EQUIVALENCE BETWEEN PAIRING AND STAGGERED-FLUX-LIKE VORTICITY OF THE CURRENT-CURRENT CORRELATION FUNCTION IN WEAKLY DOPED ANTIFERROMAGNETS*

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We show that patterns of the current-current correlation function in the form of the staggered flux indicate that spin bipolarons form in doped antiferromagnets. Holes which form a spin bipolaron reside at opposite ends of a line (string) formed by the defects in the antiferromagnetic spin background. The string is relatively highly mobile, because the motion of a hole at its end does not raise extensively the number of defects, provided that the hole at the other end of the line follows along the same track. Appropriate coherent combinations of string states realize some irreducible representations of the point group C_{4v}. Creep of strings favors d- and p-wave states. Some more subtle processes decide the symmetry of pairing. The pattern of the current correlation function, that defines the structure of flux, emerges from motion of holes at string ends and coherence factors with which string states appear in the wave function of the bound state. Condensation of bipolarons and phase coherence between them puts to infinity the correlation length of the current correlation function and establishes the flux in the system.

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1. Introduction

Already many years ago, flux phase appeared in the mean-field approaches to undoped and doped antiferromagnets described by the t-J model [1]. Until now, the physical meaning of that phase and its relation with pairing was not clear.

Ivanov, Lee and Wen [2] recently reported staggered-vorticity correlations of the Current–Current Correlation Function $\langle j_{ij} j_{kl} \rangle$ (CCCF) in the d-wave variational functions for a weakly doped antiferromagnet, where j_{ij} denotes the current flowing on the bond $\langle i, j \rangle$. Leung found a similar pattern of the CCCF for a bound state of two holes in results of an exact diagonalization [3]. It was a tempting task for us to resolve whether there exists a deep relation between binding and the formation of flux patterns, since the mechanism of hole binding in weakly doped antiferromagnets is relatively well understood [4].

The physics of weakly doped antiferromagnets is governed by two opposite tendencies, to minimize the kinetic energy (related with the off-diagonal part of the Hamiltonian in the basis of the spin up and down states) and the static, potential energy which is determined by the contribution from the Ising model. One way of finding compromise by the system is formation of spin bipolarons. A hole that moves through an antiferromagnetic background of spins which is preferred by the potential energy, shifts spins and creates magnons, which are defects in the perfect pattern of antiferromagnetically arranged spins. That effect raises the magnetic contribution to energy and tends to localize the hole, which, in addition, raises the kinetic energy. A solution for that problem is the simultaneous motion of a hole pair connected by a path (string) of magnons. If holes trace each other along the path its length is limited, the rise of the potential energy is low and the hole pair as a whole is relatively mobile. The holes at string ends may also chaotically oscillate in all possible directions but the lengthening of the string is hindered by the growth of the potential energy. In order to be more quantitative we first consider the oscillations of holes at the ends of the string by pinning it at a pair of nearest neighbor sites. The t-J model on the square lattice that we use to describe doped antiferromagnets is confined to the subspace of no doubly occupied sites and is defined by two parameters, the amplitude of electron hopping to nearest neighbor sites, $-t$, and the antiferromagnetic exchange J. The number $-J/4$ also counts the gain in the energy for a particular bond due to attraction of electrons created at nearest neighbor sites. The Schrödinger equation that describes a hole pair at end-points of a pinned string may be written in term of these parameters as

$$t\left[\alpha_{\mu-1,\nu} + (z-1)t\alpha_{\mu+1,\nu} + \alpha_{\mu,\nu-1} + (z-1)t\alpha_{\mu,\nu+1}\right] + J\left(4+\mu+\nu-\frac{1}{2}\delta_{\mu,\nu}\right)\alpha_{\mu,\nu} = E_2\alpha_{\mu,\nu}\,, \qquad (1)$$

