SUPERSYMMETRIC LEPTON FLAVOUR VIOLATION AT e^+e^- LINEAR COLLIDERS*

Jan Kalinowski

Institute of Theoretical Physics, Warsaw University
Hoża 69, 00-681 Warszawa, Poland

(Received October 16, 2001)

The neutrino experiment results suggest that neutrinos have finite masses and violate lepton flavour. In supersymmetric extensions of the Standard Model, the Yukawa and/or mass terms of the heavy neutrinos can generate radiatively the lepton flavour violating slepton masses. These new sources of lepton flavour violation may enhance the rates of charged lepton flavour violating processes, like $\mu \to e\gamma$. At future colliders they can also generate distinct final states, *e.g.* $\tau\mu + \text{jets} + \not{E}_T$. In this paper the supersymmetric lepton flavour violating processes and their signals at future e^+e^- colliders are discussed.

PACS numbers: 12.60.Jv, 14.80.Ly

1. Introduction

Earlier observations on solar neutrinos [1], the studies of the Super-Kamiokande [2] and recent results of the SNO [3] have a natural explanation in terms of neutrino oscillations. This phenomenon is Lepton Flavour Violating (LFV) and it is a first direct evidence for physics beyond the Standard Model (SM). The simplest model to account for the neutrino masses and their oscillations includes three very heavy singlet neutrinos N_i. Their Yukawa couplings to the light neutrinos ν_i explain the smallness of m_{ν_i} via the see-saw mechanism [4] in a natural way if the heavy neutrino masses are assumed in the range $M_N \sim 10^{14}$–10^{15} GeV. At the same time the flavour oscillation of light neutrinos originates from non-diagonal elements of the Yukawa couplings and/or non-diagonal mass matrix of the heavy neutrinos [5]. Radiative corrections due to these interactions also induce

* Presented at the XXV International School of Theoretical Physics "Particles and Astrophysics — Standard Models and Beyond", Ustroń, Poland, September 10–16, 2001.

flavour mixing in the charged lepton sector. An interesting question then arises whether processes with charged-lepton flavour violation, like $\mu \to e\gamma$, $\tau \to \mu\gamma$, μ-e conversion in nuclei, rare K decays *etc.*, can be generated at observable rates.

Unfortunately, due to the smallness of neutrino masses, these processes in the Standard Model are strongly suppressed via the GIM mechanism. In addition, the presence of the high energy scale M_N requires a mechanism to stabilize the electroweak scale against radiative corrections. This is commonly achieved by supersymmetrizing the theory. In the supersymmetric extension of the Standard Model with heavy scalar neutrinos, however, the situation of LFV processes may be quite different. In addition to the SM mechanism, new sources of flavour violation in the leptonic sector can be generated by non-diagonal soft supersymmetry breaking terms, *e.g.*

$$\mathcal{L}_{\text{soft}} = m^2_{Lij}\tilde{l}^*_i\tilde{l}_j + m^2_{Rij}\tilde{e}^*_i\tilde{e}_j + A^e_{ij}\tilde{e}^*_i h_1 \tilde{l}_j + \dots , \tag{1}$$

where only scalar mass and trilinear terms in the leptonic sector have been written explicitly using self-explanatory notation. As a result, virtual superpartner loops may enhance the rates of charged lepton flavour violating processes. Of course, once superpartners are discovered, it will be possible to probe lepton flavour violation directly in their production and decay [6]. For example, it has been demonstrated that sneutrino or charged slepton pair production at future e^+e^- (and/or $\mu^+\mu^-$) colliders may provide a more powerful tool to search for Supersymmetric Lepton Flavour Violation (SLFV) than the rare decay processes [7–9]. In some favourable scenarios signals of SLFV can also be searched for at the LHC [10].

The outline of the paper is as follows. In Section 2 we start with a brief review of global symmetries and lepton flavour violation in the Standard Model. Current experimental limits from rare decay processes are discussed in Section 3 as an indication of possible energy scales of lepton flavour violation. Then in Section 4 we discuss in some detail the lepton flavour violation mechanisms in the supersymmetric model, and in Section 5 their possible signals at future colliders. In Section 6, which is based on Ref. [11], we note that sneutrinos and charged sleptons may not only be directly pair-produced in e^+e^- collisions, but can also be decay products of other supersymmetric particles, like charginos and neutralinos, decaying via cascades. We find that off-diagonal chargino pair-production, overlooked earlier, can make a significant contribution to the SLFV signal. We conclude in Section 7.

2. Lepton Flavour Violation in the Standard Model

In the Standard Model renormalizability and gauge invariance restricts the Yukawa interactions of quarks, charged leptons and left-handed neutrinos to the form

$$\mathcal{L}_{\rm Y} = y_e^{ij} h \bar{l}_i e_j + y_d^{ij} h \bar{q}_i d_j + y_u^{ij} h^c \bar{q}_i u_j \,, \tag{2}$$

where i and j are generation indices. By unitary rotations in the generation space the leptonic Yukawa couplings y_e^{ij} can be diagonalized implying individual lepton flavour numbers L_e, L_μ and L_τ to be conserved in addition to the conserved baryon B and total lepton L numbers. These conservation laws are consequences of global symmetries of the Standard Model which are "accidental" in the sense that they follow from the spin and gauge quantum number assignments of the SM fields.

