ABSOLUTE NEUTRINO MASSES*

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Since the recent convincing evidence for massive neutrinos in oscillation experiments, the next task is to determine the absolute masses of neutrinos. A unique pattern of neutrino masses will be hopefully fixed in the future superbeam experiments and neutrino factories. However, the determination of the exact scale is more complicated and depends on the mass of the lightest neutrino $(m_\nu)_{\rm min}$. If $(m_\nu)_{\rm min} \gtrsim 0.35\,{\rm eV}$, the future tritium β decay experiments (*e.g.* KATRIN) will have a chance to establish absolute neutrino masses. For smaller masses, $0.004\,{\rm eV} \leq (m_\nu)_{\rm min} \leq 0.35\,{\rm eV}$, if neutrinos are Majorana particles, an additional information can be derived from the neutrinoless double β decay $(\beta\beta)_{0\nu}$ of nuclei and again the absolute neutrino masses can be fixed. If, however, $(m_\nu)_{\rm min} \leq 0.004$ eV, none of the present and foreseeable future experiments is known to be able to fix the mass scale.

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1. Introduction

The problem of the neutrino mass spectrum is the most important issue in the leptonic part of the Standard Model. It is expected that the knowledge of the absolute values of neutrino masses and their mixing pattern will put some light on the scale of new physics and the problem of particle masses in general. In astrophysics it will be possible to verify models of supernova explosion or, maybe, interpret the GZK cutoff [1,2]. Massive neutrinos may constitute the hot dark matter and can help to understand the problem of large scale structure formation in cosmology.

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The last years were very fruitful to neutrino physics. The behavior of atmospheric [3] and solar [4] neutrinos provides a rather strong evidence that neutrinos are massive particles. There are trials of alternative understanding of the observations which do not require massive neutrinos [5], but they use much more sophisticated assumptions and, more importantly, give poorer fits to the data [6]. We are, therefore, left with no other choice than to assume that neutrinos are massive particles. However, the problem of their absolute masses and mixing pattern remains unsolved. The oscillation phenomena are not able to determine their masses. Only the difference of mass squares, $\delta m_{ab}^2 = m_a^2 - m_b^2$ is fixed by this theory. Where do neutrino masses have a chance to be measured? Unfortunately, all present neutrino experiments are consistent with the Standard Model where three flavour neutrinos ν_e, ν_μ and ν_τ are massless particles and both the family $L_\alpha\,(\alpha = e, \mu, \tau)$ and the total $(L = L_e + L_\mu + L_\tau)$ lepton numbers are conserved. However, there is some chance that two kinds of experiments are "just around the corner" to determine the neutrino masses. Both are known for years — the beta decay and the neutrinoless double beta decay of nuclei. Already Fermi [7] in 1934 and Furry [8] in 1939 realized, that both processes are important for the neutrino mass determination. Astrophysics and cosmology with their model-dependent assumptions are also potential sources of information about neutrino masses. However, we do not discuss here bounds on neutrino masses which follow from such extraterrestrial experiments (see *e.g.* [2] for these issues).

In this talk we would like to shed some light on the present knowledge on neutrino masses and of their mixing pattern. We will also try to answer the question when and how precisely the absolute neutrino masses can be determined in future. Presently, the solar and atmospheric anomalies give much better fits for the case of active neutrino oscillations. The only experiment which needs an additional sterile neutrino, LSND [9] is still waiting for confirmation. That is the reason why three neutrino scenarios are considered here.

2. Neutrino masses in the light of present experimental data

Fits to solar and atmospheric anomalies give estimates on δm^2's and mixing matrix elements U_{ei}. Two different δm^2's are obtained

$$\delta m^2_{\rm solar} \ll 10^{-4}\ {\rm eV}^2 \quad [4] \tag{1}$$

and

$$\delta m^2_{\rm atm} > 10^{-3}\ {\rm eV}^2 \quad [3], \tag{2}$$

which indicates that two patterns of neutrino masses are possible. The first is known as normal mass hierarchy scheme (A_3) with $\delta m^2_{\rm solar} = \delta m^2_{21} \ll \delta m^2_{32} = \delta m^2_{\rm atm}$. The second one is so-called inverse mass hierarchy scheme ($A_3^{\rm inv}$) with $\delta m^2_{\rm solar} = \delta m^2_{21} \ll -\delta m^2_{31} = \delta m^2_{\rm atm}$ (see Fig. 1).

