an appreciable rate relative to the total virtual-photon-proton cross section, $\sigma_{\gamma^*p}(W^2, Q^2)$. The sphericity and thrust analysis [1, 2] of the diffractively produced states revealed (approximate) agreement in shape with the final state found in e^+e^- annihilation at $\sqrt{s} = M_X$. This observation of high-mass diffractive production confirms the conceptual basis of Generalized Vector Dominance (GVD) [3, 4] that extends the role of the low-lying vector mesons in photoproduction [5] to DIS at arbitrary Q^2 , provided $x_{\text{Bj}} \ll 1$.

* Presented at the XXV International School of Theoretical Physics "Particles and Astrophysics — Standard Models and Beyond", Ustroń, Poland, September 10–16, 2001.

** Supported by BMBF under Contract 05HT9PBA2.

- (ii) An increase of $\sigma_{\gamma^*p}(W^2, Q^2)$ with increasing energy considerably stronger [6] than the smooth “soft-pomeron” behavior known from photo-production and hadron–hadron scattering.

We have recently shown [7] that the data for total photon–proton cross sections, including *virtual as well as real photons*, show a scaling behavior. In good approximation,

$$\sigma_{\gamma^*p}(W^2, Q^2) = \sigma_{\gamma^*p}(\eta), \quad (1)$$

with

$$\eta = \frac{Q^2 + m_0^2}{\Lambda^2(W^2)}, \quad (2)$$

(compare figure 1). The scale $\Lambda^2(W^2)$, of dimension GeV^2 , turned out to be an increasing function of the γ^*p energy, W^2 , and may be represented by a power law or a logarithmic function of W^2

$$\Lambda^2(W^2) = \begin{cases} c_1(W^2 + W_0^2)^{c_2}, \\ c'_1 \ln\left(\frac{W^2}{W_0^2} + c'_2\right). \end{cases} \quad (3)$$

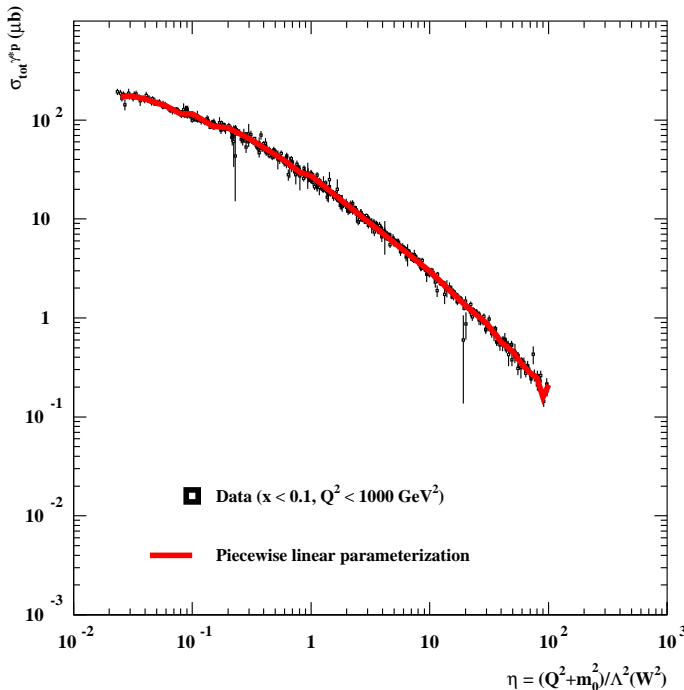


Fig. 1. The experimental data for $\sigma_{\gamma^*p}(W^2, Q^2)$ for $x \simeq Q^2/W^2 \leq 0.1$, including $Q^2 = 0$, versus the scaling variable $\eta = (Q^2 + m_0^2)/\Lambda^2(W^2)$.

In a model-independent fit to the experimental data, the threshold mass, $m_0^2 < m_\rho^2$, and the two parameters $c_2(c'_2)$ and $W_0^2(W_0'^2)$ were found to be given by $m_0^2 = 0.125 \pm 0.027 \text{ GeV}^2$, $c_2 = 0.28 \pm 0.06$, $W_0^2 = 439 \pm 94 \text{ GeV}^2$ with $\chi^2/\text{ndf} = 1.15$, and $m_0^2 = 0.12 \pm 0.04 \text{ GeV}^2$, $c'_2 = 3.5 \pm 0.6$, $W_0'^2 = 1535 \pm 582 \text{ GeV}^2$, with $\chi^2/\text{ndf} = 1.18$. The overall normalization, $c_1(c'_1)$ in (3) is irrelevant for the scaling behavior.

