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TRIVIALITY AND VACUUM STABILITY BOUNDS ON THE HIGGS BOSON MASS BEYOND THE STANDARD MODEL*

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The triviality and vacuum stability bounds on the Higgs-boson mass were revisited in presence of weakly-coupled new interactions parameterized in a model-independent way by effective operators of dimension 6. It was shown that for the scale of new physics in the region $\Lambda \simeq 0.5 \div 50$ TeV the Standard Model triviality upper bound remains unmodified whereas it is natural to expect that the lower bound derived from the requirement of vacuum stability is increased by $40 \div 60$ GeV depending on the scale Λ and strength of coefficients of effective operators. It turns out that if the Higgs-boson mass is close to its lower LEP limit then the scale of new physics that follows from the vacuum stability requirement would be decreased dramatically even for modest values of coefficients of effective operators implying new physics already at the scale of a few TeV.

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1. Introduction

In spite of a huge experimental effort, the Higgs particle, the last missing ingredient of the Standard Model (SM) of electroweak interactions has not been discovered yet. For a Higgs-boson mass $m_h \lesssim 115$ GeV the most promising production channel has been the radiation off a Z-boson at LEP2:

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$e^+e^- \to Zh$; using this reaction the LEP data provided a limit [1] on the SM Higgs-boson mass: $m_h > 113.2$ GeV. The Higgs particle also contributes radiatively to several well measured quantities, this can be used to derive an upper bound [2] on m_h: $m_h \lesssim 212$ GeV at 95 % C.L.. However, one should be aware that both limits are highly model-dependent.

There exist other theoretical restrictions of m_h based on the so-called triviality and vacuum stability arguments. As it is well known [3] the renormalized ϕ^4 theory cannot contain an interaction term ($\lambda\phi^4$) for any non-zero scalar mass: the theory must be trivial. Within a perturbative approach the statement corresponds to the fact that for any non-zero scalar mass (since the mass is $\propto \sqrt{\lambda}$ this condition corresponds to a non-vanishing initial value for the Renormalization Group (RG) evolution of λ) there exists a finite energy scale at which λ diverges (the Landau pole). Consequently, only for zero scalar mass the theory can be consistent for all energy scales. An analogous effect occurs in the scalar sector of the SM (modified to some extend by the presence of gauge and Yukawa interactions). This, however, does not necessarily implies zero Higgs-boson mass since there is no reason to believe that the SM is valid at arbitrarily high energy scale. For example, it is often assumed that the SM represents the low energy limit of some underlying more fundamental theory whose heavy excitations decouple at low energy, but become manifest at a scale Λ. Within that scenario the SM is an effective theory valid possibly only at the energy scale of the order of the Fermi scale: $G_{\rm F}^{-1/2} \simeq 300$ GeV.

If the SM is to be accurate for energies below Λ the Landau pole should occur at scale Λ or above, and this condition gives an (Λ-dependent) upper bound on m_h [4]. On the other hand, for sufficiently small m_h radiative corrections can destabilize the ground state. This occurs if the scalar self coupling constant λ becomes negative at some scale that can be identified with the scale of new physics Λ. Alternatively requiring the SM vacuum to be stable for scales below Λ implies a lower bound on m_h [5].

The consequences of the above arguments (triviality and vacuum stability) are usually discussed assuming SM interactions. However, if the scale of new physics is sufficiently low (of the order of a few TeV) one could expect that the non-standard interactions would modify the electroweak theory at the lower scale and influence the scalar potential in such a way that the above bounds on the Higgs-boson mass are changed. The problem deserves a special attention in the context of possible Higgs-boson discovery [6] at LEP2 at the mass $m_h \simeq 115$ GeV since in this case the SM constraint from vacuum stability requires $\Lambda \lesssim \mathcal{O}(100)$ TeV [7] (the precise number depends on the top quark mass) with the attractive possibility that Λ is actually much lower.

It then becomes interesting to determine the manner in which heavy physics with scales in the 10 TeV region modify the stability and triviality bounds on the Higgs-boson mass. In this lecture we address this question in a model-independent way. We parameterize the heavy physics effects using an effective Lagrangian satisfying the SM gauge symmetries. Since LHC, the future proton–proton collider, is expected to be sensitive to scales Λ of the order of a few TeV, the results will be presented for scales between 0.5 and 50 TeV.