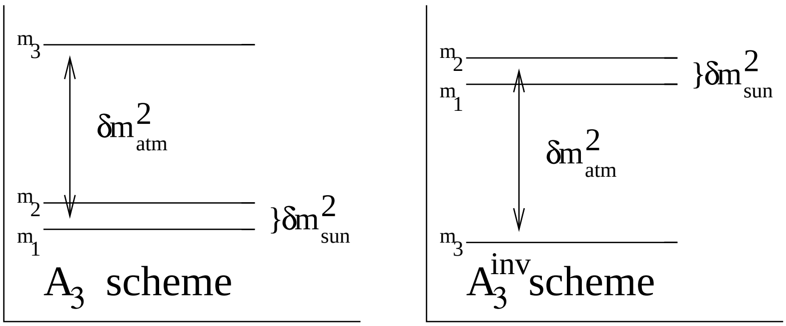

Fig. 1. Two possible mass spectra which can describe the oscillation data. The scheme A_3, normal mass hierarchy, has a small gap between m_1 and m_2 to explain the oscillation of solar neutrinos and a larger gap for the atmospheric neutrinos ($\delta m^2_{\rm sol} = \delta m^2_{21} \ll \delta m^2_{32} \simeq \delta m^2_{\rm atm}$; $m_1 < m_2 \ll m_3$). In the inverse mass hierarchy scheme $A_3^{\rm inv}$, $m_3 \ll m_1 < m_2$ and $\delta m^2_{\rm atm} \simeq -\delta m^2_{32} \gg \delta m^2_{21} \simeq \delta m^2_{\rm sol}$.

In both schemes the mass scale is determined by the mass of the lightest of neutrinos $(m_\nu)_{\rm min}$ ($= m_1$ in A_3 and $= m_3$ in $A_3^{\rm inv}$). So, together with $\delta m^2_{\rm solar}$ and $\delta m^2_{\rm atm}$ two more data must be known to solve the problem of the neutrino mass spectrum

(i) the scheme A_3 or $A_3^{\rm inv}$ and

(ii) the mass of the lightest neutrino $(m_\nu)_{\rm min}$.

Up to now the precision of experimental data is not good enough to give satisfactory answers to both of these questions. The schemes A_3 and $A_3^{\rm inv}$ are not distinguishable by present experiments. Oscillations in vacuum depend on $\sin^2\left((\delta m^2 L)/(4E)\right)$ and the sign of δm^2 is irrelevant. Fortunately, the oscillation probabilities for neutrino transitions involving ν_e or ν_μ are modified if neutrinos propagate through matter and the modification depends upon the sign of δm^2_{32}. In the leading approximation the probability of $\nu_\mu \to \nu_e$ neutrino oscillations in matter of constant density depends on the effective mixing angle $\Theta^{\rm eff}$ [10, 11]

$$\sin^2 2\Theta^{\rm eff} = \frac{\sin^2 2\Theta^{\rm vac}}{\left(\frac{A}{\delta m^2_{32}} - \cos 2\Theta^{\rm vac}\right)^2 + \sin^2 2\Theta^{\rm vac}} . \tag{3}$$