Global symmetries however are not sacred: even within the SM they are broken by non-perturbative sphaleron effects — only the $B-L$ remains conserved. Black holes are also expected to break all global symmetries. Since these mechanisms of global symmetry breaking are extremely weak, the existence of neutrino oscillations calls for physics beyond the SM. New interactions due to new gauge bosons, fermions and/or scalars may also lead to violation of global symmetries. For example, adding the right-handed heavy neutrinos N_i brings a new scale M_N and new Yukawa interactions

$$y_\nu^{ij} h^c \bar{l}_i N_j \tag{3}$$

and, as a result, the violation of charged-lepton flavour is induced in the manner parallel to the quark sector. Unfortunately, due to small neutrino masses $m_{\nu_i} \sim 0.1$ eV, which require a high $M_N \sim 10^{14}$–10^{15} GeV scale, the charged-lepton flavour violating processes are also strongly suppressed, *e.g.* $\mathrm{BR}\,(\mu \to e\gamma) \sim 10^{-48}$ [12]. With the SM particles alone at the weak scale this approach produces desired neutrino properties with no other experimentally interesting implications. However, other extensions of the SM, like supersymmetry, can provide significant enhancement of LFV processes leading to observable rates at future experiments. Before we turn to the minimal extension of the Standard Model (MSSM) let us discuss possible scales of LFV processes which are consistent with present experiments.

3. Energy scales of Lepton Flavour Violation

It is ironic that the "accidental" global symmetries are so accurately observed in nature, while local gauge symmetries are broken. To discuss the experimental limits on transitions due to global symmetry breaking it is convenient to consider the SM as an effective theory valid up to some energy scale $\sim M$. The effects of physics above the scale M can be accounted for

by a series of non-renormalizable operators of dimension greater than 4 and suppressed by powers of M. The terms in Eq. (2) serve as a first term in the series, with higher in $1/M$ non-renormalizable operators:

$$\text{(a)}: \frac{\bar{q}q\bar{q}\bar{l}}{M^2}, \qquad \text{(b)}: \frac{hhll}{M}, \qquad \text{(c)}: \frac{\bar{l}l\bar{e}e}{M^2}, \frac{h\bar{l}\sigma^{\alpha\beta}eF_{\alpha\beta}}{M^2}, \tag{4}$$

where the first operator (a) breaks B and L, the second (b) breaks L and the last two (c) conserve L but break individual L_i (the generation indices are understood). The experimental status of current limits derived from non-observation of processes violating global quantum numbers can conveniently be summarized by determining the corresponding scale M:

- The most stringent limit on the baryon number violating operator (4a) is derived from the proton life time $\tau_p > 10^{32}$ y, which translates into the limit $M > 10^{15}$ GeV.

- The non-zero neutrino masses $m_\nu \sim 0.1$ eV set the scale of the total lepton number violating operator (4b). Assuming that the Higgs field acquires the vacuum expectation value $\langle h \rangle \sim m_W$ when the electroweak gauge symmetry is broken, the scale of L violation is derived to be of the order $M \sim 10^{14}$ GeV.

- Neutrino oscillations imply the lepton flavour violation as well. However, other L-conserving, but L_i-violating processes in the charged lepton sector, like

  — $\mathrm{BR}(Z \to \mu e) < 1.7 \times 10^{-6}$ [15],
  — $\mathrm{BR}(Z \to \tau e) < 9.8 \times 10^{-6}$ [15],
  — $\mathrm{BR}(Z \to \tau \mu) < 1.2 \times 10^{-5}$ [16],
  — $\mathrm{BR}(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [17],
  — $\mathrm{BR}(\mu \to 3e) < 1.0 \times 10^{-12}$ [18],
  — $\mathrm{BR}(\mu \to e) < 6.1 \times 10^{-13}$ [19],
  — $\mathrm{BR}(\tau \to \mu\gamma) < 1.1 \times 10^{-6}$ [20],

  currently do not exclude an exciting possibility that the scale of LFV processes might be of the order $\sim 10^3 - 10^4$ GeV, that is much lower than the scales of baryon and total lepton number violation mechanisms. New experiments on $\mu \to e\gamma$ [21], $\mu \to e$ [22] or $Z \to ll'$ [23] will improve limits on M by an order of magnitude.

Thus the experimental data seem to suggest an interesting possibility of the hierarchy between

- the LFV processes which might be governed by a low scale (not much above the electroweak scale)
- the B and L violating processes which have to be governed by a high scale ($\sim M_{\rm GUT},\ M_{\rm Pl}$)

in parallel to the conventional gauge hierarchy between the electroweak and the GUT or Planck scales.

