HIGH ENERGY SCATTERING AND AdS/CFT*

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In this talk we describe the application of the AdS/CFT correspondence for a confining background to the study of high energy scattering amplitudes in gauge theory. We relate the energy behaviour of scattering amplitudes to properties of minimal surfaces of the helicoidal type. We describe the results of hep-th/0003059 and hep-th/0010069 for amplitudes with vacuum quantum number exchange and, very briefly, hep-th/0110024 on the extension of this formalism to Reggeon exchange.

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1. Introduction

It is well known that high energy scattering amplitudes with small momentum transfer (more precisely $t/s \to 0$) are *phenomenologically* well described by the exchange of Regge poles (see *e.g.* [1]). The dominant contribution to scattering processes with no exchange of quantum numbers is the exchange of the Pomeron. These amplitudes behave like $s^{1.08+0.25t}$. Other processes, involving the exchange of a Reggeon, have a different behaviour of the type $s^{0.5+1t}$. The theoretical description of such processes from first principles remains a formidable challenge, as it is of an inherently nonperturbative character. In this talk we will describe an approach developed in [2–6] which uses the AdS/CFT correspondence.

The AdS/CFT correspondence [7,8] was first proposed as an equivalence between $\mathcal{N} = 4$ supersymmetric gauge theory and type IIB string theory in a curved $AdS_5 \times S^5$ background. The statement that the two theories are

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indeed equivalent means that all (gauge-invariant) observables in the SYM should be calculable in the string theory language. At least for some observables like Wilson loops such prescriptions are available (but there is no direct proof up till now). The utility of this correspondence for nonperturbative calculations in gauge theory comes from the fact that calculations done for large 't Hooft coupling ($\lambda = g_{\mathrm{YM}}^2 N$) are translated into (semi-)classical calculations on the string theory side.

The original version of the AdS/CFT correspondence involved a conformal supersymmetric gauge theory, but later it was generalized to other cases including confining theories. We will use one such version to study the interplay between confinement and properties of soft high energy scattering amplitudes. Our result should be, however, quite generic.

Finally let us mention that other approaches to the description of the nonperturbative scattering physics exist and use various models of the nonperturbative vaccuum like the instanton vaccuum [9,10] and Stochastic Vacuum Model [11,12].

The outline of this talk is as follows. In Section 2 we will describe the basic features of the AdS/CFT correpondence, in Section 3 we will state the appropriate formulation of scattering amplitudes in gauge theories which is a convenient starting point for performing the calculation using the AdS/CFT correspondence. Then we will move on to evaluate the classical contribution to the scattering amplitude which is determined, in this framework, by solving a minimal surface problem. In Section 6 we will evaluate the contribution of quadratic fluctuations and show how it gives rise to a shift of the intercept. In Section 7 we give a brief discussion of Reggeon exchange. We close the paper with a summary and outlook.

2. The AdS/CFT correspondence

The AdS/CFT correspondence was discovered by Maldacena [7], who conjectured that two seemingly unrelated theories, namely $\mathcal{N} = 4$ SYM gauge theory and type IIB string theory in an $\mathrm{AdS}_5 \times \mathrm{S}^5$ space are in fact completely equivalent. This statement gives a very concrete realization of the old hope that gauge theories can be described by string theory in the large N limit. The novel feature is the fact that strings of the dual string theory live in 10 dimensions, the geometry of which is moreover curved. These extra dimensions could be roughly understood as some parameter space for some (unspecified at the moment) semiclassical field configurations in gauge theory. An example of such an interpretation is the identification of instantons in $\mathcal{N} = 4$ SYM with $D(-1)$ string instantons (which are points in $\mathrm{AdS}_5 \times \mathrm{S}^5$). The 5^{th} coordinate of these points represents the size of the instanton in $\mathcal{N} = 4$ SYM [13]. So the string theory side of the AdS/CFT correspondence can be thought of to be a convenient and very nontrivial choice of degrees of freedom in the original gauge theory especially suited to studying properties at large gauge coupling (see below).

A difference with the old 'effective string' picture of QCD is that this correspondence is thought to be exact and not only valid in the IR. The various properties of different gauge theories are encoded in differing background geometries for the string theory.

The utility of the AdS/CFT correspondence lies in the fact that when the gauge coupling ($g_{\mathrm{YM}}^2 N$) is large, gravity (in the 10D string background geometry — this has nothing to do with real world gravity!) becomes weak and (semi-)classical methods in string theory become applicable.