A is the matter amplitude [10] and $\Theta^{\rm vac}$ is the vacuum mixing angle. For antineutrino oscillations the sign of A is reversed. Comparison of transitions involving neutrinos and antineutrinos discriminates between the two signs of δm^2_{32}. Unfortunately, the present atmospheric neutrino data does not distinguish between neutrino and antineutrino oscillations and the sign of δm^2_{32} is not measured. A global analysis of the solar, atmospheric and reactor neutrino data determines five parameters: three mixing angles $\left(\Theta_{12}, \Theta_{13}, \Theta_{23}\ \left(0 < \Theta_{ij} < \frac{\pi}{2}\right)\right)$ and two mass square differences $\left(\delta m^2_{\rm atm}\text{ and } \delta m^2_{\rm solar}\right)$ [11]. Before the SNO data [12] four solutions where acceptable at the 95% CL [13]. However, the comparison of Solar Neutrino Signals in SNO and SuperKamiokande strongly disfavors the Small Mixing Angle (SMA MSW) and the Vacuum Oscillation (VO) solutions. Therefore, only two solutions remain [14]

(i) LMA MSW with

$$\delta m^2_{\rm solar} \approx (1.6 \div 20) \times 10^{-5}, \ \left(\delta m^2_{\rm solar}\right)_{\rm bestfit} \approx 3.3 \times 10^{-5} {\rm eV}^2, \tag{4}$$

and

$$\tan^2 \Theta_{\rm solar} \approx (0.2 \div 1)\,, \qquad \left(\tan^2 \Theta_{\rm solar}\right)_{\rm bestfit} \approx 0.36\,, \tag{5}$$

(ii) LOW MSW with

$$\delta m^2_{\rm solar} \ \simeq \ (0.08 \div 30) \times 10^{-8}, \quad \left(\delta m^2_{\rm solar}\right)_{\rm bestfit} \approx 9.6 \times 10^{-8} {\rm eV}^2, \tag{6}$$

and

$$\tan^2 \Theta_{\rm solar} \ \simeq \ (0.2 \div 3)\ , \qquad \left(\tan^2 \Theta_{\rm solar}\right)_{\rm bestfit} \approx 0.58. \tag{7}$$

The Θ_{13} mixing angle was also determined and is known to be small [15]

$$\tan^2 \Theta_{13} \simeq (0 \div 0.055) \quad \text{and} \quad \left(\tan^2 \Theta_{13}\right)_{\rm bestfit} \approx 0.005. \tag{8}$$

Future experiments will measure the oscillation parameters much more precisely [16]

$$|\Delta(\Theta_{13})| \ \sim \ 10^{-4}, \qquad \Delta(\delta m^2_{\rm atm}) \sim 1\%,$$

$$\Delta(\delta m^2_{\rm solar}) \ \sim \ 10\%, \qquad \Delta(\sin^2 2\Theta_{\rm solar}) \sim 0.1. \tag{9}$$

As we can see, in neutrino oscillation experiments, only differences of squares of neutrino masses are determined. As mentioned, much more can be achieved with the tritium β and $(\beta\beta)_{0\nu}$ decays.

In the tritium β decay

$$ {}^3_1\mathrm{H} \to {}^3_2\mathrm{He} + e^- + \bar{\nu}_e \tag{10}$$

the kinematic electron energy spectrum ($E = E_{\mathrm{tot}} - m_e \approx \frac{p^2}{2m_e}$, $E_0 = M\left({}^3_1\mathrm{H}\right) - M\left({}^3_2\mathrm{He}\right) - m_e \approx 18572.1$ eV)

$$\frac{dN}{dE} = R(E)(E_0 - E)\sum_{i=1}^{3}|U_{ei}|\sqrt{(E_0 - E)^2 - mi^2}\;\Theta\left(E_0 - E - m_i\right)\,, \tag{11}$$

can be approximated by [17]

$$\frac{dN}{dE} = R\left(E_0 - m_1\right)\left(E_0 - E\right)\sqrt{\left(E_0 - E\right)^2 - m_\beta^2}\,. \tag{12}$$

For present and future detectors with energy resolution $\Delta E > |m_3 - m_1| \approx 0.08\,\mathrm{eV}$ the effective neutrino mass m_β is given by

$$m_\beta = \sqrt{\sum_{l=1}^{3}|U_{ei}|^2 m_i^2}\,. \tag{13}$$

Two experiments in Mainz [18] and Troitsk [19] have recently found a following bound on m_β

$$m_\beta < 2.2 \text{ eV} \quad [18]\,, \qquad m_\beta < 2.5 \text{ eV} \quad [19]\,. \tag{14}$$

There are plans to improve the existing limits by a factor of ten, so within 6–7 years m_β at a level of 0.3 eV is a probable perspective [20].