The paper is organized as follows. In Section 2, we define the Lagrangian relevant for our discussion. Section 3 is devoted to the derivation of the triviality bound including effects of non-standard interactions. In Section 4, we calculate the effective potential with one insertion of an effective operator and discuss its consequences for the vacuum stability bounds. Concluding remarks are given in Section 5.

2. Non-standard interactions

Our study of the stability and triviality constraints on the Higgs-boson mass will be based on the SM Lagrangian modified by the addition of a series of effective operators whose coefficients parameterize the low-energy effects of the heavy physics [8]. Assuming that these non-standard effects decouple implies [9] that the operators appear multiplied by appropriate inverse powers of Λ. The leading effects are then generated by operators of mass-dimension 6 (dimension 5 operators necessarily violate lepton number [10] and are associated with new physics at very large scales; so we can safely ignore their effects). Given our emphasis on Higgs-boson physics the effects of all fermions excepting the top-quark can be ignored [1]. We then have

$$\begin{aligned}\mathcal{L}_{\rm tree} \;=\; & -\tfrac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} + |D\phi|^2 - \lambda\left(-\tfrac{1}{2}v^2 + |\phi|^2\right)^2 \\ & + i\bar{q}\,\not{D}q + i\bar{t}\,\not{D}t + f\left(\bar{q}\tilde{\phi}t + \mathrm{h.c.}\right) + \sum_i \frac{\alpha_i}{\Lambda^2}\mathcal{O}_i\,, \end{aligned} \tag{2.1}$$

where ϕ ($\tilde{\phi} = -i\tau_2\phi^*$), q and t are the scalar doublet, third generation left-handed quark doublet and the right-handed top singlet, respectively. D, $F^i_{\mu\nu}$ and $B_{\mu\nu}$ denote a covariant derivative and SU(2), U(1) field strength whose couplings we denote by g and g'.

The factors α_i are unknown coefficients that parameterize the low-energy effects of the non-standard interactions and we have neglected contributions $\propto 1/\Lambda^4$. In addition, for weakly coupled theories, the α_i, that can be generated only through loop effects, are sub-dominant as they are suppressed by

[1] We assume that the masses are natural in the technical sense [11] so that effective couplings containing the Higgs boson and the light fermions are suppressed by powers of the corresponding Yukawa couplings.

numerical factors $\sim 1/(4\pi)^2$ [12]; hence we will consider only those operators which can be generated at tree-level by the heavy physics. Even with all the above restrictions there remain 16 operators which involve exclusively the fields in (2.1). Of these only 5 contribute directly to the effective potential, the remaining 11 affect the results only through their RG mixing which, being suppressed by a factor $\sim (v/\Lambda)^2$ are expected to play a sub-dominant role. In the calculations below we will include only one of these operators; our results do justify the claim that the corresponding effects are small.

The following set of operators will be considered:

$$\begin{aligned} \mathcal{O}_\phi &= \tfrac{1}{3}|\phi|^6 , \qquad \mathcal{O}_{\partial\phi} = \tfrac{1}{2}\left(\partial|\phi|^2\right)^2 , \qquad \mathcal{O}_\phi^{(1)} = |\phi|^2\,|D\phi|^2 , \\ \mathcal{O}_\phi^{(3)} &= \left|\phi^\dagger D\phi\right|^2 , \quad \mathcal{O}_{t\phi} = |\phi|^2\left(\bar{q}\tilde{\phi}t + \mathrm{h.c.}\right) , \quad \mathcal{O}_{qt}^{(1)} = \tfrac{1}{2}\,|\bar{q}t|^2 , \end{aligned} \tag{2.2}$$

where $\mathcal{O}_\phi$, $\mathcal{O}_{\partial\phi}$, $\mathcal{O}_\phi^{(1)}$, $\mathcal{O}_\phi^{(3)}$, $\mathcal{O}_{t\phi}$ are the 5 operators contributing to the effective potential, while $\mathcal{O}_{qt}^{(1)}$ is included to estimate the effects of RG mixing.

Of the first five operators only $\mathcal{O}_\phi = \frac{1}{3}|\phi|^6$ contributes at the tree level to the scalar potential:

$$V^{(\mathrm{tree})} = -\eta\Lambda^2|\phi|^2 + \lambda|\phi|^4 - \frac{\alpha_\phi}{3\Lambda^2}|\phi|^6 , \tag{2.3}$$

where we have used the notation: $\eta \equiv \lambda v^2/\Lambda^2$.

