DISORDER INDUCED FLUCTUATIONS OF THE PAIRING PARAMETER IN *p*-WAVE SUPERCONDUCTORS *

Grzegorz Litak

Department of Mechanics, Technical University of Lublin
Nadbystrzycka 36, 20-618 Lublin, Poland

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We study the effect of site diagonal disorder on the pairing amplitude by a perturbation method. Using an extended Hubbard model with the inter-site attraction we analyze fluctuations of order parameter in the presence of non-magnetic disorder and discuss the instability of various solutions with *p*-wave pairing.

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1. Introduction

In case of an anisotropic superconductor non-magnetic disorder leads eventually to pair breaking effect via Abrikosov–Gorkov formula [1–5], like for magnetic impurities, for any value of a coherence length ξ. By analogy to an isotropic s-wave pairing, for superconductors with the enough large coherence length ξ, the amplitude of pairing potential $|\Delta(ij)|$ tends to be the same for all bonds but disorder induces the effect of fluctuations in the amplitude leading, in this way, to destruction of pairing. Here we will calculate fluctuating potentials standard deviations $\langle\delta|\Delta_{ij}|^2\rangle$ and $\langle\varepsilon_i^2\rangle$. Their ratio $\Gamma = \langle\delta|\Delta_{ij}|^2\rangle/\langle\varepsilon_i^2\rangle$ will be a criterion of pairing potential fluctuations [6].

2. *p*-wave solutions for a clean system

We start a single band, extended, Hubbard model with effective nearest neighbor inter-site attraction W_{ij} ($W_{ii} = 0$) [7]. Taking the Fourier transform of the Green function for a clean system we can write its equation of motion:

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$$\begin{bmatrix} (\omega - \epsilon_{\boldsymbol{k}} + \mu)\mathbf{1} & \boldsymbol{\Delta}_{\boldsymbol{k}} \\ \boldsymbol{\Delta}^*_{\boldsymbol{k}} & (\omega + \epsilon_{\boldsymbol{k}} - \mu)\mathbf{1} \end{bmatrix} \boldsymbol{G}^0(\boldsymbol{k};\omega) = \mathbf{1}\,, \tag{1}$$

where $\boldsymbol{\Delta}_{\boldsymbol{k}} = \boldsymbol{\Delta}_x \sin k_x + \boldsymbol{\Delta}_y \sin k_y$ defines 2×2 matrix of a pairing potential. For a clean system t_{ij} can be expressed in $\boldsymbol{k}$-space: $\epsilon_{\boldsymbol{k}} = \sum_j t_{ij}\, \mathrm{e}^{-\imath \boldsymbol{R}_{ij}\boldsymbol{k}} = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y$, where t represents the nearest neighbor site amplitude of electron hopping, while t' corresponds to the next nearest neighbor one. For the assumed solution [8] $\boldsymbol{\Delta}_{\boldsymbol{k}} = \imath\hat{\sigma}_y\hat{\sigma}\cdot\boldsymbol{d}(\boldsymbol{k})$, with $\boldsymbol{d}(\boldsymbol{k}) = (0,0,d^z(\boldsymbol{k}))$ and $d^z(\boldsymbol{k}) = \Delta_{\boldsymbol{k}}$. The corresponding free energy F for a finite temperature T can be found from the standard formula:

$$F = \sum_{\boldsymbol{k}} \left[-(n-1)(\epsilon_{\boldsymbol{k}} - \mu) - 2k_{\mathrm{B}}T\, \ln\left(2\cosh\frac{E_{\boldsymbol{k}}}{2k_{\mathrm{B}}T}\right) - \frac{|\Delta_{\boldsymbol{k}}(T)|^2}{W}\right], \tag{2}$$

where $E_{\boldsymbol{k}}$ denotes the eigenvalue. To perform numerical calculations we have fitted our one band system parameters to the realistic γ band structure of Sr_2RuO_4 [5,9–11]. Fig. 1(a) presents the corresponding Fermi surface. For the above assumptions we have found three solutions with p-wave pairing. Namely, depending on relative values Δ_x and Δ_y, the dipole one ($\Delta_x \neq 0$ and $\Delta_y = 0$), the real one ($\Delta_x = \Delta_y$) and the complex one ($\Delta_x = \imath\Delta_y$). They correspond to minima of free energy curves ($\delta F(|\Delta|) = F(|\Delta|) - F(0)$) in Fig. 1(b), denoted by D, R and C, respectively. The interaction parameter used in calculations ($W = -0.46\,\mathrm{t}$) were chosen to give $T_{\mathrm{c}} = 1.5\mathrm{K}$ as for clean Sr_2RuO_4. One can easily see that the complex solution reaches the global minimum of free energy F.