As it was already mentioned, the correspondence was later extended to different gauge theories, some of which were confining. Although an exact counterpart for QCD is unknown, we will perform the calculations in a specific confining background and argue that the main features of our approximation scheme are generic and do not depend on the very detailed properties of the background geometry.

Before we proceed let us briefly recall the way in which the AdS/CFT correspondence arose. The idea is to construct a gauge theory from string theory and to look at this same construction from two perspectives, each of which can be thought of to give a different picture of the same underlying theory. These two points of view become just the two sides of the AdS/CFT correspondence.

The first path to gauge theory is to consider N coincident D3 branes in flat 10D space. The D3 branes are nonperturbative objects in type IIB string theory. They are 4-dimensional hypersurfaces on which open strings may end. The massless excitations of these open strings form the multiplet of $\mathcal{N} = 4$ gauge theory. In addition there are massive excitations with the mass scale set by $m^2 \propto 1/\alpha'$. When one takes the $\alpha' \to 0$ limit, these additional states will decouple and one will be left with just the gauge theory (and a noninteracting flat 10D bulk theory).

Let us now take into account that the D3 branes are massive and charged objects. Therefore a stack of N D3 branes will curve spacetime and generate some metric for the closed strings. After an appropriate rescaling of coordinates one then performs the same $\alpha' \to 0$ limit as before. The result is the $\mathrm{AdS}_5 \times \mathrm{S}^5$ background for closed strings. Since this should be exactly the same system as before one is led to the AdS/CFT correspondence.

2.1. Wilson loops

As an example of using the AdS/CFT correspondence let us recall the prescription for calculating the expectation values of Wilson loops. We will use it later for evaluating scattering amplitudes.

The generic feature of the 10D dual geometries is that they posses a well defined boundary. The prescription for calculating $\langle W(C)\rangle$ is to place the contour C on the boundary, and consider the partition function of strings in the AdS bulk which are spanned on C [14]:

$$\langle W(C)\rangle \sim \int_{\partial\Sigma=C} \mathcal{D}X^A \mathrm{e}^{-\frac{1}{2\pi\alpha'}S_{\mathrm{string}}(\Sigma)}\,, \tag{1}$$

where Σ is the string worldsheet defined by the bosonic fields X^A, $A=0.9$ (here we suppressed the fermionic partners), S_{string} is the string action. $1/\alpha'$ typically involves $\sqrt{g_{\mathrm{YM}}^2 N}$, so at large gauge coupling the partition function is saturated by the string worldsheet spanned on C whose surface is minimal:

$$\langle W(C)\rangle \sim \mathrm{Fluctuations}\,(C)\cdot \mathrm{e}^{-\frac{1}{2\pi\alpha'}\mathrm{Area}\,(\Sigma_{\mathrm{minimal}})}\,. \tag{2}$$

We will now illustrate this prescription with a calculation of the static $q\bar{q}$ potential in the case of (a) the conformal $\mathcal{N}=4$ theory and (b) a confining theory.

2.1.1. $\mathcal{N}=4$ SYM theory

In this case the background geometry is pure $\mathrm{AdS}_5\times\mathrm{S}^5$:

$$ds^2 = \frac{dz^2}{z^2} + \frac{dx^\mu dx^\mu}{z^2} + d\Omega_5\,. \tag{3}$$

The boundary is at $z=0$. The problem of calculating the potential boils down to finding $\langle W(T\times R)\rangle$ with $T\to\infty$. Thus we have to put the two lines separated by R on the boundary at $z=0$. Due to the factors of $1/z^2$, we see that in order to minimise distances (and hence the area), it pays to increase z and go into the bulk, as far from the boundary as possible. The resulting minimal surface in the bulk was found in [14] and the (regularized) area was evaluated to yield:

$$\langle W(T\times R)\rangle \sim \mathrm{e}^{\frac{4\pi^2\sqrt{2g_{\mathrm{YM}}^2 N}}{\Gamma(1/4)^2}\frac{T}{R}}\,, \tag{4}$$

where we expressed all string-theoretic parameters in terms of the gauge coupling. In these calculations one always subtracts a divergence due to singularity of the metric near the boundary [14, 15].

We note that despite the fact that we have a string picture of a Wilson loop we obtained a *Coulombic* potential. This is because of the nontrivial curved geometry of the string background.