Many experiments were conducted in order to find neutrinoless double β decay of some even–even nuclei [21]. Up to now such a decay has not been found giving the upper limit on the decay lifetime of nuclei. The most stringent bound was obtained by the ^{76}Ge Heildelberg–Moscow experiment [22]

$$T_{1/2}\left({}^{76}\mathrm{Ge}\right) \geqslant 5.7 \times 10^{25}\,\mathrm{yr}\,, \tag{15}$$

which was translated as a bound on the effective Majorana neutrino mass [22]

$$\langle m_\nu\rangle \equiv \left|\sum_{i=1}^{3} U_{ei}^2\, m_i\right| < 0.2 \text{ eV}. \tag{16}$$

There are also plans to reach a much better sensitivity in future [23], even up to

$$\langle m_\nu\rangle \simeq 0.006\,\mathrm{eV}. \tag{17}$$

It is a well known fact that the neutrinoless double beta decay can only take place if neutrinos are Majorana particles [21]. For Dirac neutrinos the effective mass $\langle m_\nu \rangle = 0$ [24], for any m_i.

So, what can we say about neutrino masses in the light of present experimental data?

From oscillations we have

$$|m_i - m_j| < \sqrt{\left(\delta m^2_{\rm atm} + \delta m^2_{\rm solar}\right)_{\rm max}} < 0.08 \text{ eV}, \tag{18}$$

$$(m_\nu)_{\rm max} > \sqrt{\left(\delta m^2_{\rm atm} + \delta m^2_{\rm solar}\right)_{\rm min}} > 0.04 \text{ eV}. \tag{19}$$

Using the bound on m^2_β we get an upper limit on the mass of any of the neutrinos

$$m_i < \sqrt{\delta m^2_{\rm atm} + (2.2)^2} \approx 2.2 \text{ eV}\,. \tag{20}$$

With this bound, we can find

$$m_{\nu_\mu} = \sqrt{\sum |U_{\mu i}|^2 m_i^2} < 2.2 \text{ eV}, \tag{21}$$

$$m_{\nu_\tau} = \sqrt{\sum |U_{\tau i}|^2 m_i^2} < 2.2 \text{ eV}, \tag{22}$$

which imposes a much better restriction than the directly measured bounds in π^+ and $\tau^\pm$ and decays [25]

$$m_\mu < 170 \text{ keV} \quad \text{and} \quad m_\tau < 18.2 \text{ MeV}. \tag{23}$$

The conditions (18)–(20) are valid independently of the neutrinos' nature. If they are Majorana particles additional restriction (Eq. (16)) can be applied. Unfortunately, the present knowledge of mixing matrix elements U_{ei} does not allow to find better bounds on Majorana neutrino masses. In future, as we will describe in the next section, the neutrinoless double β decay with the better knowledge of the oscillation parameters can be a powerful tool of information about Majorana neutrino masses.

3. Future perspectives

The pattern of neutrino masses should be relatively easy to find in the future superbeam experiments [26] or neutrino factories [27]. In Fig. 2 (taken from [28]) the ratio R$\equiv$ $\left(N\left(\bar{\nu}_e \to \bar{\nu}_\mu\right)/N\left(\nu_e \to \nu_\mu\right)\right)$ of wrong sign muon events is shown as a function of the baseline for 20 GeV neutrino factories.