For the interpretation of the scaling law (1), we turn to the generalized vector dominance/color-dipole picture (GVD/CDP) [7, 8], of deep-inelastic scattering at low $x \ll 1$. It rests on $\gamma^*(q\bar{q})$ transitions from e^+e^- annihilation, forward scattering of the $(q\bar{q})$ states of mass $M_{q\bar{q}}$ via (the generic structure of) two-gluon exchange [9] and transition to space-like Q^2 via propagators of the $(q\bar{q})$ states of mass $M_{q\bar{q}}$. In the transverse-position-space representation [10], we have

$$\begin{aligned} \sigma_{\gamma^*p}(W^2, Q^2) &= \int dz \int d^2r_\perp \left| \psi \right|^2 (r_\perp^2 Q^2 z(1-z), Q^2 z(1-z), z) \\ &\times \sigma_{(q\bar{q})p}(r_\perp^2, z(1-z), W^2). \end{aligned} \quad (4)$$

We refer to [10] for the explicit representation of the square of the photon wave function, $|\psi|^2$. The Ansatz (4) for the total cross section *must* be read in conjunction with the Fourier representation of the color-dipole cross section,

$$\sigma_{(q\bar{q})p}(r_\perp^2, z(1-z), W^2) = \int d^2l_\perp \tilde{\sigma}_{(q\bar{q})p}(\vec{l}_\perp^2, z(1-z), W^2) \left(1 - e^{-i\vec{l}_\perp \cdot \vec{r}_\perp} \right). \quad (5)$$

The function $\tilde{\sigma}_{(q\bar{q})p}(\vec{l}_\perp^2, z(1-z), W^2)$ describes the gluon-gluon-proton-proton vertex function. Upon insertion of (5) into (4), together with the Fourier representation of the photon wave function, one indeed recovers [8] the expression for σ_{γ^*p} that displays the $x \rightarrow 0$ generic structure of two-gluon exchange¹: The resulting expression for σ_{γ^*p} is characterized by the difference of a diagonal and an off-diagonal term with respect to the transverse momenta (or masses) of the $q\bar{q}$ states the incoming and outgoing photon virtually dissociates into.

From (5), the color-dipole cross section, in the two limiting cases of vanishing and infinite interquark separation, becomes, respectively,

$$\sigma_{(q\bar{q})p}(r_\perp^2, z(1-z), W^2) = \sigma^{(\infty)} \begin{cases} \frac{1}{4} r_\perp^2 \langle \vec{l}_\perp^2 \rangle_{W^2, z}, & \text{for } r_\perp^2 \rightarrow 0, \\ 1, & \text{for } r_\perp^2 \rightarrow \infty. \end{cases} \quad (6)$$

¹ It is precisely the identical structure [8] that justifies the GVD/CDP (4), (5) from QCD.

The proportionality to r_{\perp}^2 for small interquark separation is known as “color transparency” [10]. For large interquark separation the color-dipole cross section should behave as an ordinary hadronic one. Accordingly,

$$\sigma^{(\infty)} = \pi \int d\vec{l}_{\perp}^2 \tilde{\sigma}(l_{\perp}^2, z(1-z), W^2), \quad (7)$$

must be independent of the configuration variable z and has to fulfill the restrictions from unitarity on its energy dependence. The average gluon transverse momentum $\langle \vec{l}_{\perp}^2 \rangle_{W^2, z}$ in (6), is defined by

$$\langle \vec{l}_{\perp}^2 \rangle_{W^2, z} = \frac{\int d\vec{l}_{\perp}^2 \vec{l}_{\perp}^2 \tilde{\sigma}_{(q\bar{q})p}(\vec{l}_{\perp}^2, z(1-z), W^2)}{\int d\vec{l}_{\perp}^2 \tilde{\sigma}_{(q\bar{q})p}(\vec{l}_{\perp}^2, z(1-z), W^2)}. \quad (8)$$