From the theoretical model-building point of view, one can consider the following scenarios:

- SM-like: all global symmetries are broken at a high scale. In this case the LFV processes are strongly suppressed and far from the reach of laboratory explorations, with the only exception of neutrino oscillations and proton decay experiments.
- Large extra dimensions: this idea has been suggested as a solution of the gauge hierarchy problem. In this approach the Planck scale is an artifact of extra dimensions, and the "true" scale of gravity is of the order of electroweak scale. In this case all global symmetries are broken at a low scale. As a result, the proton decays very fast and it is a challenge in such a scenario to find an elegant solution to stabilize the proton.
- Supersymmetry: the hierarchy between the electroweak scale and the Planck scale is stabilized by introducing a superpartner for every SM particle. By the same token the hierarchy between the B and L violation and the LFV processes can arise naturally, as discussed in the next chapter.

4. Lepton Flavour Violation in the MSSM

In supersymmetric models, the hierarchy between L_i flavour violation on the one hand and B and L violation on the other arises as a natural consequence of the basic motivations for supersymmetry, namely the gauge hierarchy and the existence of dark matter. The introduction at the weak scale of a superpartner to every particle allows us to write additional renormalizable and gauge-invariant interactions. Using capital letters to denote superfields, the Yukawa superpotential is given by

$$\begin{aligned} W \ = \ & y_e H_1 L E + y_d H_1 Q D + y_u H_2 Q U \\ & + \lambda L L E + \lambda' L Q D + \lambda'' U D D \,, \end{aligned} \tag{5}$$

where generation indices are suppressed. The interactions of Eq. (2) are recovered by choosing terms with two fermions (quarks or leptons) and a

scalar Higgs from the first line (with the only exception that supersymmetry requires two Higgs fields). However, any combination of two fermions and one scalar from any term in Eq. (5) is allowed in the Lagrangian. As a result, global B and L symmetries of the Standard Model are broken since the terms in the second line of (5) violate both B and L. The presence of all λ-type terms at the electroweak scale not only implies fast proton decay but also allows all superpartners to decay eliminating the possibility of supersymmetric dark matter. The proton and the lightest supersymmetric particle can be made stable by requiring R-parity conservation, where $R \equiv (-1)^{B+L+2S}$, which eliminates all interactions in the second line of Eq. (5). At the same time B and L are again allowed to be broken only at a high scale.

The remaining soft supersymmetry breaking interactions, however, still allow L_i-violating processes at the weak scale, leading to the hierarchy discussed above. The most important for our discussion are slepton masses

$$\tilde{l}^*_{Li} m^2_{Lij} \tilde{l}_{Lj} + \tilde{e}^*_{Ri} m^2_{Rij} \tilde{e}_{Rj} \,, \tag{6}$$

where the $m^2_{L,R}$ are *a priori* arbitrary Hermitian matrices, and trilinear terms

$$A^e_{ij} \tilde{e}^*_i h_1 \tilde{l}_j \quad \Rightarrow \quad m^2_{LRij} \tilde{e}^*_i \tilde{e}_j \tag{7}$$

which, after electroweak symmetry breaking, couple left- and right-handed charged sleptons through the mass matrix $m^2_{LRij} \equiv A^e{}_{ij} \langle h_1 \rangle$.

These terms lead to charged LFV, as can easily be seen in the basis in which the leptons are already rotated to diagonalize the lepton Yukawa coupling, and the sleptons are rotated to preserve flavour-diagonal gaugino couplings. No additional freedom in flavour space is then left to diagonalize the terms of Eqs (6) and (7). If we now work in the mass eigenstate basis for all fields, the slepton-mass diagonalization matrices $W_{i\alpha}$, where $m^2_{\tilde{l}} = W^\dagger m^2_{\tilde{l}\,\mathrm{diag}} W$, enter the chargino and neutralino couplings

$$\tilde{e}_i W^*{}_{i\alpha} \bar{e}_\alpha \tilde{\chi}^0 + \tilde{\nu}_i W^*{}_{i\alpha} \bar{e}_\alpha \tilde{\chi}^- + \dots \,, \tag{8}$$

where the Latin and Greek subscripts are generational indices for scalars and fermions, respectively.

Non-trivial W matrices violate lepton flavour conservation and may give sizeable contributions to rates for rare processes such as $\mu \to e\gamma$. For nearly degenerate sleptons, however, these contributions are suppressed through the superGIM mechanism by $\Delta m_{\tilde{l}} / \bar{m}_{\tilde{l}}$ with the mass difference $\Delta m_{\tilde{l}}$ and the average mass $\bar{m}_{\tilde{l}}$ of the sleptons; in the limit of exact degeneracy, the W matrices can be rotated to the unit matrix, and the gaugino interactions conserve lepton flavour. Actually, a high degree of degeneracy of the first

two generation of sleptons is needed in order to evade bounds from $\mu \to e\gamma$ if the mixing between them is not very small. This is an example of the supersymmetric flavour-changing problem.

Even if the off-diagonal slepton mass terms are assumed to vanish at tree level to avoid the supersymmetric flavour-changing problem, they can be induced radiatively in the framework of a seesaw mechanism by the right-handed heavy neutrinos [13]. In such a case a substantial $\nu_\mu - \nu_\tau$ mixing leads to [7, 14] large $\tilde{\mu}_L - \tilde{\tau}_L$ and $\tilde{\nu}_\mu - \tilde{\nu}_\tau$ mixings. In the rest of the paper we will not discuss these theories but we will concentrate on the question how well SLFV can be probed in a model independent way at future e^+e^- colliders .