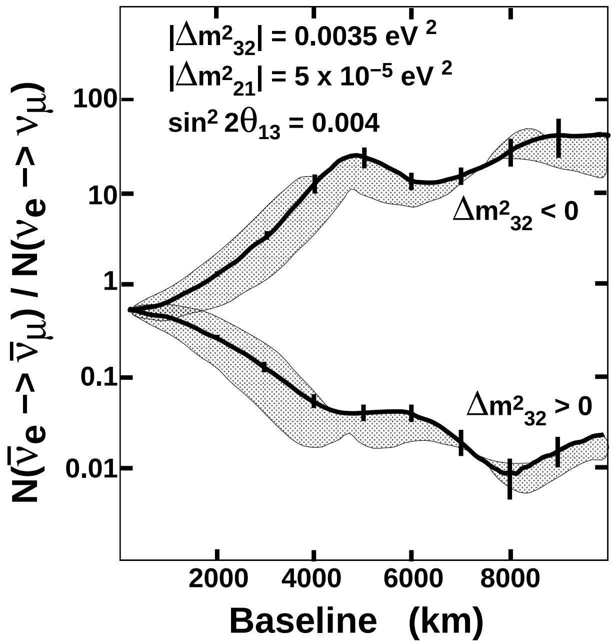

Fig. 2. The lower and upper bands for $\delta m^2_{32} > 0$ and $\delta m^2_{32} < 0$, which correspond to the schemes A_3 and $A_3^{\rm inv}$ in Fig. 1, respectively, (taken from [28]). Ratios have been calculated for a 20 GeV neutrino factory. The widths of the shaded bands describe how predictions vary with the CP phase δ. The thick lines give the results for $\delta = 0$.

The figure shows two bounds. The upper and lower correspond to $\delta m^2_{32} < 0$ and $\delta m^2_{32} > 0$, respectively, within these bounds the CP phase is varying. At large distances matter effects enhance R if $\delta m^2_{32} < 0$ and reduce R if $\delta m^2 > 0$, so the bands diverge. For L exceeding about 2000 km the matter effects change the bands significantly enough in order to see the difference. For more details concerning statistical errors and dependence on other neutrino parameters, especially $\sin^2 2\Theta_{13}$, see [29].

The problem of establishing the scale of neutrino masses *i.e.* the value of $(m_\nu)_{\rm min}$ is more complicated and depends on how large $(m_\nu)_{\rm min}$ is.

If $(m_\nu)_{\rm min} > 0.3$ eV, then the planed KATRIN experiments should provide the answer. m^2_β (Eq. (14)) depends on $(m_\nu)_{\rm min}$ in the following way

$$m^2_\beta = (m_\nu)^2_{\rm min} + \Omega_{\rm scheme} \tag{24}$$

where $\Omega_{\rm scheme}$ is a scheme dependent quantity and for the A_3 scheme is given by [30]

$$\Omega\,(A_3) = \left(1 - |U_{ei}|^2\right) \delta m^2_{\rm solar} + |U_{e3}|^2\, \delta m^2_{\rm atm}\,. \tag{25}$$

The measurement of m_β and $\Omega_{\rm scheme}$ gives the value of $(m_\nu)_{\rm min}$. The relative error of $(m_\nu)_{\rm min}$ is

$$\frac{\Delta\,(m_\nu)_{\rm min}}{(m_\nu)_{\rm min}} = \frac{m_\beta}{(m_\nu)^2_{\rm min}}\Delta m_\beta + \frac{1}{2\,(m_\nu)^2_{\rm min}}\Delta\,(\Omega_{\rm scheme})\,. \tag{26}$$

Already now $\Delta(\Omega_{\rm scheme})$ which comes from the uncertainties of neutrino parameters is small, *e.g.* for A_3 scheme

$$\Delta\,(\Omega_{A_3}) \approx 3.4\times 10^{-4}\,{\rm eV}^2, \tag{27}$$

and, for $(m_\nu)_{\rm min} \gtrsim 0.3$ eV is negligible in Eq. (26). We can see that the error of $(m_\nu)_{\rm min}$ comes merely from Δm_β

$$\Delta\,(m_\nu)_{\rm min} \equiv \frac{m_\beta}{(m_\nu)_{\rm min}}\Delta m_\beta\,, \tag{28}$$

and future ${}^3_1$H decay experiments should fix the scale of light neutrinos. Up to now there are no ideas on how to find neutrino masses in a direct kinematic way (so regardless their nature) for $(m_\nu)_{\rm min} < 0.3\,$eV. In this situation the only way to establish smaller values of $(m_\nu)_{\rm min}$ seems to be through neutrinoless double β decay experiments. Already now the probing values of effective neutrino masses $\langle m_\nu\rangle \approx 0.2\,$eV are one order of magnitude better than $m_\beta \simeq 2.2\,$eV, and there are plans to reach much smaller values $\langle m_\nu\rangle \approx 0.006$ eV.