3. Triviality bound

In order to test the high energy behavior of the scalar potential one has to derive the RG running equations for λ, η and α_ϕ. The β functions for these parameters are influenced by all the operators in (2.2) and by the gauge and Yukawa interactions, so the full RG evolution also requires the β function for the corresponding couplings. Both for the β functions and then for the effective potential we will adopt dimensional regularization and $\overline{\mathrm{MS}}$ renormalization scheme. We will restrict ourselves to the one-loop approximation keeping SM contributions and terms linear in the effective operators, defined by Eq. (2.2). The evolution equations for the running coupling constants are the following:

$$\begin{aligned} 8\pi^2\frac{d\lambda}{dt} &= 12\lambda^2 - 3f^4 + 6\lambda f^2 - 2\eta\left[2\alpha_\phi + \lambda\left(7\alpha_{\partial\phi} + 8\alpha_\phi^{(1)} + 5\alpha_\phi^{(3)}\right)\right] \\ &\quad + \frac{3}{2}\left[-\lambda\left(3g^2 + g'^2\right) + \frac{g'^4 + 2g^2g'^2 + 3g^4}{8}\right] , \\ 8\pi^2\frac{d\eta}{dt} &= \eta\left[6\lambda + 3f^2 - 2\eta\left(\alpha_{\partial\phi} + 2\alpha_\phi^{(1)} + \alpha_\phi^{(3)}\right) - \frac{3}{4}\left(3g^2 + g'^2\right)\right] , \end{aligned}$$

$$
\begin{aligned}
8\pi^2 \frac{df}{dt} &= \frac{9}{4} f^3 + \frac{1}{2}\eta \left[6\alpha_{t\phi} - f\left(\alpha_{\partial\phi} + 2\alpha_\phi^{(1)} + \alpha_\phi^{(3)} + 3\alpha_{qt}^{(1)} \right) \right] \\
&\quad - \frac{f}{8}\left(9g^2 + \frac{17}{3} g'^2 \right) , \\
8\pi^2 \frac{d\alpha_\phi}{dt} &= 9\alpha_\phi \left(6\lambda + f^2 \right) + 12\lambda^2 \left(9\alpha_{\partial\phi} + 6\alpha_\phi^{(1)} + 5\alpha_\phi^{(3)} \right) + 36\alpha_{t\phi} f^3 \\
&\quad - \frac{9}{4}(3g^2 + g'^2)\alpha_\phi \\
&\quad - \frac{9}{8}\left[\alpha_\phi^{(1)}(3g^4 + 2g^2 g'^2 + g'^4) + \alpha_\phi^{(3)}(g^2 + g'^2)^2 \right] , \\
8\pi^2 \frac{d\alpha_{\partial\phi}}{dt} &= 2\lambda \left(7\alpha_{\partial\phi} - \alpha_\phi^{(1)} + \alpha_\phi^{(3)} + 3\frac{\alpha_{\partial\phi} f^2}{\lambda} - 3\frac{\alpha_{t\phi} f}{\lambda} \right) , \\
8\pi^2 \frac{d\alpha_\phi^{(1)}}{dt} &= 2\lambda \left(\alpha_{\partial\phi} + 5\alpha_\phi^{(1)} + \alpha_\phi^{(3)} + 3\frac{\alpha_\phi^{(1)} f^2}{\lambda} - 3\frac{\alpha_{t\phi} f}{\lambda} \right) , \\
8\pi^2 \frac{d\alpha_\phi^{(3)}}{dt} &= 6(\lambda + f^2)\alpha_\phi^{(3)} , \\
8\pi^2 \frac{d\alpha_{t\phi}}{dt} &= -3f(f^2 + \lambda)\alpha_{qt}^{(1)} + \frac{3}{4}(5f^2 - 16\lambda)\alpha_{t\phi} \\
&\quad - \frac{1}{2} f^3 \left(2\alpha_{\partial\phi} + \alpha_\phi^{(1)} + \alpha_\phi^{(3)} \right) , \\
8\pi^2 \frac{d\alpha_{qt}^{(1)}}{dt} &= \frac{3}{2}\alpha_{qt}^{(1)} f^2 ,
\end{aligned}
\tag{3.1}
$$

where $t \equiv \log(\kappa/m_Z)$ and κ denotes the renormalization scale.