To be more specific, we take a pure 2–3 intergeneration mixing between $\tilde{\nu}_\mu$ and $\tilde{\nu}_\tau$, generated by a near-maximal mixing angle θ_{23}, and ignore any mixings with $\tilde{\nu}_e$. This means that scalar mass matrices are not diagonal in the same basis as fermion mass matrices. For example, the scalar neutrino mass matrix $m^2_{\tilde{\nu}}$, restricted to the 2–3 generation subspace, can be written in the fermion mass-diagonal basis as

$$m^2_{\tilde{\nu}} = \begin{pmatrix} \cos\theta_{23} & -\sin\theta_{23} \\ \sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} m_{\tilde{\nu}_2} & 0 \\ 0 & m_{\tilde{\nu}_3} \end{pmatrix} \begin{pmatrix} \cos\theta_{23} & \sin\theta_{23} \\ -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}, \qquad (9)$$

where $m_{\tilde{\nu}_2}$ and $m_{\tilde{\nu}_3}$ are the physical masses of $\tilde{\nu}_2$ and $\tilde{\nu}_3$ respectively. In the following we take the mixing angle θ_{23} and $\Delta m_{23} = |m_{\tilde{\nu}_2} - m_{\tilde{\nu}_3}|$ as free, independent parameters. The same goes for the charged slepton sector, modulo standard LR mixing, where θ_{23} and Δm_{23} are then the corresponding parameters for charged sleptons. A parallel discussion for the e-τ mixing case [24] is obtained by replacing μ by e everywhere.

5. Collider signals of Supersymmetric Lepton Flavour Violation

Once supersymmetric particles are discovered, it will be possible to probe lepton flavour violation directly in their production and decay processes at future colliders. A flavour-violating signal is generated by the production of real sleptons, followed by their oscillation into a different flavoured slepton, and subsequent decay to a lepton. For example, at e^+e^- colliders events with

$$\tau\mu + 4j + \not{E}_T\,, \qquad \tau\mu l + 2j + \not{E}_T\,, \qquad \tau\mu l\bar{l} + 2j + \not{E}_T \qquad (10)$$

can be expected. Searching for such signals has several advantages: first these processes are at tree level while rare decays are generated by loop corrections. Second, the SLFV processes in decays of sleptons are suppressed only as $\Delta m_{\tilde{l}}/\Gamma_{\tilde{l}}$ [8], where $\Gamma_{\tilde{l}}$ is the slepton decay width, as compared to the

$\Delta m_{\tilde{l}}/\bar{m}_{\tilde{l}}$ for rare decays. Since $\bar{m}_{\tilde{l}}/\Gamma_{\tilde{l}}$ is typically of the order 10^2–10^3, one may expect spectacular signals for possible discovery in future e^+e^-, $\mu^+\mu^-$ or pp collider experiments. Last, but not least, the SM background with two or more leptons with different flavours is quite small.

In discussing SLFV collider signals one has to consider two cases in which oscillation of lepton flavour can occur in processes with single or correlated slepton pair production processes. To derive expressions for the signal cross sections we will follow the field-theoretic method of Ref. [8]

5.1. Single slepton production

Processes involving single slepton production may be produced in cascade decays of heavier superparticles. They are very important for hadron colliders, where single slepton can be a product of a gluino or squark decay, but they may also be relevant for lepton colliders where single slepton can be a product of chargino or neutralino decay. The amplitude for such a process, $f\,f' \to e^+_\alpha\, X\, \tilde{e}^-_i \to e^+_\alpha\, X\, e^-_\beta\, Y$, has the form

$$\mathcal{M}_{\alpha\beta} = \sum_i \mathcal{M}_{\rm P} W_{i\alpha} \frac{i}{q^2 - m_i^2 + i m_i \Gamma_i} W^*_{i\beta} \mathcal{M}_{\rm D}\,, \tag{11}$$

where $\mathcal{M}_{\rm P}$ and $\mathcal{M}_{\rm D}$ are the amplitudes for production and decay of the slepton $\tilde{e}_i$ in the absence of LFV, and $W_{i\alpha}$ is the lepton-flavour mixing matrix element. Summing over all slepton generations that can be produced on-shell and assuming negligible generation dependence of these amplitudes, the cross section can schematically be written as

$$\sigma_{\alpha\beta} = \int dq^2 \sum_{ij} W_{i\alpha} W^*_{i\beta} W_{j\beta} W^*_{j\alpha}\, A_{ij}(q^2) \;\times [\text{production}] \times [\text{decay}]\,. \tag{12}$$