2.1.2. Confining black hole background

The first and simplest background for a confining theory was proposed by Witten in [16]. The metric has the form:

$$ds^2 = \frac{16}{9}\frac{1}{z^{2/3}(1-(z/R_0)^4)}\frac{dz^2}{z^2} + \frac{\eta_{\mu\nu}dx^\mu dx^\nu}{z^2} + \ldots\,, \tag{5}$$

where R_0 is the scale of the horizon and ... represent additional dimensions which do not play any role for the problem at hand. Again it pays to increase z as much as possible, but now the range of possible z is limited by the horizon radius R_0. We, therefore, have two qualitatively different regimes. When the size of the Wilson loop $W(C) \equiv W(T \times R)$ is greater than R_0, the minimal surface will rise up to the horizon and then will extend at a fixed value of z_{fixed} close to R_0 (see Fig. 1). The area of this minimal surface will be proportional to the area of C, since it is measured in the effectively flat metric

$$ds^2 = \frac{1}{z_{\text{fixed}}^2} dx^\mu dx^\mu\,. \tag{6}$$

Therefore, we have

$$\langle W(T\times R)\rangle \sim \mathrm{e}^{-\frac{1}{2\pi\alpha'_{\text{eff}}}TR}\,, \tag{7}$$

where we absorbed all the scale factors in the effective string tension. We see that in this regime the details of the metric (powers/coefficients) do not matter and we expect a large degree of universality.

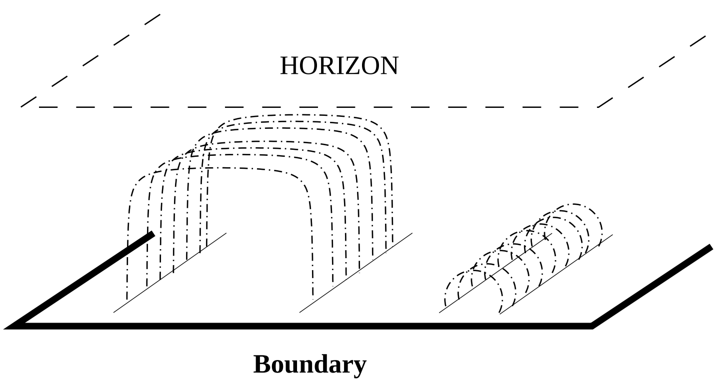

Fig. 1. Static $q\bar{q}$ potential using the black hole geometry.

On the other hand when the size of C is small compared to R_0, the minimal surface problem will depend very much on the structure of the metric for small z (*cf.* the smaller loop in Fig. 1), and, therefore, it will depend on the detailed nature of the gauge theory.

2.1.3. Fluctuations

In the above examples we included just the action of the classical solution of the equations of motion for the string (the minimal surface equations). It is often interesting to go further, and calculate the contribution of quadratic fluctuations around this solution. In general this is a very difficult problem, so we will now state the result for the confining black hole background in the flat space approximation. The answer in flat space is [17–20]

$$\text{Fluctuations}\,(T \times R) \sim (\det \Delta)^{-\frac{n_\perp}{2}} = \mathrm{e}^{n_\perp \cdot \frac{\pi}{24} \cdot \frac{T}{R}}\,, \tag{8}$$

where $\det \Delta$ is the determinant of the Laplacian with Dirichlet boundary conditions, and $n_\perp$ is the effective number of massless transverse degrees of freedom for the string. For the superstring in the black hole background the expected number is $n_\perp = 7$ [21] (see Section 6). The contribution (8) is called the Lüscher term and gives a Coulombic correction to the linear confining potential. It is independent of the string tension, and in fact quite universal, in the sense of being independent of most of the details of the specific string theory.

3. Gauge-theoretic scattering amplitudes

In this section we will rewrite scattering amplitudes in gauge theory in a form adapted to calculation using the techniques of the AdS/CFT correspondence.