From this set of equations it is straightforward to obtain the triviality constraints on m_h as a function of Λ requiring that the position of the Landau pole is beyond the scale Λ. There is a comment here in order, namely, in actual calculations the position of the Landau pole cannot be accurately determined to any finite order in perturbation theory. Therefore, the triviality bound on m_h will be obtained by requiring λ and α_ϕ to become smaller than specified values (as opposed from requiring an actual divergence) up to the scale Λ:

$$
\lambda(t) \leq \lambda_{\max} \quad \text{and} \quad |\alpha_i(t)| \leq 1.5 \quad \text{for} \quad 0 \leq t < \log\left(\frac{\Lambda}{m_Z} \right) , \tag{3.2}
$$

where we considered $\lambda_{\max} = \pi$ and $\pi/2$. We have verified that our results are quite insensitive to the values chosen as upper limits for the α_i.

In order to solve equations (3.1) we have to specify appropriate boundary conditions. For the SM parameters these are determined by requiring that the correct physical parameters (such as the Higgs-boson and top-quark masses) are obtained at the electroweak scale. These initial conditions should also insure that the correct SM ground state is realized, in which the scalar field has the expectation value $\langle\bar{\varphi}\rangle \equiv v_0/\sqrt{2} = 246/\sqrt{2}$ GeV. Although we will discuss the effective potential in more detail later, it will be useful to provide here the general 1-loop relation between the SM tree-level vacuum v and the physical electroweak vacuum in the theory defined by equation (2.1) v_0:

$$v_0 = v + \delta v \quad \text{for} \quad \delta v \equiv -\frac{1}{4\lambda(0)v^2}\left.\frac{\partial\left(V_{\text{eff}} - V_{\text{SM}}^{(\text{tree})}\right)}{\partial\left(\frac{\bar{\varphi}}{\sqrt{2}}\right)}\right|_{\bar{\varphi}=v/\sqrt{2}}, \tag{3.3}$$

where $V_{\text{SM}}^{(\text{tree})}$ is the tree-level SM potential and V_{eff} is the 1-loop effective potential that includes effective operator contributions; $\lambda(0)$ denotes the running coupling constant evaluated at the scale $\kappa = m_Z$. Having the vacuum determined by the above equation, the following low-scale relations will be adopted to fix initial conditions at $\kappa = m_Z$ for the RG equations for λ, η and f.

$$\begin{aligned} m_h^2 &= 2\lambda v_0^2\left[1 - \frac{v_0^2}{4\Lambda^2}\left(4\alpha_{\partial\phi} + \alpha_\phi^{(1)} + \alpha_\phi^{(3)} + \frac{2\alpha_\phi}{\lambda}\right)\right] + m_h^{(1)}\,, \\ m_t &= \frac{v_0}{\sqrt{2}}\left(f + \alpha_{t\phi}\frac{v_0^2}{\Lambda^2}\right) + m_t^{(1)}\,, \end{aligned} \tag{3.4}$$

where $m_h^{(1)}$, $m_t^{(1)}$ denote the 1-loop radiative corrections to the corresponding masses. In the calculations bellow we use the expression for $m_h^{(1)}$ of Ref. [7]. For the top-quark the deviations from the tree-level value are smaller than the experimental error and so, for simplicity we will use the expression $m_t = v_0 f/\sqrt{2}$. The initial conditions are non-linear functions of the Higgs-boson mass, and so the solutions to (3.1) will depend on both Λ and m_h.

The boundary conditions for α_i are naturally specified at the scale $\kappa = \Lambda$ since below this scale it is appropriate to describe the effects of the heavy excitations in terms of the coefficients α_i. According to Ref. [12] it is natural to assume that $\alpha_i|_{\kappa=\Lambda} \simeq \mathcal{O}(1)$.

The triviality bound is obtained by solving equations (3.1) with the mixed (defined in part at the electroweak scale m_Z and at the new-physics scale Λ) boundary conditions described above and requiring that at least one of the inequalities in Eq. (3.2) is saturated. This provides a relationship between m_h and Λ that we plot in Fig. 1(a) for two values of λ_{max}. In order to understand qualitatively the corrections to the triviality bound we have obtained, it is useful to switch off all α_i but α_ϕ. Then, as it is seen from Eq. (3.1) a Landau pole in the evolution of $\lambda(t)$ causes a singularity in

