NEUTRINO MASSES MEASUREMENT IN FUTURE TRITIUM BETA DECAY EXPERIMENT* **

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The end of the electron energy distribution $\frac{dN}{dE}$ in β decays of nuclei depends on neutrino masses and mixing angles. Various approximate parametrization of the $\frac{dN}{dE}$, proposed in literature, and the definition of effective neutrino masses m_β are investigated. Bounds or future measured values of m_β together with the oscillation parameters are a source of information about the mass of the lightest neutrino.

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1. Introduction

Among various processes, which give information about absolute values of neutrino masses, the tritium β decay plays particular role. Currently, measurements of the end of the electron energy distribution $\frac{dN}{dE}$ in ${}^3_1$H-decay are the only known source of information about scale of neutrino masses, which are independent of their nature. There are future plans to measure much more precisely the end of energy spectrum [1] which could result in finding the effective neutrino mass m_β or new better bound on it.

The state of produced ν_e neutrino in weak β^+ decay is the coherent combination of various neutrino mass states $|\nu_i\rangle$

$$|\nu_e\rangle = \sum_{i=1}^{N} U_{ei} |\nu_i\rangle \ , \tag{1}$$

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where U_{ei} are the elements of the mixing matrix. Then $\frac{dN}{dE}$ depends on the light neutrinos masses m_i and on the mixings U_{ei}. The existence of tree light neutrinos ($N = 3$) is experimentally confirmed. If we combine the LSND experiment data and the solar and atmospheric neutrino anomalies, we are forced to introduce larger number of light neutrinos ($N > 3$) [2]. As the results of the recent experiment wait for confirmation, we now consider N=3 case only. Then $\frac{dN}{dE}$ depends on five neutrino parameters which is too many to specify them all from β^+ decay. One parameter, the effective electron neutrino mass m_β is usually introduced as

$$m_\beta = f(|U_{ei}|, m_i)\,, \tag{2}$$

but there is no agreement how m_β should depend on m_i and U_{ei}. In this article we consider two definitions of m_β and try to decide which is the more appropriate one. Then having experimentally measured m_β we check how precisely we are able to determine neutrino spectrum. In the next Section, we briefly describe electron energy distribution from tritium β decay together with recent experimental determination of m_β. Then in Section 3, two parametrization of m_β (which can be found in literature) are presented. In Section 4, problem of neutrino mass determination is discussed and finally we give conclusions.

2. Brief description of beta decay experiment

We consider the transition between tritium and helium ${}^3_1\mathrm{H}\to{}^3_2\mathrm{He}+e^-+\bar{\nu}_e$. The energy distribution of outgoing electron, known as a Curie plot, is almost parabolic-shaped with deviation at the end. This deviation refers to non- zero neutrino mass. In this case, energy distribution is given by [3]:

$$\left(\frac{dN}{dE}\right) = R(E)(E_0 - E)\sqrt{(E_0 - E)^2 - m_\beta^2}\,, \tag{3}$$

where $E = E_{\mathrm{tot}} - m_e \approx \frac{p^2}{2m_e}$ is electron kinetic energy, and

$$R(E) = G_{\mathrm{F}}^2 \frac{m_e^5 \cos\theta_c}{2\pi^3} |M|^2 F(E) \sqrt{2m_e E}(E + m_e)\,, \tag{4}$$

where G_{F} — Fermi constant, θ_c — Cabibbo angle, M — nuclear matrix element. $F(E)$ is neutrino mass independent, smooth function of E which describes the interaction of the produced electron in the final state and the radiative corrections. The tritium β decay process is very convenient to analysis since:

- the value of maximal kinetic energy of electron in case where neutrino has zero mass ($E_0 = M(^3_1\mathrm{H}) - M(^3_2\mathrm{He}) - m_e \approx 18572.1$ eV) is quite small,
- life time of tritium is short,
- nuclear structure of tritium and atomic corrections are not very complicated and in principle calculable.