For nearly degenerate in mass and narrow sleptons, Δm^2_{ij}, $m\Gamma \ll m^2$, the product of slepton propagators

$$A_{ij}(q^2) = \frac{i}{q^2 - m_i^2 + im\Gamma}\, \frac{i}{q^2 - m_j^2 + im\Gamma} \tag{13}$$

can be simplified as

$$A_{ij}(q^2) \sim \frac{1}{1 + i\,\frac{\Delta m}{\Gamma}}\, \frac{\pi}{m\Gamma}\, \delta(q^2 - m^2)\,, \tag{14}$$

where $m^2 = (m_i^2 + m_j^2)/2$ and $\Delta m^2_{ij} = (m_i^2 - m_j^2)/2 \sim 2m\Delta m$. In the case of 2–3 intergeneration mixing

$$W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \tag{15}$$

we arrive finally at

$$\sigma_{\alpha\beta} = \chi_{23} \sin^2 2\theta_{23} \times \sigma_0 \times \varepsilon_{\rm BR}\,, \tag{16}$$

where

$$\chi_{23} = \frac{x_{23}^2}{2(1+x_{23}^2)}\,, \qquad x_{23} = \frac{\Delta m_{23}}{\Gamma} \tag{17}$$

and $\sigma_0 \times \varepsilon_{\rm BR} = \sigma(f\,f' \to e_\alpha^+\,X\,\tilde{e}_\alpha^-) \times {\rm BR}\,(\tilde{e}_\alpha^- \to e_\alpha^-\,Y)$ is the analogous cross section in the absence of flavour violation.

5.2. Correlated slepton pair production

Correlated slepton pair production is the dominant slepton production mechanism at lepton colliders, but it may also occur at hadron colliders when sleptons are produced via Drell–Yan process. Assuming for simplicity only the s-channel production mechanism, the amplitude for the process $\bar{f}\,f \to \tilde{e}_i^+\,\tilde{e}_i^- \to e_\alpha^+\,X\,e_\beta^-\,Y$ has the form

$$\mathcal{M}_{\alpha\beta} = \sum_i \mathcal{M}_{\rm P} \frac{i}{q^2 - m_i^2 + i m_i \Gamma_i} W_{i\alpha} \mathcal{M}_{\rm D}^+ \frac{i}{p^2 - m_i^2 + i m_i \Gamma_i} W_{i\beta}^* \mathcal{M}_{\rm D}^-\,, \tag{18}$$

where $\mathcal{M}_{\rm P}$ is the production amplitude for $\bar{f}\,f \to \tilde{e}_i^+\,\tilde{e}_i^-$ and $\mathcal{M}_{\rm D}^\pm$ are the decay amplitudes for $\tilde{e}^\pm$. Applying the same approximations as in the previous case, the cross section can be written as

$$\sigma_{\alpha\beta} = \int \sum_{ij} W_{i\alpha} W_{i\beta}^* W_{j\beta} W_{j\alpha}^*\, A_{ij}(q^2) A_{ij}(p^2)\ \times [\text{production}] \times [\text{decay}] \tag{19}$$

and, again for the 2–3 intergeneration mixing of Eq. (15), we finally have

$$\sigma_{\alpha\beta} = \chi_{23}(3 - 4\chi_{23}) \sin^2 2\theta_{23} \times \sigma_0 \times \varepsilon_{\rm BR}\,, \tag{20}$$

where $\sigma_0 \times \varepsilon_{\rm BR} = \sigma(\bar{f}\,f \to \tilde{e}_\alpha^+\,\tilde{e}_\alpha^-) \times {\rm BR}\,(\tilde{e}_\alpha^+ \to e_\alpha^+\,X) \times {\rm BR}\,(\tilde{e}_\alpha^- \to e_\alpha^-\,Y)$ is the analogous cross section in the absence of flavour violation.

6. The role of charginos

At e^+e^- linear colliders the SLFV signals can be looked for in decays of sleptons which are produced in pairs, for example

$$\begin{aligned} e^+e^- &\to \tilde{\ell}_i^-\tilde{\ell}_i^+ \to \tau^+\mu^-\tilde{\chi}_1^0\tilde{\chi}_1^0\,, \\ e^+e^- &\to \tilde{\nu}_i\tilde{\nu}_i^c \to \tau^+\mu^-\tilde{\chi}_1^+\tilde{\chi}_1^- \end{aligned} \tag{21}$$

with $i = 2, 3$. These processes have been discussed in detail, showing that they may be competitive to rare decay processes in searches for SLFV signals [7, 9].

However, if the chargino $\tilde{\chi}_2^{\pm}$ is not much heavier, as is the case in a substantial region of the MSSM parameter space, then off-diagonal chargino or neutralino pair production $e^+e^- \to \tilde{\chi}_1^{\pm}\tilde{\chi}_2^{\mp}$, $\tilde{\chi}_1^0\tilde{\chi}_2^0$ can take place. The heavier chargino and/or neutralino can decay via the SLFV chain,

$$e^+e^- \to \tilde{\chi}_2^+\tilde{\chi}_1^- \to \tau^+\mu^-\tilde{\chi}_1^+\tilde{\chi}_1^-\,, \tag{22}$$

$$e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_1^0 \to \tau^+\mu^-\tilde{\chi}_1^0\tilde{\chi}_1^0\,, \tag{23}$$