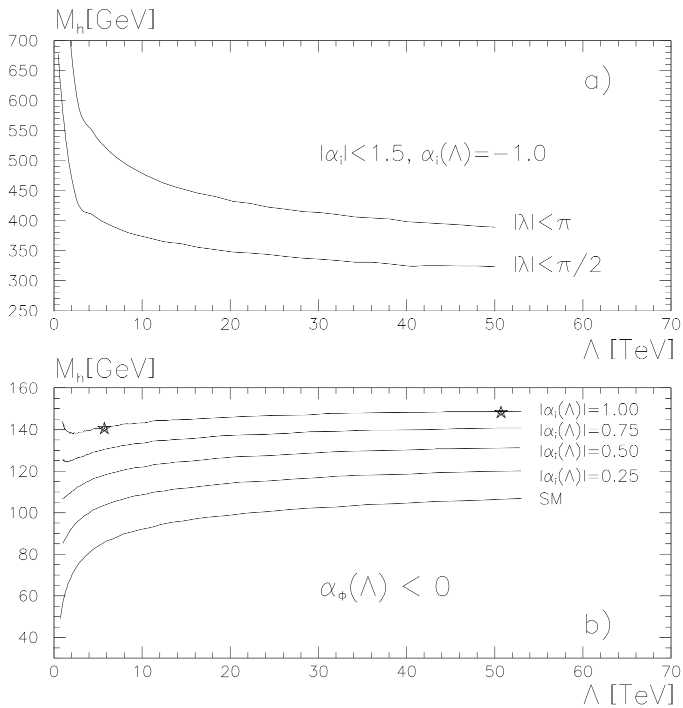

Fig. 1. The upper (a) (originating from the perturbativity requirement, Eq. (3.2)), and the lower (b) (from the condition of the electroweak vacuum stability, Eq. (4.6)) bounds on the Higgs-boson mass m_h as a function of the new-physics scale Λ. The lower bounds were obtained for $\alpha_\phi(\Lambda) < 0$ and $m_t = 175$ GeV.

evolution of α_ϕ at *this same* energy scale. However, as we have just mentioned it is natural to assume $\alpha_\phi|_{\kappa=\Lambda} \simeq 1$, it is clear that strictly speaking it is impossible to satisfy that condition. Nevertheless, since we are using a perturbation expansion, we must stop the evolution at a scale that corresponds to a large but finite value $\lambda_{\max}$, therefore, we can satisfy $\alpha_\phi|_{\kappa=\Lambda} \simeq 1$. However, since $d\log\alpha_\phi/dt$ is positive[2], therefore, in the evolution from the scale Λ down, $\alpha_\phi(t)$ decreases reaching typically $10^{-1} \div 10^{-2}$ at the scale $\kappa = m_Z$. That explains screening of the effects generated by operator $\mathcal{O}_\phi$: even if $\alpha_\phi|_{\kappa=\Lambda} \simeq 1$ it can not grow any larger[3]. So concluding, the corrections to the SM triviality bound from the non-standard physics (embedded in the coefficients α_i) are negligible.

[2] Here we consider heavy Higgs bosons, therefore, λ remains positive in the whole integration region, it addition $f \gtrsim g, g'$ what guarantees that $d\log\alpha_\phi/dt > 0$.
[3] For strongly coupled new-physics corrections to this bound see [13].

4. Vacuum stability bound

In order to investigate the vacuum structure of the effective theory we will first calculate the effective potential:

$$V_{\rm eff} = -\sum_N \frac{1}{N!}\Gamma^{(N)}(0)\bar{\varphi}^n, \tag{4.1}$$

where $\Gamma^{(N)}(0)$ are N-point one-particle-irreducible Green's functions with zero external momenta and $\bar{\varphi}$ is the classical scalar field. Adopting the Landau gauge[4] we obtained:

$$\begin{aligned}V_{\rm eff}(\bar{\varphi}) &= -\eta\Lambda^2|\bar{\varphi}|^2 + \lambda|\bar{\varphi}|^4 - \frac{\alpha_\phi|\bar{\varphi}|^6}{3\Lambda^2}\\ &+\frac{1}{64\pi^2}\left[H^2\left(\ln\frac{H}{\kappa^2}-\frac{3}{2}\right)+3G^2\left(\ln\frac{G}{\kappa^2}-\frac{3}{2}\right)+6W^2\left(\ln\frac{W}{\kappa^2}-\frac{5}{6}\right)\right.\\ &\left.+3Z^2\left(\ln\frac{Z}{\kappa^2}-\frac{5}{6}\right)-12T^2\left(\ln\frac{T}{\kappa^2}-\frac{3}{2}\right)-4\eta^2\Lambda^4\left(\ln\frac{\eta\Lambda^2}{\kappa^2}-\frac{3}{2}\right)\right],\end{aligned} \tag{4.2}$$