Replacing the integration variable r_{\perp}^2 in (4) by the dimensionless variable

$$u \equiv r_{\perp}^2 \Lambda^2 (W^2) z(1-z), \quad (9)$$

the photon wave function becomes a function $|\psi|^2(u(Q^2/\Lambda^2), (Q^2/\Lambda^2), z)$. The requirement of scaling (1), in particular for $Q^2 \gg m_0^2$, implies that the color-dipole cross section in (4) be a function of u ,

$$\sigma_{(q\bar{q})p}(r_{\perp}^2, z(1-z), W^2) = \sigma_{(q\bar{q})p}(u). \quad (10)$$

Taking into account (6), with (10), we find

$$\langle \vec{l}_{\perp}^2 \rangle_{W^2, z} = \Lambda^2 (W^2) z(1-z), \quad (11)$$

and upon averaging over z

$$\langle \vec{l}_{\perp}^2 \rangle_{W^2} = \frac{1}{6} \Lambda^2 (W^2). \quad (12)$$

The quantity $\Lambda^2 (W^2)$ in the scaling variable (2) is, accordingly, identified as the average gluon transverse momentum, apart from the factor $1/6$ due to the averaging over z .

Inserting $\langle \vec{l}_{\perp}^2 \rangle_{W^2, z}$ from (11) into (6), we have

$$\sigma_{q\bar{q}p} = \sigma^{(\infty)} \begin{cases} \frac{1}{4} r_{\perp}^2 \Lambda^2 (W^2) z(1-z), & \text{for } \Lambda^2 r_{\perp}^2 \rightarrow 0, \\ 1, & \text{for } \Lambda^2 r_{\perp}^2 \rightarrow \infty. \end{cases} \quad (13)$$

The dependence of the photon wave function in (4) on $r_{\perp}^2 Q^2$ requires small r_{\perp}^2 at large Q^2 in order to develop appreciable strength; for large Q^2 , the $r_{\perp}^2 \rightarrow 0$ behavior in (13), with its associated strong W dependence, becomes relevant until, finally, for sufficiently large W , the soft W dependence of $\sigma^{(\infty)}$ will be reached.

Thus, by interpreting the empirically established scaling, $\sigma_{\gamma^*p} = \sigma_{\gamma^*p}(\eta)$, in the GVD/CDP, we have obtained the dependence of the color-dipole cross section on the dimensionless variable u in (10) and, consequently, with (13), qualitatively, the dependence on η as shown in figure 1. Conversely, assuming a functional form for the color-dipole cross section according to (10), one recovers the scaling behavior (1).

In [7], we have shown that approximating the distribution in the gluon momentum transfer by its average value, (11)

$$\tilde{\sigma}_{(q\bar{q})p} = \sigma^{(\infty)} \frac{1}{\pi} \delta \left(\vec{l}_\perp^2 - \Lambda^2 (W^2) z(1-z) \right), \quad (14)$$

allows one to analytically evaluate the expression for σ_{γ^*p} in (4) in momentum space. The threshold mass $m_0 \lesssim m_\rho$ enters via the lower limit of the integration over the masses appearing in the propagators of the ingoing and outgoing $q\bar{q}$ states. For details we refer to [7], and only note the approximate result

$$\sigma_{\gamma^*p}(\eta) \simeq \frac{2\alpha}{3\pi} \sigma^{(\infty)} \begin{cases} \ln(1|\eta), & \text{for } \eta \rightarrow \eta_{\min} = \frac{m_0^2}{\Lambda^2(W^2)}, \\ 1|2\eta = \frac{1}{2} \frac{\Lambda^2(W^2)}{Q^2}, & \text{for } \eta \gg 1. \end{cases} \quad (15)$$

Note that for any fixed value of Q^2 , with $W^2 \rightarrow \infty$, the soft logarithmic dependence as a function of η^{-1} is reached. We arrive at the important conclusion that in the $W^2 \rightarrow \infty$ limit virtual and real photons become equivalent [11]

$$\lim_{\substack{W^2 \rightarrow \infty \\ Q^2 \text{ fixed}}} \frac{\sigma_{\gamma^*p}(W^2, Q^2)}{\sigma_{\gamma p}(W^2)} = 1. \quad (16)$$