It is convenient to consider scattering amplitudes in impact parameter space which are defined through

$$A(s,t) = \frac{is}{2\pi} \int d^2 l\, \mathrm{e}^{iq\cdot l}\, \tilde{A}(s, L = |l|)\,. \tag{9}$$

Since we want to study soft processes with small momentum transfer (and no exchange of quantum numbers), we may use the eikonal approximation [12]. In this approximation the impact parameter $q\bar{q}$ scattering amplitude is given by a correlation function of two Wilson lines which follow classical straight line trajectories (see Fig. 2):

$$A(s,L) = \left\langle \mathrm{e}^{i\int_{L_1} A} \mathrm{e}^{i\int_{L_2} A} \right\rangle\,. \tag{10}$$

This expression suffers from two drawbacks. It is IR divergent and it is not gauge invariant. The first property requires us to work with an IR cut off *i.e.* a finite temporal length T of the lines. In addition we also have to add a gauge connector between the end points in order to obtain a gauge invariant quantity. Alternatively we may choose to work with a gauge invariant and

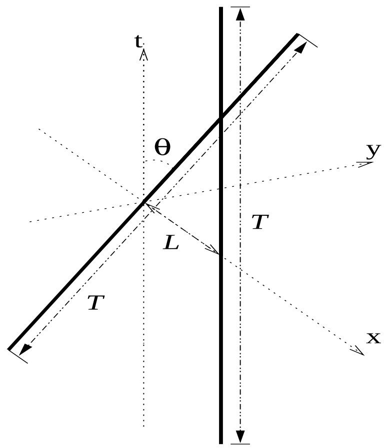

Fig. 2. Geometry of the Wilson lines in Euclidean space.

IR finite amplitude for the scattering of two $q\bar{q}$ *pairs*. Then the lines in (10) are replaced by Wilson loops, closed at infinity by mesonic wave functions.

The latter quantity has a very clear physical significance, however, it is more difficult to calculate. Therefore, we will concentrate on the $q\bar{q}$ scattering with an explicit IR cut-off. In the context of a confining theory which we will consider, such an IR cut-off may have a direct physical significance as a scale on which colour strings start to break. Another argument in favour of studying this simpler situation is that a variational treatment of the scattering of two $q\bar{q}$ pairs in the same framework [3], essentially leads to a superposition of $q\bar{q}$ scatterings with small IR cut-off T.

It will turn out to be convenient to perform one more reformulation of the problem and to go to *Euclidean* signature. There we will calculate the correlation function of the lines as a function of the relative angle θ, staying within Euclidean gauge theory. The result $A(\theta, L)$ will then be continued back to *Minkowski* space using the substitutions

$$\theta \longrightarrow -i\chi \sim -i\log s\,, \qquad\qquad T \to iT\,. \tag{11}$$

The above procedure was first used within the eikonal approximation in perturbative QED and QCD in [22].

Since the scattering amplitude is expressed as a correlation function of two Wilson lines, we may apply now the methods of the AdS/CFT correspondence in a confining background to calculate it.

4. The evaluation of scattering amplitudes

Let us now evaluate the correlation function of two Wilson lines inclined at an angle θ and separated by a transverse distance L. To this end, following the prescription of Section 2, we have to place the two lines on the boundary and find the minimal surface in the black hole geometry. Again two regimes will appear, depending on the relation of the impact parameter to the confinement scale R_0. When the impact parameter is sufficiently large the same arguments should apply as for the study of the confining potential: the minimal surface problem is essentially transported up to the horizon $z \sim R_0$, where the minimization occurs in flat space. This is the case that we are going to study. We thus have to find a minimal surface in *flat space* between two lines at an angle.

5. Minimal surface

The appropriate minimal surface is indeed well known — it is the helicoid. It can be parametrised by

$$t = \tau \cos p\sigma \,, \tag{12}$$

$$y = \tau \sin p\sigma \,, \tag{13}$$

$$x = \sigma \,, \tag{14}$$

where $\tau = -T \dots T$, $\sigma = -L/2 \dots L/2$ and $p = \theta/L$. According to the AdS/CFT prescription the correlation function is equal to

$$\mathrm{e}^{-\frac{1}{2\pi\alpha'_{\mathrm{eff}}}\mathrm{Area}} = \mathrm{e}^{-\frac{1}{2\pi\alpha'_{\mathrm{eff}}}\int d\sigma d\tau \sqrt{\det g_{ab}}} \,, \tag{15}$$

where $g_{ab} = \partial_a X^\mu \partial_b X^\mu$ is the induced metric. The explicit formula for the area is [3]

$$\mathrm{Area} = \int d\sigma d\tau \sqrt{\det g_{ab}} = \int_{-L/2}^{L/2} d\sigma \int_{-T}^{T} d\tau \sqrt{1+p^2\tau^2} \tag{16}$$

$$= \int_{-L/2}^{L/2} d\sigma \left\{ T\sqrt{1+p^2T^2} + \frac{1}{p}\log\left(pT + \sqrt{1+p^2T^2}\right)\right\} . \tag{17}$$