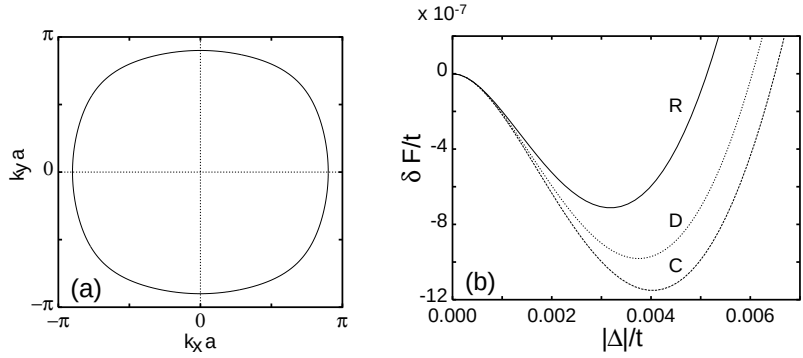

Fig. 1. (a) The Fermi surface and (b) the free energy δF as a function of pairing potential $|\Delta|$ $\left(|\Delta| = \sqrt{\Delta_x^2 + \Delta_y^2}\right)$ for different solutions: dipole (D) real (R) and complex (C) order parameters, respectively; for γ band of Sr_2RuO_4 electronic structure ($t' = 0.45\,t$, $n/2 = 0.66$). The inter-site attraction $W = -0.46\,\mathrm{t}$, temperature $T = 0$.

3. Fluctuations of pairing potential

In this section we investigate the stability of superconducting p-wave states in presence of a weak disorder. Here, we apply the same strategy as in [6] and we treat random site energies ϵ_i as perturbations. To proceed we write the Dyson equation for a Green function $\boldsymbol{G}(i,j;\omega)$ evaluated at a frequency ω:

$$\boldsymbol{G}(i,j;\omega) = \boldsymbol{G}^0(i,j;\omega) + \sum_l \boldsymbol{G}^0(i,j;\omega)\boldsymbol{V}_l\boldsymbol{G}(l,j;\omega)\,, \tag{3}$$

where $\boldsymbol{V}_l = \epsilon_l\,\boldsymbol{\sigma}_3$ is the impurity potential matrix.

Following Eq. (3) we express quantity order parameter Δ_{ij}, in the lowest order of ϵ_i perturbations by means of disordered Green function and calculate the mean square deviation of the pairing Δ_{ij} along the bond of nearest neighbor sites i and j. For site independent energies ϵ_i:

$$\left\langle \delta|\Delta_{ij}|^2 \right\rangle = \Gamma_{ij}\left\langle \epsilon_i^2 \right\rangle. \tag{4}$$

Finally, we calculate the coefficient Γ_{ij} [6]:

$$\Gamma_{ij} = \frac{1}{N}\sum_{\boldsymbol{q}} \left| \frac{W_{ij}}{2N} \sum_{\boldsymbol{k}} \frac{\Delta_{\boldsymbol{k}}\tilde{\epsilon}_{\boldsymbol{k}} + \Delta_{\boldsymbol{k}}\tilde{\epsilon}_{\boldsymbol{k}-\boldsymbol{q}}}{(E_{\boldsymbol{k}} + E_{\boldsymbol{k}-\boldsymbol{q}})E_{\boldsymbol{k}}E_{\boldsymbol{k}-\boldsymbol{q}}}\, \mathrm{e}^{\mathrm{i}(\boldsymbol{R}_i - \boldsymbol{R}_j)\boldsymbol{k}} \right|^2 . \tag{5}$$

Having found the pairing potentials (Fig. 1(b), Eq. (1)) for the clean system, we calculated Γ (Eq. (5)) for all three solutions we obtained. In all cases Γ has a very small value (of the order $\sim 10^{-8}$). This implies that fluctuations of Δ_{ij} are relatively small in this system. Interestingly, a real type solution is characterized by the smallest fluctuations ($\Gamma = 5.15 \times 10^{-9}$) while $\Gamma = 13.93 \times 10^{-9}$ for the dipole solution and $\Gamma = 8.12 \times 10^{-9}$ for the complex one. This could mean that the real solution is favored by disorder.

4. Conclusions and discussion

We have analyzed the effect of a weak disorder on a p-wave superconductor in context of newly discovered superconductor Sr_2RuO_4 [11,12]. Unfortunately the order parameter structure in this compound is still unknown [12]. Fitting our system parameters to its γ band structure we have asked about the stability of various solutions in the presence of disorder. We have found three solutions with the same critical temperature $T_c = 1.5\,\mathrm{K}$: dipole and real solutions with line nodes in the gap and complex one with a finite gap in any direction. Note that all these solutions have the same critical temperature T_c and the Abrikosov–Gorkov formula [2,4] does not differentiate

any of them. Our preliminary results at $T = 0$ indicate that the complex type of solution has the global minimum of free energy F but the real one is favored by disorder. That result was obtained in one band model in the lowest order of perturbation method and should be confirmed by a more sophisticated method like Coherent Potential Approximation [4,5] considering more realistic three bands structure of Sr_2RuO_4.

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