Measurements of the effective neutrino mass m_β have a very long tradition. First experiment was performed in 1940s [4]. Later, the problem with negative m_β^2 has appeared [5]. Currently, the best value of m_β has been obtained in two experiments:
⋆ **Mainz** [6]

$$\begin{aligned} m_\beta &< 2.2 \text{ eV} \qquad m_\beta^2 = -1.6 \pm 2.5_{\text{stat.}} \pm 2.1_{\text{syst.}} \text{ eV}^2 \,, \\ m_\beta &< 2.8 \text{ eV} \qquad m_\beta^2 = 0.6 \pm 2.8_{\text{stat.}} \pm 2.5_{\text{syst.}} \text{ eV}^2 \,. \end{aligned} \tag{5}$$

⋆ **Troisk** [7]

$$m_\beta < 2.5 \text{ eV} \qquad m_\beta^2 = -1.0 \pm 3.0_{\text{stat.}} \pm 2.1_{\text{syst.}} \text{ eV}^2 \,. \tag{6}$$

As we can see the problem of negative m_β^2 still exists, but it is not as severe as it was before. The next experiment KATRIN [1] proposes:

$$m_\beta < 0.3 \div 0.35 \text{ eV} \,. \tag{7}$$

3. Effective neutrino mass approximation [8]

Electron neutrino state is the combination of massive states (Eq. (1)), then the energy distribution is given by:

$$\begin{aligned} \left(\frac{dN}{dE}\right)_0 &= R(E)(E_0 - E) \sum_{i=1}^{3} |U_{ei}|^2 \sqrt{(E_0 - E)^2 - m_i^2} \\ &\quad \times \Theta(E_0 - E - m_i) \,, \end{aligned} \tag{8}$$

where $\Theta(E_0 - E - m_i)$ is a step function. Effective neutrino mass m_β (2) must be defined in such a way, so that the distribution (3) correctly approximates the full one (8). Two possible approximations can be found in literature. The most popular one [9]:

$$m_\beta^{(1)} = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 m_i^2} \,, \tag{9}$$

and lately found in [10]

$$m_\beta^{(2)} = \sum_{i=1}^{3} |U_{ei}|^2 m_i \,. \tag{10}$$

In what way both approximations have been obtained is described in [9,10]. Now we analyze, which one is more appropriate to particular experiment. Let us define following function:

$$f_i(E) \equiv \frac{1}{R(E_0 - m_1)} \left(\frac{dN}{dE}\right)_i \tag{11}$$

which for i=0 is connected with full energy distribution (see Eq. (8))

$$f_0 \approx (E_0 - E) \sum_{i=1}^{3} |U_{ei}|^2 \sqrt{(E_0 - E)^2 - m_i^2} \,, \tag{12}$$

and for $i = 1, 2$ with distribution (3) for $m_\beta^{(1)}$ and $m_\beta^{(2)}$ respectively, so

$$\begin{aligned} f_1 &\approx (E_0 - E)\sqrt{(E_0 - E)^2 - (m_\beta^{(1)})^2} \,, \\ f_2 &\approx (E_0 - E)\sqrt{(E_0 - E)^2 - (m_\beta^{(2)})^2}. \end{aligned} \tag{13}$$

Our results does not qualitatively depend on oscillation parameters uncertainties and the mass scheme. We take into account the LMA MSW solution of the solar neutrino anomaly [11], so

$$|U_{e1}|^2 = 0.55 \,,\ |U_{e2}|^2 = 0.43 \,,\ |U_{e3}|^2 = 0.02 \,,\ \delta m^2_{\rm solar} = 3.5 \times 10^{-5}\ {\rm eV}^2 \tag{14}$$

and from atmospheric neutrinos we obtain

$$\delta m^2_{\rm atm} = 3.1 \times 10^{-3}\ {\rm eV}^2 \,. \tag{15}$$

The so called normal mass scheme is used

$$\begin{aligned} m_2 &= \sqrt{m_1^2 + \delta m^2_{\rm solar}} \,, \\ m_3 &= \sqrt{m_1^2 + \delta m^2_{\rm solar} + \delta m^2_{\rm atm}} \end{aligned} \tag{16}$$

where m_1 is minimal neutrino mass. In order to compare full energy distribution with effective one, ratio

$$g(E) = \frac{|f_0(E) - f_2(E)|}{|f_0(E) - f_1(E)|} \tag{17}$$

have to be defined. It can be plotted as an energy function for specified minimal neutrino mass (Fig. 1). If $g(E) > 1$ then $m_\beta^{(1)}$ approximation is more appropriate, if $g(E) < 1$, $m_\beta^{(2)}$ should be chosen. Fig. 1 shows that for certain range of E, the ratio is greater then 1, and for other it is smaller. Thus, we can see that it is difficult to decide which approximation is more suitable in particular experiment.