In order to obtain the scattering amplitude we have to perform analytical continuation to Minkowski space. A naive continuation would give a result which is a pure phase. However, we see that the formula for the area involves a logarithm which has a cut in the complex plane. *A priori* we cannot rule out the possibility of moving onto a different sheet of the logarithm when doing the analytical continuation, especially as we are dealing with an inherently nonperturbative Euclidean correlation function. It is therefore interesting to explore the physical consequences of moving to a different Riemann sheet. Consequently we have to perform the substitution

$$\log(\ldots) \longrightarrow \log(\ldots) - 2\pi i n \tag{18}$$

with n being some integer number. Under this transformation, the amplitude gets a contribution:

$$\mathrm{e}^{\frac{1}{2\pi\alpha'_{\mathrm{eff}}}\frac{L^2}{\theta}2\pi i n} \longrightarrow \mathrm{e}^{-\frac{1}{\alpha'_{\mathrm{eff}}}\frac{L^2}{\log s}n} \tag{19}$$

which is *independent* of the IR cut-off T. In the following we will neglect the T dependent terms assuming that T is small (some justification for this assumption was given in [3]). After Fourier transform we obtain an inelastic amplitude with a linear Regge trajectory:

$$(\text{prefactor}) \cdot s^{1+\frac{\alpha'_{\mathrm{eff}}}{4}t}\,. \tag{20}$$

The prefactor here includes a $\log s$, further such contributions may come from α' corrections. In the following we concentrate on the dominant terms which give rise to a power-like s^α behaviour.

An interesting feature of this result is that the linear slope arose through the analytic structure of the helicoid area. Moreover, the slope $\alpha'_{\mathrm{eff}}/4$ characteristic of soft Pomeron exchange appeared in a natural way[1].

In the next section we will see that quadratic fluctuations of the string worldsheet around the helicoidal minimal surface give rise to a shift of the intercept.

6. Fluctuations

In order to perform the calculation of the contribution of the quadratic fluctuations we have to decompose the fields:

$$X^A(\sigma,\tau) = X^A_{\mathrm{helicoid}}(\sigma,\tau) + x^A(\sigma,\tau) \tag{21}$$

and expand the string action to second order in $x^A(\sigma,\tau)$. We assume, as is the case for string theories arising in the AdS/CFT correspondence, that the action for quadratic fluctuations is just the Polyakov action.

[1] In the black hole background α'_{eff} is a free parameter, but it is directly linked to the static $q\bar{q}$ potential. Hence we may take the phenomenological value of QCD string tension as defining α'_{eff}.

In order to simplify the calculation [4], it is convenient to change the parametrisation of the helicoid and replace the variable τ in (12)–(14) by

$$\rho = \frac{1}{p} \log(p\tau + \sqrt{1+p^2\tau^2}) \ . \tag{22}$$

The advantage of doing this is that the induced metric on the helicoid w.r.t the variables ρ, σ is conformally flat *i.e.*

$$g_{ab} = (\cosh^2 p\rho)\ \delta_{ab} \,. \tag{23}$$

Therefore, since string theory in the AdS background is *critical*, we may perform the calculation for the conformally equivalent flat metric $g_{ab} = \delta_{ab}$. The path integral with the Polyakov action gives

$$(\det \Delta)^{-\frac{D-2}{2}} \equiv (\det \Delta)^{-\frac{n_\perp}{2}} \ , \tag{24}$$

where $\det \Delta$ is the determinant of the Laplacian operator, with Dirichlet boundary conditions, on the rectangle defined by the range of variation of ρ and σ. D is the number of massless modes (typically the dimension of space time in which the string theory lives *i.e.* 10 for the superstring) while the -2 stands for the contribution of the ghosts. In the AdS black hole background, the effective number of transverse massless modes is $n_\perp = 10 - 2 - 1 = 7$. The additional subtraction of -1 comes from the fact that due to the curved character of the metric one bosonic mode becomes massive [21]. For ordinary superstring theory in flat 10 dimensions, we would have to include also the contribution of worldsheet fermions which would exactly cancel the bosonic one. However, as was argued in [21], due to the appearance of a nonvanishing Ramond–Ramond background field in the AdS black hole geometry, all the fermions become massive and thus give subleading contributions.