where $\tilde{\chi}_1^{\pm} \to \tilde{\chi}_1^0 f\bar{f}'$, and $\tilde{\chi}_1^0$ escapes detection. The signature therefore would be the same as in slepton pair production, *i.e.* $\tau^{\pm}\mu^{\mp}$ + jets + $\not{E}_T$, $\tau^{\pm}\mu^{\mp} + \ell + \not{E}_T$, or $\tau^{\pm}\mu^{\mp} + \not{E}_T$, depending on hadronic or leptonic $\tilde{\chi}_1^{\pm}$ decay mode. As a result, the processes (22) and (23) provide a new source for the signal in addition to those discussed in Ref. [9]. Moreover, the production of two charginos in e^+e^- collision has both the s-channel and t-channel exchange contributions and hence expected somewhat larger cross sections at higher collider energies. Other production processes, like $\tilde{\chi}_2^{\pm}\tilde{\chi}_2^{\mp}$, $\tilde{\chi}_i^0\tilde{\chi}_j^0$, may also be open at higher energies (depending on the mass pattern) and contribute to the same final states not only via the SLFV mechanisms, but through lepton flavour conserving decay chains as well contributing thereby to the background.

To illustrate the role of charginos for the SLFV process at an e^+e^- linear collider, such as the proposed TESLA, we estimate the signal and background rates at $\sqrt{s} = 500$ GeV for one of the representative points [23] in the MSSM parameter space. This point is given in terms of a mSUGRA scenario defined by:

$$m_0 = 100\,; \quad M_{1/2} = 200\,; \quad A_0 = 0\,; \quad \tan\beta = 3\,; \quad \mathrm{sgn}(\mu) = + \tag{24}$$

which has been chosen for detailed case studies at the ECFA/DESY linear collider workshop. Here the masses and A_0 are in GeV, and standard notation is used. The corresponding masses of chargino, neutralino and slepton states relevant for SLFV processes at $\sqrt{s} = 500$ GeV, along with some branching ratios, are listed in the Table (the branching ratios are given for the case of no slepton mixing)

As it turns out that processes with one or both $\tilde{\chi}_1^{\pm}$ decaying leptonically in (21)–(23) are overwhelmed by background, in our analyses we consider only their hadronic decay modes. Given the mass spectrum, the off-diagonal $\tilde{\chi}_1^{\pm}\tilde{\chi}_2^{\mp}$ pair is the only possibility for the SLFV signal in chargino production at $\sqrt{s} = 500$ GeV. The slepton flavour violation can occur in the heavier chargino $\tilde{\chi}_2^{\mp}$ cascade decay chain as shown

S1: $e^+e^- \to \tilde{\chi}_2^\pm \tilde{\chi}_1^\mp$, $\tilde{\chi}_2^+ \to \tau^+ \tilde{\nu}_{2,3}$, $\tilde{\nu}_{2,3} \to \mu^- \tilde{\chi}_1^+$,

S2: $e^+e^- \to \tilde{\chi}_2^\pm \tilde{\chi}_1^\mp$, $\tilde{\chi}_2^+ \to \mu^+ \tilde{\nu}_{2,3}$, $\tilde{\nu}_{2,3} \to \tau^- \tilde{\chi}_1^+$,

followed by $\tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 + q + \bar{q}'$. There is another sequence with the charges reversed. The other process for the same final state, which was discussed in [9], is the following:

S3: $e^+e^- \to \tilde{\nu}_i \tilde{\nu}_i^c$, $\tilde{\nu}_i \to \tilde{\chi}_1^- \tau^+$, $\tilde{\nu}_i^c \to \tilde{\chi}_1^+ \mu^-$,

where $i = 2, 3$. Notice that in [S1] and [S2] the slepton flavour violating decay occurs in two ways leading to the the same final state so that eventually the signal rate gets doubled.

TABLE

The masses (in GeV) and the branching ratios (only for significant decay modes) for light supersymmetric particles which are relevant to our study. No slepton mixing is assumed. ℓ denotes e or μ, and τ unless the entry for τ is explicitly shown.

	Mass	Decay	BR		Mass	Decay	BR
$\tilde{\chi}_1^+$	128	$\tilde{\chi}_1^0 \ell^+ \nu_\ell$	0.15×3	$\tilde{\chi}_1^0$	72		
		$\tilde{\chi}_1^0 q \bar{q}'$	0.56				
$\tilde{\chi}_2^+$	346	$\tilde{\chi}_2^0 W^+$	0.29	$\tilde{\chi}_2^0$	130	$\tilde{\chi}_1^0 \tau^+ \tau^-$	0.24
		$\tilde{\chi}_1^+ Z$	0.22			$\tilde{\chi}_1^0 e^+ e^-$	0.20
		$\tilde{\chi}_1^+ h$	0.14			$\tilde{\chi}_1^0 \mu^+ \mu^-$	0.20
		$\tilde{t}_1 \bar{b}$	0.14			$\tilde{\chi}_1^0 \nu_\ell \bar{\nu}_\ell$	0.04×3
		$\tilde{\ell}^+ \nu_\ell$	0.04×3				
		$\ell^+ \tilde{\nu}_\ell$	0.03×3				
$\tilde{\ell}_L^-$	176	$\tilde{\chi}_1^- \nu_\ell$	0.53	$\tilde{\nu}_\ell$	161	$\tilde{\chi}_1^+ \ell^-$	0.48
		$\tilde{\chi}_2^0 \ell^-$	0.32			$\tilde{\chi}_2^0 \nu_\ell$	0.12
		$\tilde{\chi}_1^0 \ell^-$	0.15			$\tilde{\chi}_1^0 \nu_\ell$	0.40
$\tilde{\tau}_1^-$	131	$\tilde{\chi}_1^0 \tau^-$	1.00				
$\tilde{\tau}_2^-$	177	$\tilde{\chi}_1^- \nu_\tau$	0.53	$\tilde{\nu}_\tau$	161	$\tilde{\chi}_1^+ \tau^-$	0.48
		$\tilde{\chi}_2^0 \tau^-$	0.31			$\tilde{\chi}_2^0 \nu_\tau$	0.12
		$\tilde{\chi}_1^0 \tau^-$	0.16			$\tilde{\chi}_1^0 \nu_\tau$	0.40