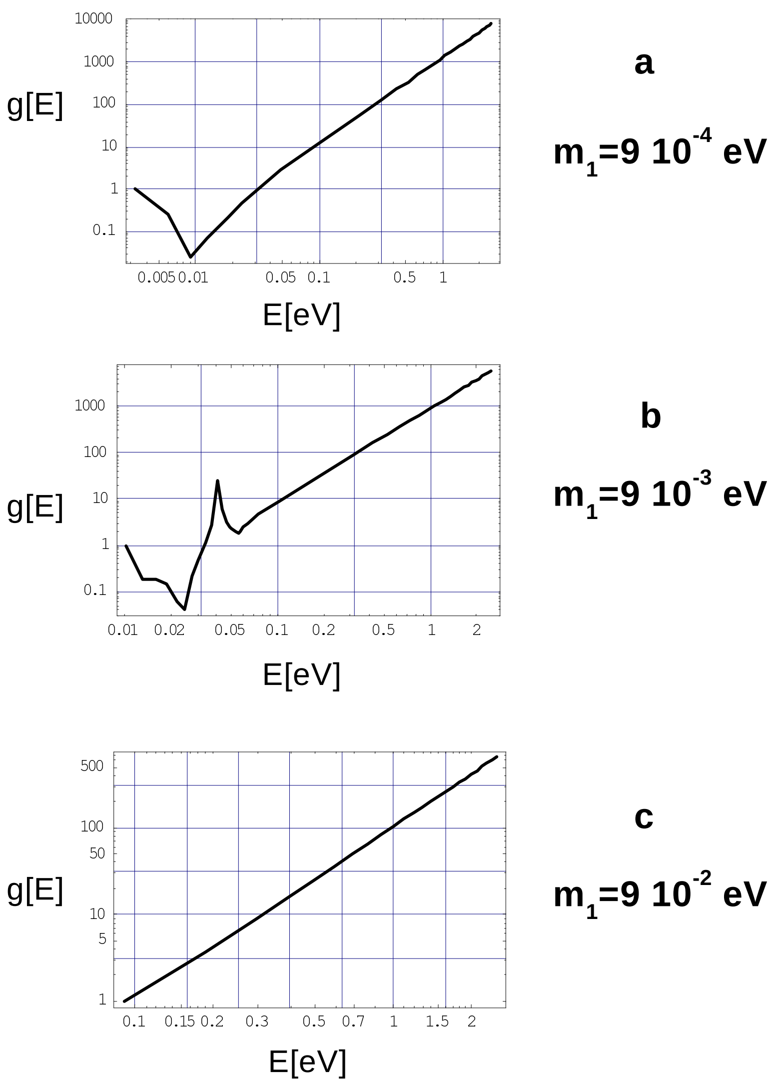

Fig. 1. g(E) as a function of electron energy E for minimal neutrino mass.

There is another possibility for above analysis [8, 10], which does not depend on energy value but only on spectrometer sensitivity. Let us assume that ΔE is the smallest energy interval which can be probed by the detector. Then the number of observed events in the last energy bin

$$(E_0 - m_1 - \Delta E, E_0 - m_1)$$

is given by

$$N_i(\Delta E) = \int\limits_{E_0-m_1-\Delta E}^{E_0-m_1} \left(\frac{dN}{dE}\right)_i dE \,. \tag{18}$$

For small interval ΔE, the $R(E)$ smooth function of E can be approximated by

$$R(E) \approx R(E_0 - m_1) \,. \tag{19}$$

Then for our purpose we can use scaled energy distribution

$$n_i(\Delta E) = \frac{1}{R(E_0 - m_1)} N_i(\Delta E) = \int\limits_{E_0-m_1-\Delta E}^{E_0-m_1} f_i(E) \,. \tag{20}$$