Even though convergence towards unity is extremely slow (compare figure 2), such that it may be difficult to be ever verified experimentally, the universality of real and virtual photons contained in (16) is remarkable. It is an outgrowth of the HERA results which are consistent with the scaling law (1) with η from (2) and $\Lambda^2(W^2)$ from (3). Note that the alternative of $\Lambda^2 = \text{const.}$ that implies Björken scaling of the structure function $F_2 \sim Q^2 \sigma_{\gamma^*p}$ for sufficiently large Q^2 , leads to a result entirely different from (16),

$$\lim_{\substack{W^2 \rightarrow \infty \\ Q^2 \text{ fixed}}} \frac{\sigma_{\gamma^*p}(W^2, Q^2)}{\sigma_{\gamma p}(W^2)} = \frac{\Lambda^2}{2Q^2 \ln \frac{\Lambda^2}{m_0^2}} \quad (\text{assuming } \Lambda = \text{const.}), \quad (17)$$

i.e. a suppression of the virtual-photon cross section by a power of Q^2 .

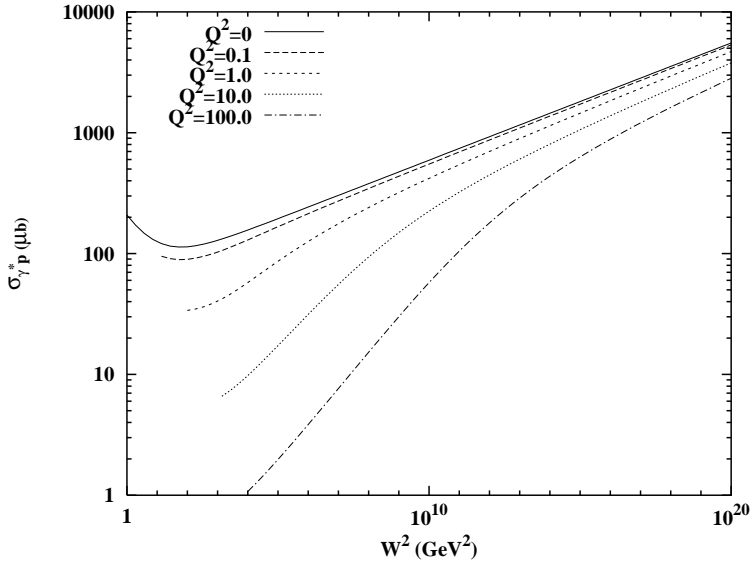


Fig. 2. The virtual-photon-proton cross section, $\sigma_{\gamma^*p}(W^2, Q^2)$, including $Q^2 = 0$ photoproduction, as a function of W^2 for fixed Q^2 . The figure demonstrates the asymptotic behavior, $\sigma_{\gamma^*p}(W^2, Q^2)/\sigma_{\gamma p}(W^2) \rightarrow 1$ for $W^2 \rightarrow \infty$, that follows from the scaling in η contained in the GVD/CDP.

In figure 3, we show $\Lambda^2(W^2)$ as obtained from the fit [7] of σ_{γ^*p} to the experimental data. The figure shows the result of fits based on the power law and the logarithm in (3), as well as the results of a point-like fit, $\Lambda^2(W_i^2)$. Using (12), one finds that the average gluon transverse momentum increases from $\langle \vec{l}_\perp^2 \rangle \simeq 0.5 \text{ GeV}^2$ to $\langle \vec{l}_\perp^2 \rangle \simeq 1.25 \text{ GeV}^2$ for W from $W \simeq 30 \text{ GeV}$ to $W \simeq 300 \text{ GeV}$. In figure 4, we show the agreement between theory and experiment for σ_{γ^*p} as a function of η . For further details we refer to Ref. [7].