The background may originate from the flavour-conserving SUSY processes:

B1: $e^+e^- \to \tilde{\chi}_2^+\tilde{\chi}_1^-$, $\tilde{\chi}_2^+ \to \tau^+\tilde{\nu}_\tau$, $\tilde{\nu}_\tau \to \tau^-(\to \mu^-)\tilde{\chi}_1^+$,

B2: $e^+e^- \to \tilde{\chi}_2^+\tilde{\chi}_1^-$, $\tilde{\chi}_2^+ \to \tau^+(\to \mu^+)\tilde{\nu}_\tau$, $\tilde{\nu}_\tau \to \tau^-\tilde{\chi}_1^+$,

B3: $e^+e^- \to \tilde{\nu}_i\tilde{\nu}_i^c$, $\tilde{\nu}_i \to \tilde{\chi}_1^-\tau^+$, $\tilde{\nu}_i^c \to \tilde{\chi}_1^+\tau^-(\to \mu^-)$,

B4: $e^+e^- \to \tilde{\tau}_2^+\tilde{\tau}_2^-$, $\tilde{\tau}_2^+ \to \tau^+\tilde{\chi}_2^0$, $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\tau^+(\to \text{jets})\tau^-(\to \mu^-)$, $\tilde{\tau}_2^- \to \nu_\tau\tilde{\chi}_1^-$.

Note that in all background cases B1–B4 the μ comes from τ decay after the $\tau\tau X$ events are produced. Allowing two quark jets to overlap, an important SM background to the final states with $\tau^\pm\mu^\mp + \geq 3\text{jets} + \not{E}_T$ also comes from

B5: $e^+e^- \to t\bar{t}g$.

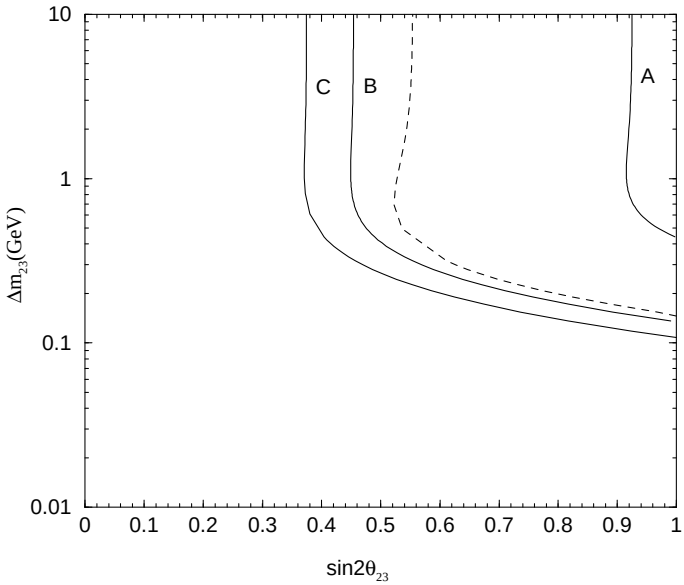


Fig. 1. The significance contours (for the SUSY point mentioned in the text) in $\Delta m_{23} - \sin 2\theta_{23}$ plane for $\sqrt{s}$ =500 GeV and for different luminosity options, contours A, B and C being for 50 fb^{-1}, 500 fb^{-1} and 1000 fb^{-1}, respectively. The dashed line is for only $\tilde{\nu}\tilde{\nu}^c$ contribution with luminosity 500 fb^{-1}. The upper-right side of these contours can be explored or ruled out at the 3σ level.

To assess the sensitivity to the SLFV processes the signal and background events have been selected using simple parton level simulation with a number of simple kinematic cuts listed in [11]. Using Poisson distributions, the significance is given by $\sigma_d = \frac{N}{\sqrt{N+B}}$ where N and B is the number of signal and background events respectively for a given luminosity. Fig. 1 shows the region (to the right of the curve) in the Δm_{23}–$\sin 2\theta_{23}$ plane that can be explored or ruled out at a 3σ level by the linear collider of energy 500 GeV for the given integrated luminosity. The contours are drawn for three luminosity options, namely 50 fb^{-1}, 500 fb^{-1} and 1000 fb^{-1}, whereas the dashed line shows the reach of the process $\tilde{\nu}_i\tilde{\nu}_i^c$ alone using our cuts and assuming luminosity of 500 fb^{-1}. Comparing the dashed line with line B we see that that the chargino contribution, [S1] and [S2], increases the sensitivity range to $\sin^2\theta_{23}$ by 10–20 % while the sensitivity to Δm_{23} does not change appreciably.