If we look at the definition of $\langle m_\nu\rangle$ we can see that the phases of mixing matrix elements U_{ei} are important. Two new Majorana $\phi_1$and ϕ_2 phases must be taken into account and

$$\begin{aligned}\langle m_\nu\rangle \;=\; & \left|\cos^2\Theta_{13}\left(\cos^2\Theta_{12}\,(m_\nu)_{\rm min} + \sin^2\Theta_{12}\sqrt{(m_\nu)^2_{\rm min}+\delta m^2_{\rm solar}}\,{\rm e}^{i2\phi_1}\right)\right.\\ & \left. + \sin^2\Theta_{13}\sqrt{(m_\nu)^2_{\rm min}+\delta m^2_{\rm solar}+\delta m^2_{\rm atm}}\,{\rm e}^{i2\phi_2}\right|\,,\end{aligned} \tag{29}$$

where we use the standard parametrization of the mixing matrix [31]. As Majorana ϕ_i phases are unknown we are not able to predict values of $\langle m_\nu\rangle$ even if $(m_\nu)_{\rm min}$ is specified. We can, however, find the largest $(\langle m_\nu\rangle_{\rm max})$ and smallest $(\langle m_\nu\rangle_{\rm min})$ values of $\langle m_\nu\rangle$ for a given $(m_\nu)_{\rm min}$.

The value of $\langle m_\nu\rangle_{\rm max}$ is simple

$$\langle m_\nu\rangle_{\rm max} = \left(\cos^2\Theta_{12}m_1 + \sin^2\Theta_{12}m_2\right)\cos^2\Theta_{13} + m_3\sin^2\Theta_{13}, \tag{30}$$

but $\langle m_\nu\rangle_{\rm min}$, where two Majorana phases play a role, is more complicated. For $(m_\nu)_{\rm min} > \sqrt{\delta m^2_{\rm solar}} \approx 0.08$ eV, where the spectrum is almost degenerate $(m_1\approx m_2\approx m_3)$ we can find

$$\langle m_\nu \rangle_{\min} = \begin{cases} (m_\nu)_{\min} \left(\varepsilon \cos^2 \Theta_{13} - \sin^2 \Theta_{13} \right) , & \text{if } \varepsilon > \tan^2 \Theta_{12}, \\ 0 & \text{otherwise,} \end{cases} \tag{31}$$

where the new parameter ε has been introduced as

$$\varepsilon = \sqrt{1 - \sin^2 2\Theta_{12}} \, . \tag{32}$$

For $(m_\nu)_{\min} \leq \sqrt{\delta m^2_{\rm atm}}$, $\langle m_\nu \rangle_{\min}$ is more complicated and is given in Fig. 3 where $\langle m_\nu \rangle_{\max}$ and $\langle m_\nu \rangle_{\min}$ is presented for a full range of $(m_\nu)_{\min}$. For a given value of $(m_\nu)_{\min}$ the oscillation data determine the range of possible $\langle m_\nu \rangle$ and, opposite, the knowledge of $\langle m_\nu \rangle$ gives some information on $(m_\nu)_{\min}$. The interdependence between $\langle m_\nu \rangle$ and $(m_\nu)_{\min}$ is described by the oscillation parameters (especially the ε) and the experimental error bars. In Fig. 3 we consider the central value of $\tan^2 \Theta_{\rm solar}$ given by the present LMA MSW solution of the solar anomaly ($\varepsilon = 0.47$) and the anticipated error bars for the oscillation parameters in future experiments (Eq. (9)).