So far we have exclusively concentrated on a representation of σ_{γ^*p} in terms of the color-dipole cross section, $\sigma_{(q\bar{q})p}(\vec{r}_\perp^2, W^2, z(1-z))$. For sufficiently large Q^2 and non-asymptotic W^2 , such that the $\Lambda^2(W^2) \vec{r}_\perp^2 \rightarrow 0$ limit in (13) is valid, one may alternatively parameterize the gluon interaction with the proton target in terms of the gluon density of the proton. The corresponding formula has indeed been worked out in [12]. It reads

$$\sigma_{(q\bar{q})p}(r_\perp^2, x, Q^2) = \frac{\pi^2}{3} r_\perp^2 x g(x, Q^2) \alpha_s(Q^2). \quad (18)$$

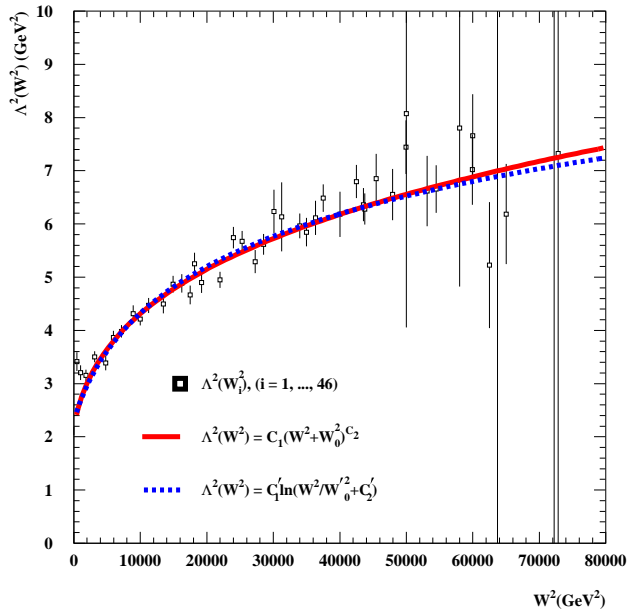


Fig. 3. The dependence of Λ^2 on W^2 , as determined by a fit of the GVD/CDP predictions for σ_{γ^*p} to the experimental data.

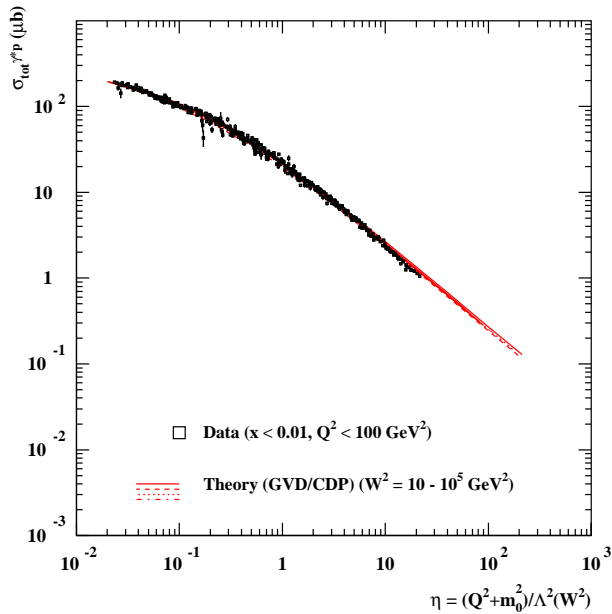


Fig. 4. The GVD/CDP scaling curve for σ_{γ^*p} compared with the experimental data for $x < 0.01$.

Identifying (18) with the $\Lambda^2 (W^2) r_\perp^2 \rightarrow 0$ form of $\sigma_{(q\bar{q})p}$ from (13), upon averaging over $z(1-z)$ as in (12)

$$\bar{\sigma}_{(q\bar{q})p} (r_\perp^2, W^2) = \sigma^{(\infty)} \frac{1}{24} r_\perp^2 \Lambda^2 (W^2) , \quad (19)$$

we deduce

$$xg(x, Q^2) \alpha_s(Q^2) = \frac{1}{8\pi^2} \sigma^{(\infty)} \Lambda^2 \left(\frac{Q^2}{x} \right) . \quad (20)$$

The functional behavior of $\Lambda^2 (W^2) = \Lambda^2 (Q^2/x)$ responsible for the $r_\perp^2 \rightarrow 0$ dependence of the color-dipole cross section thus determines (or provides a model for) the gluon density. We note that the result (18) is also obtained [11] by assuming gluon dominance at low x in DGLAP evolution [13], thus extracting the gluon distribution by taking the logarithmic derivative of the expression for the structure function F_2 corresponding to σ_{γ^*p} for $\eta \gg 1$ in (15). This explicitly demonstrates the consistency of the interpretation of the GVD/CDP in terms of the gluon density.