The determinant of the Laplacian in (24) has to be calculated for a rectangle of size $a \times b$ where

$$a \ = \ L\,, \tag{25}$$

$$b \ = \ \frac{2L}{\theta} \log\left(pT + \sqrt{1+p^2T}\right) \ . \tag{26}$$

This determinant can be evaluated using ζ function regularisation in a calculation equivalent to the Lüscher term computation (*cf.* (8)), and for high energies we obtain [4]

$$\text{Fluctuations} = \exp\left(n_\perp \frac{\pi}{24}\frac{a}{b}\right) = \exp\left(n_\perp \frac{\pi}{24} \frac{\theta}{2\log\left(pT + \sqrt{1+p^2T^2}\right)}\right) . \tag{27}$$

Here we kept the piece which dominates after continuation to Minkowski space (then $a/b = \mathcal{O}(\log s) \gg 1$).

We now have to perform the same analytical continuation to Minkowski space as we did for the area in the preceding section. Namely, we let $\log \to \log -2\pi i n$. Furthermore, we neglect the logarithmic T dependent terms. The outcome is

$$\text{Fluctuations} = \mathrm{e}^{\frac{n_\perp}{96}\log s} = s^{\frac{n_\perp}{96}}\,. \tag{28}$$

Putting together the above result and the contribution obtained in the previous section we obtain finally for the trajectory

$$(\text{prefactors}) \cdot s^{1+\frac{n_\perp}{96}+\frac{\alpha'_{\text{eff}}}{4}t}\,. \tag{29}$$

The values for $n_\perp$ suggested by the AdS/CFT correspondence would give an intercept of 1.073 (or 1.083 for $n_\perp = 8$), very close to the observed soft Pomeron intercept of 1.08. The phenomenological value of α'_{eff} extracted from the static quark-antiquark potential is $\alpha'_{\text{eff}} \sim 0.9\,\text{GeV}^{-2}$. This gives the slope 0.225 in comparison with the observed one of 0.25.

7. Reggeon exchange

It is interesting to see, whether in the same framework one can obtain an analogous description of Reggeon exchange, which should dominate for processes *with* an exchange of quantum numbers in the t channel. The problem is interesting as phenomenologically Reggeon trajectories are quite different from the one of the Pomeron. The slope is four times larger, while the dominant intercepts are around 0.5 (they depend on the trajectory) instead of 1.08. The Reggeon amplitudes typically behave like $s^{0.5+\alpha'_{\text{eff}}t}$.

We will now briefly relate the relevant results of [5]. Since Reggeon exchange always involves an exchange of quarks in the t channel (see the schematic spacetime picture of the process in Fig. 3), the eikonal approximation which allowed to express the scattering amplitude as a correlation function of Wilson *lines*, is no longer valid. The approach used in [5] uses the worldline formalism which expresses the fermionic propagator in an external gauge field $\mathcal{A}$ as a path integral over quark trajectories [23–26]:

$$S(x,y|\mathcal{A}) = \int \mathcal{D}x^\mu(\tau)\,\mathrm{e}^{-m\cdot\text{Length}} \cdot \{\text{Spin Factor}\} \cdot \mathrm{e}^{i\int_{\text{trajectory}} A}\,, \tag{30}$$

where the {Spin Factor} keeps track of the spin 1/2 nature of the quarks. In the above expression the colour and spin parts do factorise, which is very convenient for calculations using various models of the nonperturbative gluonic vacuum.

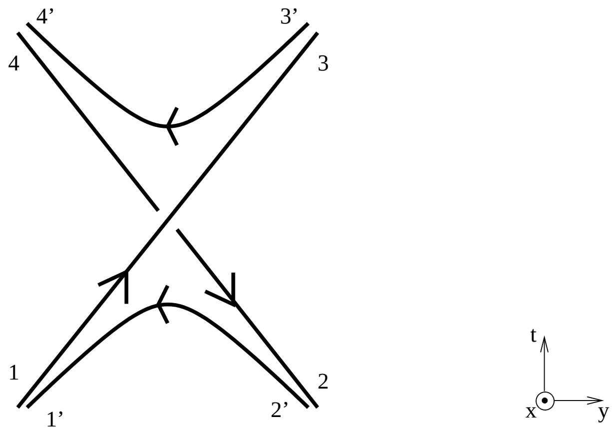

Fig. 3. Spacetime picture of a meson-meson scattering process mediated by Reggeon exchange. The impact parameter axis is perpendicular to the longitudinal $t-y$ plane.