7. Conclusions

Neutrino oscillations imply the violation of individual lepton flavour numbers and raise an interesting possibility of observing processes with a violation of lepton flavour between two charged leptons. Current experimental data seem to suggest a hierarchy between the energy scales that govern the lepton flavour violation and the B or L breaking. Supersymmetry provides a natural framework for such a hierarchy and many interesting signals of SLFV processes may be expected at future colliders.

I am grateful to M. Guchait and P. Roy for many stimulating discussions. The work was supported by the Polish State Committee for Scientific Research (KBN) Grant no. 5 P03B 119 20 (2001-2002) and the European Commission 5-th framework contract HPRN-CT-2000-00149.

REFERENCES

[1] R.J. Davis, D.S. Harmer, K.C. Hoffman, *Phys. Rev. Lett.* **20**, 1205 (1968).

[2] S. Fukuda *et al.* [SuperKamiokande Collaboration], *Phys. Rev. Lett.* **86**, 5656 (2001); *Phys. Rev. Lett.* **85**, 3999 (2000), and references therein.

[3] Q.R. Ahmad *et al.* [SNO Collaboration], *Phys. Rev. Lett.* **87**, 071301 (2001).

[4] M. Gell-Mann, P. Ramond, R. Slansky, in *Supergravity*, eds P. van Nieuwenhuizen, D.Z. Freedman, North Holland Publ. Co., 1979; T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe, eds O. Sawata, A. Sugamoto, KEK, Tsukuba, Japan 1979.

[5] See *e.g.* talks by W. Buchmüller, *Acta Phys. Pol.* **32**, 3707 (2001); W. Grimus, *Acta Phys. Pol.* **32**, 3719 (2001), and references therein.

[6] N.V. Krasnikov, *Mod. Phys. Lett.* **A9**, 791 (1994).

[7] N. Arkani-Hamed, J.L. Feng, L.J. Hall, H. Cheng, *Phys. Rev. Lett.* **77**, 1937 (1996).

[8] N. Arkani-Hamed, J.L. Feng, L.J. Hall, H. Cheng, *Nucl. Phys.* **B505**, 3 (1997).

[9] J. Hisano, M.M. Nojiri, Y. Shimizu, M. Tanaka, *Phys. Rev.* **D60**, 055008 (1999).

[10] K. Agashe, M. Graesser, *Phys. Rev.* **D61**, 075008 (2000), `hep-ph/9904422`; I. Hinchliffe, F.E. Paige, *Phys. Rev.* **D63**, 115006 (2001).

[11] M. Guchait, J. Kalinowski, P. Roy, *Eur. Phys. J.* **C21**, 163 (2001).

[12] S.M. Bilenkii, S.T. Petcov, B. Pontecorvo, *Phys. Lett.* **B67**, 309 (1977); T.P. Cheng, L. Li, *Phys. Rev. Lett.* **45**, 1908 (1980); W.J. Marciano, A.I. Sanda, *Phys. Lett.* **B67**, 303 (1977); B.W. Lee, S. Pakvasa, R.E. Shrock, H. Sugawara, *Phys. Rev. Lett.* **38**, 937 (1977); [Erratum, *Phys. Rev. Lett.* **38**, 937 (1977)].

[13] F. Borzumati, A. Masiero, *Phys. Rev. Lett.* **57**, 961 (1986); L.J. Hall, V.A. Kostelecky, S. Raby, *Nucl. Phys.* **B267**, 415 (1986); J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi, T. Yanagida, *Phys. Lett.* **B357**, 579 (1995).

[14] J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi, *Phys. Rev.* **D53**, 2442 (1996).

[15] R. Akers *et al.* [OPAL Collaboration], *Z. Phys.* **C67**, 555 (1995).

[16] P. Abreu *et al.* [DELPHI Collaboration], *Z. Phys.* **C73**, 243 (1997).

[17] M.L. Brooks *et al.* [MEGA Collaboration], *Phys. Rev. Lett.* **83**, 1521 (1999).

[18] U. Bellgardt *et al.* [SINDRUM Collaboration], *Nucl. Phys.* **B299**, 1 (1988).

[19] J. Kaulard *et al.* [SINDRUM II Collaboration], *Phys. Lett.* **B422**, 334 (1998).

[20] S. Ahmed *et al.* [CLEO Collaboration], *Phys. Rev.* **D61**, 071101 (2000).

[21] L.M. Barkov *et al.*, Research Proposal PSI, R-99-05.1 (1999).

[22] J.L. Popp [MECO Collaboration], `hep-ex/0101017`.

[23] J.A. Aguilar-Saavedra *et al.* [ECFA/DESY LC Physics Working Group], `hep-ph/0106315`.

[24] D. Nomura, *Phys. Rev.* **D64**, 075001 (2001).