where $\alpha_{\mu,\nu} = 0$ for $\mu < 0$ or $\nu < 0$ and z=4 and the total length of the string is given by $\mu + \nu$. The solution of that equation defines a spin bipolaron $|\Psi_{\langle i,j\rangle}\rangle$ at the sites i, j to which the string is pinned. s-, d- and p-wave symmetries may be realized as coherent sums of bipolaronic states [4]: $\sum_{\langle i,j\rangle} S_{\langle i,j\rangle}|\Psi_{\langle i,j\rangle}\rangle$, where $S_{\langle i,j\rangle} = 1$ if $\langle i,j\rangle$ is horizontal and $S_{\langle i,j\rangle} = -1$ provided that $\langle i,j\rangle$ is vertical for the d-wave, while for the p-wave $S_{\langle i,j\rangle}$ vanishes if $\langle i,j\rangle$ is vertical, $S_{\langle i,j\rangle} = 1$ provided that j is on the right side of i and $S_{\langle i,j\rangle} = -1$ if on the left. Tracing of holes by each other gives rise to shifts of bipolarons from a pair of sites to another and contributes to off-diagonal terms in the Hamiltonian in the basis of bipolarons states. The application of the spin–polaron scenario to binding of holes in doped antiferromagnets gave rise to better than qualitative agreement with results of numerical diagonalizations [4].

2. Vorticity of currents in the *d*-wave paired state

To the CCCF contributes motion of holes at string ends. In order to give rise to a non-vanishing contribution the product of the current operators should couple bipolaron created at (or in another language strings pinned to) different pairs of sites. Examples of such process are presented in Fig. 1. By counting similar contributions to the CCCF for links $\langle k,l\rangle$ and

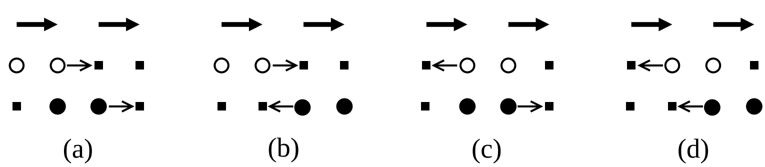

Fig. 1. Some processes which contribute to the CCCF function when holes are shifted by a product of current operators in the same direction on outer bonds.

$\langle m,n\rangle$ related to strings spanned along a path that connects these links, and includes them, we may derive the following formula

$$2\sum_{m=1}^{l-1}\sum_{n=2}^{l}(-1)^{m+n+1}S_mS_nt^2\alpha_{m-1,l-m-1}\alpha_{n-2,l-n} + 2\sum_{m=1}^{l}\sum_{n=2}^{l-1}(-1)^{m+n}S_mS_nt^2\alpha_{m-1,l-m}\alpha_{n-2,l-n-1}\,, \qquad (2)$$

where S_m is a coherence factor with which the bipolaron at m-th bond in the path appears in a coherent sum that represents a bound state of a hole pair. Since amplitudes α diminish with the length of strings, dominant contributions to the CCCF is related with shortest paths. After evaluating all such contributions we collect the data in Fig. 2. We observe a good agreement with results of numerical diagonalization performed by Leung [3].

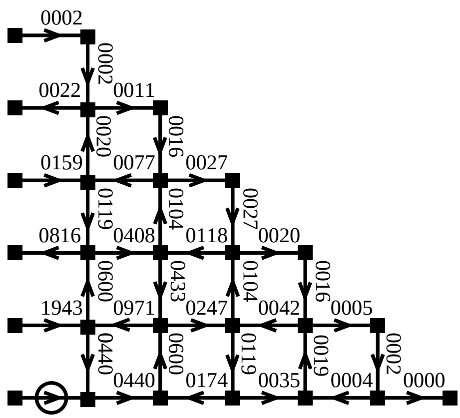

Fig. 2. $\langle j_{kl} j_{mn} \rangle / x$ in units 10^{-5}, where x is the hole concentration at $J = 0.3\,t$. The reference bond has been marked by a circle.

The calculation presented in this paper suggests that both local pairs which are probably responsible for pseudogap phenomena and circulating currents which may give rise to the marginal Fermi liquid behavior of cuprates can be attributed to formation of spin bipolarons.

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