If $\langle m_\nu \rangle \geqslant 0.02\,$eV and the neutrinos are Majorana particles, the future $(\beta\beta)_{0\nu}$ experiments should find $\langle m_\nu \rangle = \kappa \pm \Delta\kappa \, (\neq 0)$. In such a case the value of $(m_\nu)_{\min}$ must belong to the interval

$$(m_\nu)_{\min} \in \left(\kappa - \Delta\kappa, (\kappa + \Delta\kappa) \, \frac{1}{\varepsilon} \right) . \tag{33}$$

We can see that the precision of $(m_\nu)_{\min}$ determination depends on uncertainties of κ ($\Delta\kappa$) and the mixing angle for solar neutrino (ε). Precision is better for larger ε (smaller $\sin^2 2\Theta_{\rm solar}$). The method becomes useless for $\varepsilon \to 0$ ($\sin^2 2\Theta_{\rm solar} \to 1$). The other problem is to determine $\Delta\kappa$ which can have large systematic errors (*e.g.* coming from problems with determination of nuclear transition amplitudes or other than light Majorana neutrino exchange mechanisms effects on transition rates).

For $(m_\nu)_{\min} \in (0.004 \div 0.02)\,$eV there is some chance that future $(\beta\beta)_{0\nu}$ experiments will find $\langle m_\nu \rangle \neq 0$ and as a result a better interval for $(m_\nu)_{\min}$ can be determined.

For $(m_\nu)_{\min} \leq 0.004\,$eV and $\sin^2 \Theta_{13} = 0.02$, the largest values of $\langle m_\nu \rangle_{\max}$ $\simeq$ $(0.003 \div 0.006)\,$eV and planed $(\beta\beta)_{0\nu}$ experiments will not resolve the problem of neutrino mass scale. New ideas for the mass measurements are needed in such a case.