In figure 5 we show the gluon density obtained from (20) upon inserting the appropriate values of $\alpha_s(Q^2)$ from the PDG [14]. There is a remarkable consistency between our results in figure 5 and the results for the gluon density obtained in QCD fits by the H1 and ZEUS collaborations. More specifically, it is gratifying that the results in figure 5 are consistent with the ones of the H1 and ZEUS collaborations [15] based on the LO analysis [13]

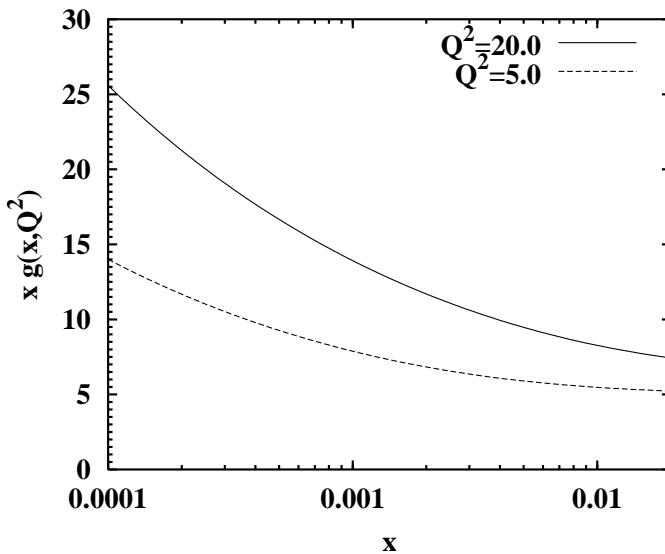


Fig. 5. The gluon density corresponding to the color-dipole cross section of the GVD/CDP for $x < 0.01$ including $Q^2 = 0$ photoproduction.

also used in our extraction of the gluon density. A comparison with the results of the more sophisticated NLO-QCD fit [16] reveals consistency with figure 5 for $x \cong 10^{-4}$. For $x \cong 10^{-2}$, the NLO-QCD fit lies below the LO analysis and, consequently, it lies somewhat below our results in figure 5.

The essential differences between the GVD/CDP presented here and related approaches [17,18] were briefly touched in [7]. As additional distinctive feature, we note our aforementioned straightforward connection between the GVD/CDP and the gluon density of the proton. A further remark concerns the scaling behavior of σ_{γ^*p} . From the above discussion, it is clear that scaling in Q^2/Λ^2 (W^2), assuming $Q^2 \gg m_0^2$ for simplicity, is intimately connected with the color-dipole approach. It is a consequence of the $r_\perp^2 \Lambda^2$ (W^2) dependence of the color-dipole cross section in (9). A different Ansatz for the color-dipole cross section, such as the one in Ref. [18] that contains an $r_\perp^2/R_0^2(x)$ dependence, accordingly, is bound to also imply a scaling behavior for σ_{γ^*p} , which is expected to be based on a different scaling variable. In [18] the scaling variable τ was found. For $Q^2 \neq 0$, the data in the presently explored kinematic domain do not discriminate between scaling in η and scaling in τ . It is the very existence of scaling that supports the color-dipole Ansatz for DIS at low x . The variable τ , however, does not allow one to consistently include $Q^2 = 0$ photoproduction.

In summary, we have shown that the HERA data on DIS in the low- x diffraction region, including $Q^2 = 0$ photoproduction, find a natural interpretation in the GVD/CDP that rests on the generic structure of two-gluon exchange from QCD. The gluon density that in the appropriate limit corresponds to the color-dipole cross section is consistent with the results from QCD fits. The cross sections for real and virtual photons on protons become identical in the limit of infinite energy.

It is a pleasure to thank G. Cvetic, B. Surrow, M. Tentyukov for a fruitful collaboration that led to the results reported here. The stimulating atmosphere at the workshop and the warm hospitality by our Polish friends and colleagues in Ustroń are gratefully acknowledged.