where

$$\begin{aligned}H &= \lambda(-v^2+6|\bar{\varphi}|^2) - \left[\lambda(-v^2+6|\bar{\varphi}|^2)(2\alpha_{\partial\phi}+\alpha_\phi^{(1)}+\alpha_\phi^{(3)})+5\alpha_\phi|\bar{\varphi}|^2\right]\frac{|\bar{\varphi}|^2}{\Lambda^2},\\ G &= \lambda(-v^2+2|\bar{\varphi}|^2) - \left[\lambda(-v^2+2|\bar{\varphi}|^2)\frac{1}{3}(3\alpha_\phi^{(1)}+\alpha_\phi^{(3)})+\alpha_\phi|\bar{\varphi}|^2\right]\frac{|\bar{\varphi}|^2}{\Lambda^2},\\ W &= \frac{g^2}{2}|\bar{\varphi}|^2\left(1+\frac{|\bar{\varphi}|^2\alpha_\phi^{(1)}}{\Lambda^2}\right),\\ Z &= \frac{g^2+g'^2}{2}|\bar{\varphi}|^2\left(1+\frac{|\bar{\varphi}|^2(\alpha_\phi^{(1)}+\alpha_\phi^{(3)})}{\Lambda^2}\right),\\ T &= f^2|\bar{\varphi}|^2\left(1+\frac{2\alpha_{t\phi}|\bar{\varphi}|^2}{f\Lambda^2}\right),\end{aligned}$$

where g and g' denote the SU(2) and U(1) running gauge coupling constants, respectively. The *form* of the effective potential is precisely the same as

[4] As it has been noticed in Ref. [14] the effective potential (as a sum of off-shell Greens functions) is gauge dependent. Therefore the bounds on the Higgs-boson mass derived from vacuum stability arguments can depend on the gauge parameter adopted in the loop calculation [15]. However, since the β functions and the tree-level potential $V_{\rm eff}^{(\rm tree)}$ are gauge-independent, a consistent RG improved tree-level effective potential is in fact gauge independent. For the one-loop SM RG improved effective potential, the error caused by the gauge dependence has been estimated in Ref. [7] at $\Delta m_h \lesssim 0.5$ GeV.

the one in the pure SM, the whole effect of the effective operators can be absorbed in a re-definition of the SM quantities H, G, *etc.*[5] It should be noticed here that the last term in Eq. (4.2) is a constant that is needed to construct a scale invariant effective potential, for details see Ref. [17]. The constant term chosen here is consistent with the diagrammatic definition of the effective potential Eq. (4.1), which implies $V_{\rm eff}(\bar{\varphi}=0)=0$.

Since we will consider values of $\bar{\varphi}$ substantially larger then the electroweak scale v_0, we shall chose an appropriate renormalization scale $\kappa \sim \bar{\varphi}$ in order to moderate the logarithms that appear in the effective potential. As in the previous section we shall use the RG running equations to relate the coupling constants renormalized at the high scale $\bar{\varphi}$ to the low-scale parameters v_0, m_t and m_h.

Finally, (and unlike the pure ϕ^4) the interaction of the scalars with the fermions and gauge bosons, generate a non-trivial scalar field anomalous dimension γ. We, therefore, also include the corresponding scale dependence of $\bar{\varphi}$:

$$\bar{\varphi}(t)=\exp\left\{-\int_{t_z}^{t}\gamma[\lambda(t'),\eta(t'),f(t'),\alpha_i(t')]dt'\right\}\bar{\varphi}(t_z)\,, \tag{4.3}$$

where

$$\gamma=\frac{1}{16\pi^2}\left[3f^2-\frac{9}{4}g^2-\frac{3}{4}g'^2-\eta\left(\alpha_{\partial\phi}+2\alpha_\phi^{(1)}+\alpha_\phi^{(3)}\right)\right]. \tag{4.4}$$

Hereafter we will consider the RG improved effective potential $V_{\rm eff}(\bar{\varphi}(t))$.