So explicitly

$$\begin{aligned} n_0(\Delta E) \;=\; & \frac{1}{3R(E_0 - m_1)} \{|U_{e1}|^2 B^{3/2} \\ & + |U_{e2}|^2 \left(B - \delta m^2_{\rm solar}\right)^{3/2} \Theta\left(\Delta E - (m_2 - m_1)\right) \\ & + |U_{e3}|^2 \left(B - \delta m^2_{\rm solar} - \delta m^2_{\rm atm}\right)^{3/2} \Theta\left(\Delta E - (m_3 - m_1)\right)\} \,, \end{aligned} \tag{21}$$

and

$$n_i(\Delta E) = \left(B - (m_\beta^{(i)})^2\right)^{3/2} \Theta\left(\Delta E - (m_\beta^{(i)} - m_1)\right) , \tag{22}$$

with

$$B = \Delta E(\Delta E + 2m_1)$$

are found. Analogously to the case which have been discussed before, the following ratio is considered

$$h(\Delta E) = \frac{|n_0(\Delta E) - n_2(\Delta E)|}{|n_0(\Delta E) - n_1(\Delta E)|} \,. \tag{23}$$

This function is depicted in Fig. 2. We can see that independently of the lightest neutrino mass for $\Delta E > 0.1$ eV we obtain $h(\Delta E) > 0$. It means that $m_\beta^{(1)}$ approximation is better and should be used in the future KATRIN experiment data analysis. In this experiment the spectrometer sensitivity is estimate to be of about 1 eV.

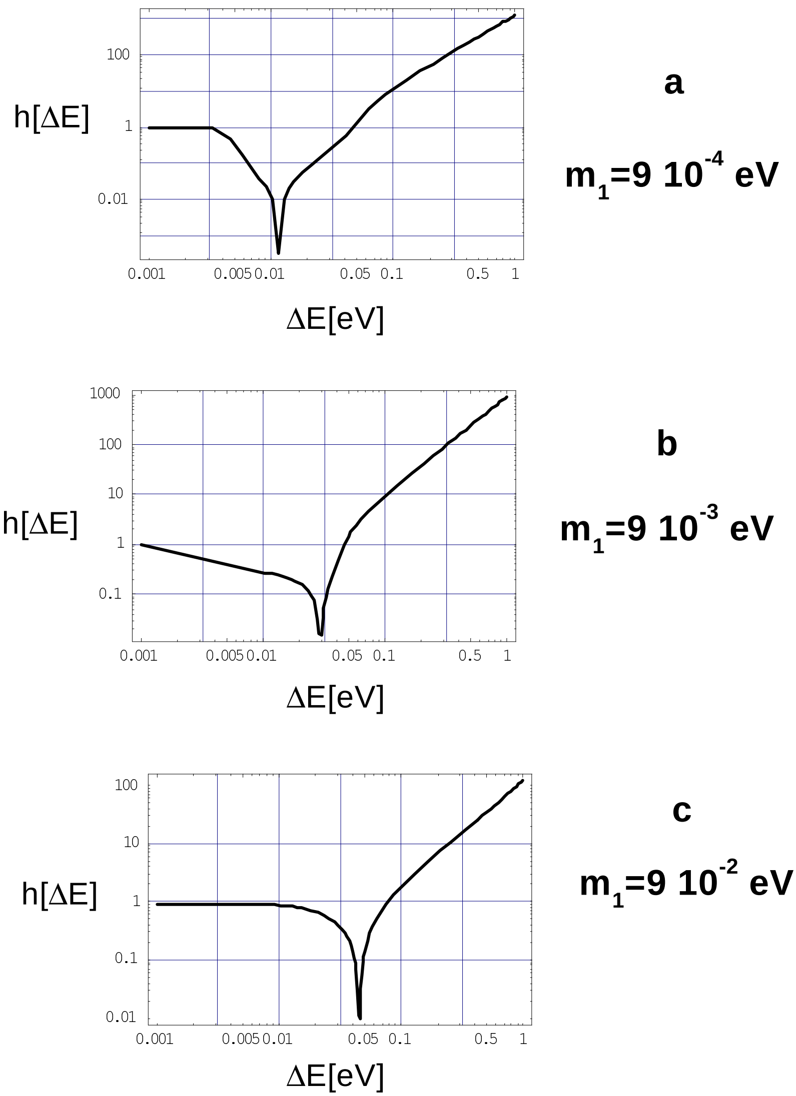

Fig. 2. h(ΔE) as a function of ΔE for minimal neutrino mass.