In [5] we used the expressions (30) for the exchanged quarks and eikonal approximation for the spectator quarks. The gauge field dependent part became a Wilson loop expectation value, which was calculated using the AdS/CFT correspondence. The result again involved the helicoid (assuming that the exchanged quarks were light). At this stage one obtains an effective action for the trajectories of the exchanged quarks. Finally, we performed this remaining path integral over the trajectories of the exchanged quarks (now constrained to lie on the helicoid) by saturating it with the classical saddle point solution.

The resulting saddle point was imaginary leading to *(i)* an inelastic amplitude, and *(ii)* a linear Regge slope $\alpha'_{\mathrm{eff}}t$. The contribution of the spin factors gave a $1/s$ suppression so at this stage the energy dependence of the Reggeon exchange amplitude was

$$s^{0+\alpha'_{\mathrm{eff}}t}\,. \tag{31}$$

When we include the contribution of the fluctuations of the string worldsheet around the helicoid, and continue analytically this expression to the saddle point configuration we obtain a shift of the intercept by $n_\perp/24$. The final result is thus:

$$s^{0+\frac{n_\perp}{24}+\alpha'_{\mathrm{eff}}t}\,. \tag{32}$$

We note that within this framework, we obtain the factor of 4 between the slopes of the Reggeon and the 'Pomeron'-like trajectories. The intercept of the Reggeon is also four times larger than the difference $\alpha_{\mathrm{Pomeron}}(0)-1$.

8. Discussion

In this paper we described the application of the AdS/CFT correspondence to the study of high energy scattering amplitudes in the Regge limit. In the eikonal approximation these amplitudes were reduced to the evaluation of a Wilson loop correlation function. Using the AdS/CFT correspondence for a confining theory this reduced the problem to studying properties of minimal surfaces of the helicoidal type. The linear slope of the Regge trajectory arose when making analytical continuation from Euclidean to Minkowski space [3], while a shift of the intercept appeared after including the contribution of quadratic fluctuations of the string worldsheet around the helicoid [4]. The resulting trajectory is very close to the experimentally observed soft Pomeron trajectory.

In the above approach it was crucial that we considered a theory with confinement, which in the setting of the black hole background allows one to use a flat space approximation for solving the minimal surface problems. For smaller impact parameters, where confinement is not important the resulting formulas would be quite different and would also depend on the specific type of gauge theory considered (which would translate to different properties of the metric near the boundary).

Finally we briefly reported on the work of [5] where amplitudes with Reggeon exchange were considered. These scattering processes involve an exchange of a quark-antiquark pair in the t channel and thus necessarily require going beyond the eikonal approximation. In order to overcome this difficulty we used the worldline formalism which expresses (Euclidean) fermionic propagators in a background gauge field as a path integral over quark trajectories. The averaging over gauge fields was done using again AdS/CFT correspondence and helicoidal minimal surfaces. The remaining path integral was done by saddle point leading to an inelastic amplitude with a linear Regge trajectory with *four* times larger slope than the one for the Pomeron. Fluctuations led to an intercept of the order of 0.33. This intercept is lower than the dominant intercepts observed experimentally. A possible reason for this discrepancy was our assumption that the exchanged quark trajectories do not deform the helicoid minimal surface. In addition we did not evaluate fluctuations of the boundaries of the helicoid. Due to high nonlinearity it is difficult to estimate whether this may or may not modify the intercept.

It seems that the framework of AdS/CFT correspondence is very convenient to study the interplay of soft scattering amplitudes and confinement. The familiar Regge trajectories arise here in a nonstandard manner through the properties of the helicoid minimal surface. In addition two types of Regge trajectories appear naturally, corresponding qualitatively (and partly quantitatively) to the observed Pomeron and Reggeon trajectories. It would be interesting to address a number of further problems in this framework. In particular the transition to smaller impact parameters which would necessarily involve a departure from the flat metric approximation used here. Another very interesting but difficult problem would be to understand how unitarization occurs within the framework of the AdS/CFT correspondence.

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