We note that the RG improved effective potential given by Eq. (4.2) is scale invariant. That is, to one loop and ignoring terms quadratic in the α_i, $V_{\rm eff}$ obeys the renormalization group equation:

$$\kappa\partial_\kappa V_{\rm eff}^{(1-\rm loop)}+\left(\sum_i\beta_i\partial_{\lambda_i}-\gamma\bar{\varphi}\partial_{\bar{\varphi}}\right)V_{\rm eff}^{(\rm tree)}=0\,, \tag{4.5}$$

where $V_{\rm eff}^{(\rm tree)}$ and $V_{\rm eff}^{(1-\rm loop)}$ denote, respectively, the tree, Eq. (2.3), and 1-loop, Eq. (4.2), contributions to $V_{\rm eff}$, and β_i are defined in (3.1). We note that terms quadratic (and higher) in the α_i are associated with contributions of the order of $1/\Lambda^4$ to the effective Lagrangian and are sub-dominant.

Fig. 2 (a) illustrates the behavior of the effective potential renormalized at the scale $\kappa=\bar{\varphi}$. Since the minimum at $\langle\bar{\varphi}\rangle=v_0/\sqrt{2}$ is very shallow,

[5] The same result (in the leading order in α_i) for the effective potential have been obtained adopting the diagrammatic approach (with one insertion of an effective operator) according to Eq. (4.1) and also using the functional definition of the effective potential proposed by Jackiw [16].

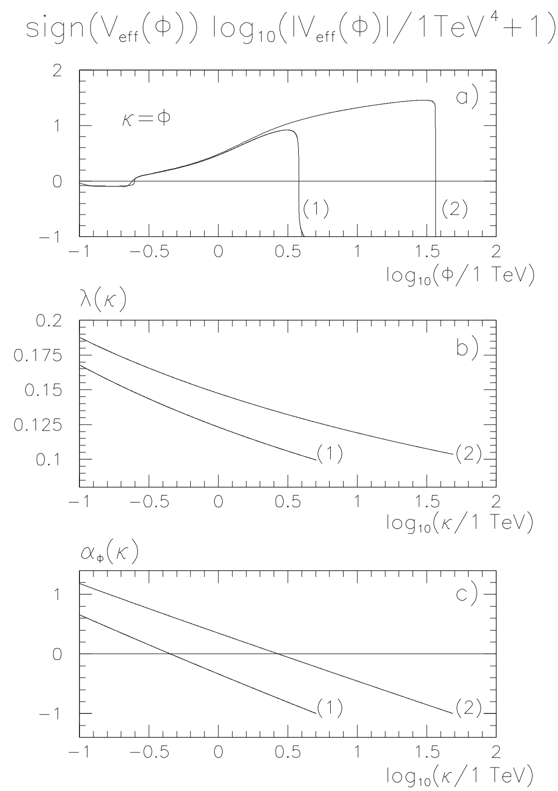

Fig. 2. The effective potential renormalized at the scale $\kappa = \phi$ (a), the running of λ (b) and α_ϕ (c) for the parameter sets (1) and (2) defined in the text.

in order to make it visible we plot the following function of the effective potential: $\mathrm{sign}(V_{\mathrm{eff}})\log_{10}[(V_{\mathrm{eff}}/1\ \mathrm{TeV}^4)+1]$. To show the relevance of RG running of effective-potential parameters we also plot in Fig. 2 the evolution of λ (b) and α_ϕ (c). The curves contained in the figure correspond to two sets of initial conditions (1) and (2) that lead to the Higgs-boson mass and the new-physics scale marked in Fig. 1 by $\star$'s. As it is seen from the figure effects of the running are substantial, *e.g.* for the set (2) λ changes by almost 100% while α_ϕ by more than 200 %. At the electroweak scale, α_ϕ's start with positive values, however then, through the evolution they switch signs and eventually reach $\alpha_\phi = -1$. That should illustrate the fact that the RG running of the coefficients α_i is crucial for the stability of the system[6].

[6] Corrections to the SM vacuum stability bound that emerge in presence of the operator $\mathcal{O}_\phi$ has been previously discussed in Ref. [18]. However, there the authors did not consider one-loop contributions to the effective potential that are generated by insertions of effective operators. RG running of α_ϕ has also been neglected.