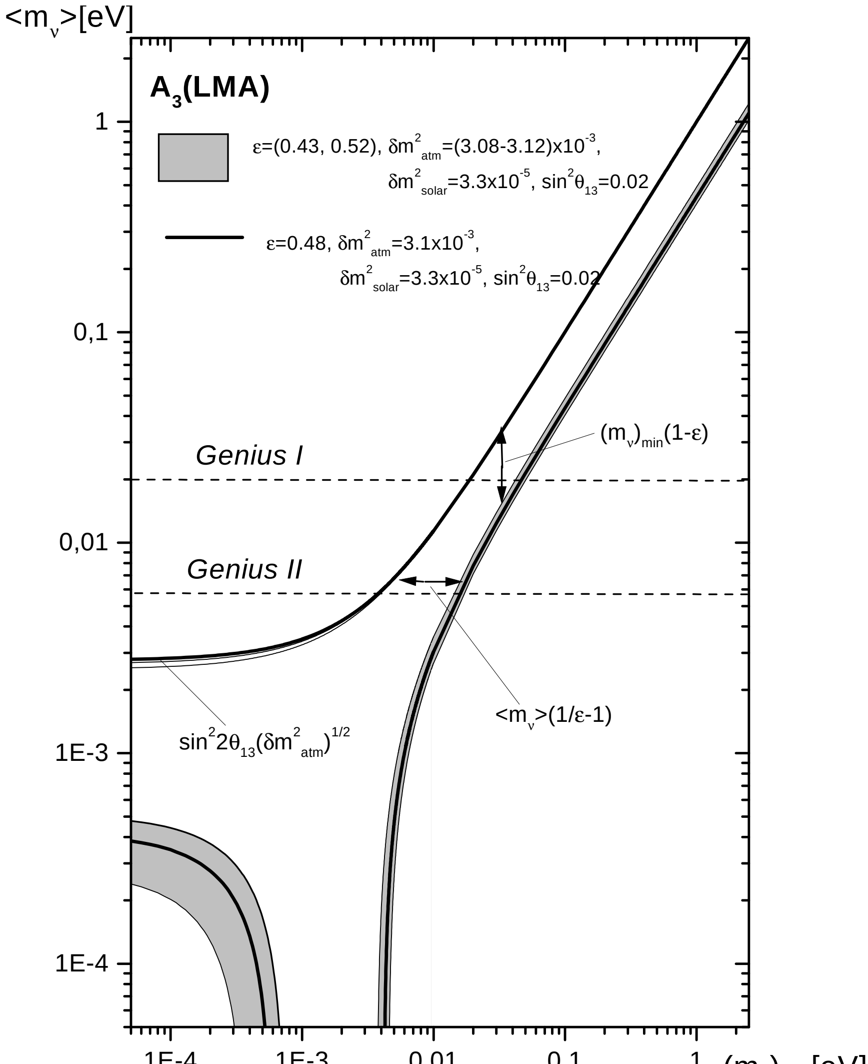

Fig. 3. $\langle m_\nu \rangle_{\rm max}$ and $\langle m_\nu \rangle_{\rm min}$ as a function of $(m_\nu)_{\rm min}$ for the A_3 mass scheme and the LMA MSW solution of the solar neutrino problem. The present best fit value of $\tan^2 \Theta_{12} = 0.36$ and $\sin^2 \Theta_{13} = 0.02$ are taken. Future anticipated errors of all the oscillation parameters (Eq. (9)) are used (shaded areas). For $(m_\nu)_{\rm min} \geq 0.03\,{\rm eV}$, $\langle m_\nu \rangle_{\rm max} \approx (m_\nu)_{\rm min}$ and $\langle m_\nu \rangle_{\rm min} = (m_\nu)_{\rm min} \left(\varepsilon \cos^2 \Theta_{13} - \sin^2 \Theta_{13}\right) \approx \varepsilon\ (m_\nu)_{\rm min}$. Then the vertical and horizontal widths of the $\langle m_\nu \rangle$ band are $\langle m_\nu \rangle_{\rm min} (1-\varepsilon)$ and $\langle m_\nu \rangle_{\rm min} (1/\varepsilon - 1)$, respectively. The shape of $\langle m_\nu \rangle_{\rm min\ (max)}$ bands are universal, the central values of ε and $\sin^2 \Theta_{13}$ can change in the future.

4. Conclusions

The hypothesis that neutrinos are massive particles has now a very strong support. It is almost sure that anomalies observed in solar and atmospheric experiments are due to neutrino oscillations. These experiments determine the so-called oscillation parameters: elements of the mixing matrix $|U_{\alpha i}|$ and two differences of square masses $\delta m^2_{\rm solar}$ and $\delta m^2_{\rm atm}$. Reconstruction of the full mass spectrum requires to determine the lightest neutrino mass and the mass scheme.

From present data we are unable to find which of two mass schemes $\left(A_3, A_3^{\rm inv}\right)$ is correct. About values of light neutrino masses we can say only that they are smaller than 2.2 eV.

Future neutrino oscillation experiments will be able to determine with much better precision the neutrino mass parameters. Moreover, a unique mass scheme can be found. Two other experiments, tritium β decay and neutrinoless double β decay of some even–even nuclei can say something about the mass scale — the value of the lightest neutrino mass.

The future project KATRIN has a chance to determine $(m_\nu)_{\rm min}$ if $(m_\nu)_{\rm min} \gtrsim 0.35\,{\rm eV}$. For smaller masses there is an additional possibility, the $(\beta\beta)_{0\nu}$ decay which in future can search for effective Majorana neutrino masses as small as $\langle m_\nu \rangle \simeq 0.006\,{\rm eV}$.

If the Majorana neutrinos have a mass $(m_\nu)_{\rm min} \geq 0.02$ eV then the future $(\beta\beta)_{0\nu}$ experiments will be able to find more precisely the possible interval of $(m_\nu)_{\rm min}$. For $(m_\nu)_{\rm min} \in (0.004 \div 0.02)$ eV the future $(\beta\beta)_{0\nu}$ measurement can give (but not necessarily will give) $\langle m_\nu \rangle \neq 0$. Then better interval for $(m_\nu)_{\rm min}$can be specified.

There is no way to say anything about $(m_\nu)_{\rm min} \lesssim 0.004\,{\rm eV}$, at least in the light of planed experiments.

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