REFERENCES

- [1] T. Ahmed *et al.*, (H1 Collab.), *Nucl. Phys.* **B429**, 477 (1994); M. Derrick *et al.*, (ZEUS Collab.), *Phys. Lett.* **B315**, 481 (1993); R. Wichmann, (on behalf of ZEUS and H1 Collab.), *Nucl. Phys. B Proc. Suppl.* **82**, 268 (2000).
- [2] ZEUS Collab., DESY 01-097, [hep-ex/0107052](#).
- [3] J.J. Sakurai, D. Schildknecht, *Phys. Lett.* **B40**, 121 (1972); B. Gorczyca, D. Schildknecht, *Phys. Lett.* **B47**, 71 (1973).

- [4] H. Fraas, B.J. Read, D. Schildknecht, *Nucl. Phys.* **B86**, 346 (1975); R. Devenish, D. Schildknecht, *Phys. Rev.* **D19**, 93 (1976).
- [5] L. Stodolsky, *Phys. Rev. Lett.* **18**, 135 (1967); H. Joos, *Phys. Lett.* **B24**, 103 (1967).
- [6] M. Derrick *et al.*, (ZEUS Collab.), *Z. Phys.* **C72**, 399 (1996); J. Breitweg *et al.*, (ZEUS Collab.), *Eur. Phys. J.* **C7**, 609 (1999); J. Breitweg *et al.*, (ZEUS Collab.), *Phys. Lett.* **B407**, 432 (1997); J. Breitweg *et al.*, (ZEUS Collab.), *Phys. Lett.* **B487**, 53 (2000); C. Adloff *et al.*, (H1 Collab.), *Nucl. Phys.* **B497**, 3, (1997); S. Aid *et al.*, (H1 Collab.), *Nucl. Phys.* **B470**, 3 (1996).
- [7] D. Schildknecht, B. Surrow, M. Tentyukov, *Phys. Lett.* **B499**, 116 (2001); D. Schildknecht, *Nucl. Phys. B Proc. Suppl.* **B99**, 121 (2001); G. Cvetic, D. Schildknecht, B. Surrow, M. Tentyukov, *Eur. Phys. J.* **C20**, 77 (2001).
- [8] G. Cvetič, D. Schildknecht, A. Shoshi, *Eur. Phys. J.* **C13**, 301 (2000); *Acta Phys. Pol.* **B30**, 3265 (1999); D. Schildknecht, Contribution to DIS 2000, Liverpool, April 2000, Eds. J.A. Gracey and T. Greenshaw, World Scientific 2001, p. 111; [hep-ph/0006153](#).
- [9] F.E. Low, *Phys. Rev.* **D12**, 163 (1975); S. Nussinov, *Phys. Rev. Lett.* **34**, 1286 (1975); *Phys. Rev.* **D14**, 246 (1976); J. Gunion, D. Soper, *Phys. Rev.* **D15**, 2617 (1977).
- [10] N.N. Nikolaev, B.G. Zakharov, *Z. Phys.* **C49**, 607 (1991).
- [11] D. Schildknecht, B. Surrow, M. Tentyukov, *Mod. Phys. Lett.* **A16**, 1829 (2001); ([hep-ph/0107178](#)); D. Schildknecht, presented at DIS 2001, Bologna, Italy, April–May, 2001, [hep-ph/0106188](#).
- [12] L. Frankfurt, A. Radyushkin, M. Strikman, *Phys. Rev.* **D55**, 98 (1997).
- [13] K. Prytz, *Phys. Lett.* **B311**, 286 (1993).
- [14] Particle Data Group, *Eur. Phys. J.* **C3**, 81 (1998).
- [15] A.M. Cooper-Sarkar, R.C.E. Devenish, A. de Rock, *Int. J. Mod. Phys.* **A13**, 3385 (1998).
- [16] H1 Collab., *Eur. Phys. J.* **C21**, 33 (2001); ZEUS Contributed paper 628 to the European Internat. Conf. on High Energy Physics, Budapest 2001.
- [17] J. Forshaw, G. Kerley, G. Shaw, *Phys. Rev.* **D60**, 074012 (1999).
- [18] K. Golec-Biernat, M. Wüsthoff, *Phys. Rev.* **D59**, 014017 (1999); *Phys. Rev.* **D60**, 114023 (1999); A.M. Stasto, K. Golec-Biernat, J. Kwiecinski, *Phys. Rev. Lett.* **86**, 596 (2001).