The initial conditions for the running couplings guarantee that the electroweak vacuum is at $\langle\bar{\varphi}\rangle = v_0/\sqrt{2}$. However, if $V_{\rm eff}$ at some large value of the field $\bar{\varphi}_{\rm high}$ is smaller than $V_{\rm eff}(\langle\bar{\varphi}\rangle)$ this vacuum becomes unstable (as there would be a possibility of tunneling[7] towards the region of lower energy). This will occur when the Higgs-boson mass is sufficiently small (corresponding to a small value of $\lambda(0)$), and will provide a lower bound on m_h. In this case $\bar{\varphi}_{\rm high}$ defines a scale at which the theory breaks down, so that $\bar{\varphi}_{\rm high} \sim \Lambda$. In actual calculations we took $\bar{\varphi}_{\rm high} = 0.75\Lambda$ since (2.1) is valid for scales below Λ, hence the stability bound on m_h is determined by the condition

$$V_{\rm eff}(\bar{\varphi} = 0.75\Lambda)|_{\kappa=0.75\Lambda} = V_{\rm eff}\left(\bar{\varphi} = \frac{v_0}{\sqrt{2}}\right)\Bigg|_{\kappa=v_0/\sqrt{2}} \tag{4.6}$$

where, as mentioned previously, we have chosen the renormalization scale κ to tame the effects of the logarithmic contributions to $V_{\rm eff}(\bar{\varphi})$. The resulting bound on m_h as a function of Λ for various choices of $\alpha_i(\Lambda)$ is plotted in Fig. 1(b).

In obtaining the stability bounds of Fig. 1(b) we assumed all couplings α_i had the same magnitude at the high scale Λ, and $\alpha_\phi < 0$ (the results are insensitive to the sign of the other α_i except $\alpha_{t\phi}$). For other values of α_i we found that when $\Lambda > 300$ GeV there is a curve in the $\alpha_\phi - \alpha_{t\phi}$ plane below which either $\bar{\varphi} = 174$ GeV is not a minimum or, if it is, then there is another deeper minimum at a scale 174 GeV $< \bar{\varphi} < 0.75\Lambda$; we can roughly say that this unphysical scenario can be avoided if $\alpha_\phi \lesssim -0.1$ [8].

There is an important remark here in order. If the Higgs boson mass, as suggested by LEP data, is indeed [6] $\simeq 115$ GeV, then the SM vacuum stability bound implies $\Lambda \lesssim \mathcal{O}(100)$ TeV. As it is seen in Fig. 1(b) presence of effective operators could dramatically change the SM picture. Even for the modest values of the coefficient $|\alpha_i| = 0.25,\ 0.50,\ 0.60$ the upper bound on Λ is significantly reduced to $\Lambda \simeq 20,\ 4,\ 1$ TeV, respectively!

Other limits on the scale Λ could be obtained form the so called precision observables. The most elegant approach is to calculate the oblique parameters S, T and U [20] within the effective theory[9] and then fit their

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7 The tunneling time will not be calculated here, it can be obtained using the procedure described in [19]; we assume that it is smaller than the age of the Universe.

8 We do not expect this result to be modified significantly when terms of order $1/\Lambda^4$ are included: a contribution $\sim \alpha^{(8)}\bar{\varphi}^8/\Lambda^4$ can balance the destabilizing effect of $\mathcal{O}_\phi$ only when $\bar{\varphi} \sim \Lambda$ which again leads to $\Lambda \sim 300$ GeV.

9 It should be noticed that among operators considered here only $\mathcal{O}_\phi^{(3)}$ contributes to the oblique parameters (T) and, therefore, is constrained by the precision data, however, as it has been shown here the operator that is most relevant for the triviality and vacuum stability bound is $\mathcal{O}_\phi$ and contributions from $\mathcal{O}_\phi^{(3)}$ are much less important.

experimental values [21, 22]. The limits obtained that way depend also on the Higgs-boson mass m_h, therefore, it would be interesting to superimpose precision-measurement limits, the direct LEP limit and those obtained here, consistently taking into account higher dimensional operators, that is, however, beyond the scope of this paper[10].

5. Summary and conclusions

We have considered restrictions on the Higgs-boson mass that emerge form requirement of perturbative behavior of the quartic coupling constant (the triviality bound) and from the condition of stable electroweak vacuum taking into account possible non-standard interactions described by effective operators of dimension $\leq$ 6. It was shown that for the scale of new physics in the region $\Lambda \simeq 0.5 \div 50$ TeV the Standard Model triviality upper bound remains unmodified whereas the lower bound from requirement of vacuum stability is naturally increased by $40 \div 60$ GeV depending on the scale Λ and strength of coefficients of effective operators. Therefore, the allowed region of the Higgs-boson mass is reduced substantially. If the Higgs-boson mass is close to its lower LEP limit then the upper bound on the scale of new physics that follows from the vacuum stability requirement could be decreased dramatically even for modest values of coefficients of effective operators implying new physics already at the scale of $\sim 1 \div 2$ TeV.

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