4. Tritium β decay and neutrino mass determination [12]

Now, let us discuss how future bound on m_β or measured value of this parameter are able to determine the neutrino masses. First of all, it is obvious that tritium β decay alone is not able to give us full information. It is easy to prove that

$$(m_\nu)_{\rm min} < m_\beta < (m_\nu)_{\rm max}\,, \tag{24}$$

from which it follows that the minimal neutrino mass is smaller then future bound or measured of m_β. Much more can be inferred if results of tritium β decay are taken together with neutrino oscillation data. Then we can find (when $m_\beta = m_\beta^{(1)}$)

$$m_\beta^2 = (m_\nu)^2_{\rm min} + \Omega\,, \tag{25}$$

and

$$(m_\nu)^2_{\rm max} = m_\beta^2 + \Lambda\,, \tag{26}$$

where the Ω and Λ are quantities fully determined from oscillation experiments and the neutrino mass scheme. For normal mass hierarchy scheme they are

$$\begin{aligned} \Omega &= (1-|U_{e1}|^2)\delta m^2_{\rm solar} + |U_{e3}|^2\delta m^2_{\rm atm}\,,\\ \Lambda &= |U_{e1}|^2\delta m^2_{\rm solar} + (1-|U_{e3}|^2)\delta m^2_{\rm atm}\,. \end{aligned} \tag{27}$$

Taking the present value of oscillation parameters for LMA MSW (see Table I) we get

$$\begin{aligned} \Omega &= 0.9\times 10^{-4}\ {\rm eV}^2\,,\\ \Lambda &= 30\times 10^{-4}\ {\rm eV}^2\,. \end{aligned} \tag{28}$$

Even now the oscillation data gives relatively small error of the Ω quantity

$$\Delta\Omega = 0.3\times 10^{-3}{\rm eV}^2\,. \tag{29}$$

In such case, we can see that the effective neutrino mass m_β, measured in tritium β decay together with Ω (or Λ) parameters calculated from oscillation experiments, determine the neutrino masses. For larger $(m_\nu)_{\rm min}$ uncertainties of Ω are negligible and error of $(m_\nu)_{\rm min}$ comes merely from Δm_β

$$\Delta(m_\nu)_{\rm min} \equiv \frac{m_\beta}{(m_\nu)_{\rm min}}\Delta m_\beta\,. \tag{30}$$

TABLE I

The allowed ranges of neutrino parameters from global analysis [11].

		min.	best fit	max.
$\tan^2 \Theta_{13}$		0	0.005	0.055
δm^2_{32} $[\times 10^3\ \mathrm{eV}^2]$		1.4	3.1	6.1
$\tan^2 \Theta_{23}$		0.39	1.4	3.0
δm^2_{21} $[\mathrm{eV}^2]$	LMA $\times 10^5$	~ 1.6	3.3	~ 20
	LOW $\times 10^8$	~ 0.08	9.6	~ 30
	SMA $\times 10^6$	~ 4	5.1	~ 9
$\tan^2 \Theta_{12}$	LMA	0.2	0.36	~ 1
	LOW-QVO	0.2	0.58	3
	SMA	$\sim 10^{-4}$	6.8×10^{-4}	$\sim 2 \times 10^{-3}$

5. Conclusion

We have discussed and compared two parametrization of the electron energy distribution for tritium β decay used in literature. We have found that for energy resolution ΔE of future detector which is larger then $|m_3 - m_1|$, the effective neutrino mass m_β

$$m_\beta^{(1)} = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 m_i^2}$$

should be used. If energy resolution is smaller then $|m_3 - m_1|$, the other parametrization

$$m_\beta^{(2)} = \sum_{i=1}^{3} |U_{ei}|^2 m_i$$

better approximates the electron energy distribution.

For almost degenerated neutrino masses scheme $m_1 \approx m_2 \approx m_3$, the differences between both parametrizations are negligible and

$$m_\beta^{(1)} = m_\beta^{(2)} = m_\beta\,. \tag{31}$$

The value of m_β if it is measured in future β decay experiment, taken into account together with $\delta m^2_{\mathrm{solar}}$, $\delta m^2_{\mathrm{atm}}$ and $|U_{ei}|^2$ quantities determined from the oscillation experiments, have a chance to find more precise neutrino masses.

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