SUPERCONDUCTIVITY IN THE PRESENCE OF MAGNETIC FIELD*

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We study the influence of a strong magnetic field on a superconducting state of electron gas in a two-dimensional square lattice. The Harper equation is extended in order to include pairing interactions between electrons. We examine the effects of superconductivity with different pairing symmetries on the Hofstadter energy spectra.

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1. Introduction

The energy spectrum of a system of electrons on a square lattice in a perpendicular uniform magnetic field exhibits multifractal properties and the band spectrum for rational values of the magnetic flux is known as the "Hofstadter butterfly" [1]. This spectrum can be modified by *e.g.*, the inclusion of Coulomb interaction [2] or the antiferromagnetic correlations [3]. The aim of this paper is to study the effect of pairing interactions between electrons on the the Hofstadter energy spectra.

We start with a mean-field Hamiltonian which describes electrons in a 2D square lattice in a perpendicular magnetic field,

$$\hat{H} = \sum_{\langle ij\rangle,\sigma} t_{ij}\left(\boldsymbol{A}\right) c_{i\sigma}^{\dagger} c_{j\sigma} - V \sum_{i,j} \left(\Delta_{ij} c_{i\uparrow}^{\dagger} c_{j\downarrow}^{\dagger} + \text{h.c.} \right). \tag{1}$$

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In the pairing term we put $i = j$ for on-site pairing, and (i, j) — nearest neighbor sites for inter-site pairing. The magnetic field appears in the hopping amplitudes,

$$t_{ij}\left(\boldsymbol{A}\right) = t\,\exp\left(\frac{ie}{\hbar c}\int\limits_{\boldsymbol{R}_j}^{\boldsymbol{R}_i}\boldsymbol{A}\cdot d\boldsymbol{l}\right),$$

where t is the nearest neighbor hopping amplitude in zero field and $\nabla\times\boldsymbol{A}(\boldsymbol{r}) = \boldsymbol{B}(\boldsymbol{r})$. We consider an uniform magnetic field in z-direction *i.e.*, $\boldsymbol{B} = (0,0,B)$. Using the Landau gauge the corresponding vector potential is given by $\boldsymbol{A}(\boldsymbol{r}) = (0, Bx, 0)$. We neglect the Zeeman term.

In the momentum space the Hamiltonian (1) takes the form

$$\begin{aligned}\hat{H} \;=\; & -t\sum_{\boldsymbol{k},\sigma}\left(2\,\cos k_y c^{\dagger}_{\boldsymbol{k},\sigma}c_{\boldsymbol{k},\sigma} + \mathrm{e}^{-ik_x}c^{\dagger}_{\boldsymbol{k}-\boldsymbol{g},\sigma}c_{\boldsymbol{k},\sigma} + \mathrm{e}^{ik_x}c^{\dagger}_{\boldsymbol{k}+\boldsymbol{g},\sigma}c_{\boldsymbol{k},\sigma}\right)\\ & -\sum_{\boldsymbol{k}}\left(\Delta_{\boldsymbol{k}}c^{\dagger}_{\boldsymbol{k}\uparrow}c^{\dagger}_{-k\downarrow} + \mathrm{h.c.}\right), \quad a = 1, \end{aligned} \tag{2}$$

where

$$\boldsymbol{g} = \left(0, 2\pi\frac{p}{q}\right).$$

$p/q \equiv \phi/\phi_0$ is the number of flux quanta per plaquette ($\phi = Ba^2$, $\phi_0 = hc/e$). The energy gap is given by

$$\Delta_{\boldsymbol{k}} = \sum_{\boldsymbol{k}'} V_{\boldsymbol{k},k'}\left\langle c_{-k'\downarrow}c_{\boldsymbol{k}'\uparrow}\right\rangle.$$

Introducing multicomponent Nambu notation the Hamiltonian (2) can be written as

$$\hat{H} = {\sum_{\boldsymbol{k}}}' \Psi^{\dagger}_{\boldsymbol{k}}\mathcal{H}_{\boldsymbol{k}}\Psi_{\boldsymbol{k}},$$

where the prime denotes summation over the reduced Brillouin zone:

$$-\pi \le k_x \le \pi, \quad -\frac{\pi}{q} \le k_y \le \frac{\pi}{q},$$

and

$$\begin{aligned}\Psi^{\dagger}_{\boldsymbol{k}} \;=\; & \Big(c^{\dagger}_{\boldsymbol{k},\uparrow},\; c^{\dagger}_{\boldsymbol{k}-\boldsymbol{g},\uparrow},\; c^{\dagger}_{\boldsymbol{k}-2\boldsymbol{g},\uparrow},\; \ldots,\\ & c^{\dagger}_{\boldsymbol{k}-(q-1)\boldsymbol{g},\uparrow},\; c_{-\boldsymbol{k},\downarrow},\; c_{-\boldsymbol{k}+\boldsymbol{g},\downarrow},\; c_{-\boldsymbol{k}+2\boldsymbol{g},\downarrow},\; \ldots,\; c_{-\boldsymbol{k}+(q-1)\boldsymbol{g},\downarrow}\Big). \end{aligned} \tag{3}$$

The Hamiltonian matrix $\mathcal{H}_{\boldsymbol{k}}$ has a block structure:

$$\mathcal{H}_{\boldsymbol{k}} = \left(\begin{array}{c|c} \hat{\boldsymbol{T}}_{\boldsymbol{k}} & \hat{\boldsymbol{\Delta}}_{\boldsymbol{k}} \\ \hline \hat{\boldsymbol{\Delta}}^{*}_{\boldsymbol{k}} & -\hat{\boldsymbol{T}}_{-\boldsymbol{k}} \end{array} \right), \tag{4}$$

where the matrices $\hat{\boldsymbol{T}}_{\boldsymbol{k}}$ and $\hat{\boldsymbol{\Delta}}_{\boldsymbol{k}}$ are:

$$\hat{\boldsymbol{T}}_{\boldsymbol{k}} = -t \begin{pmatrix} M_0 & \mathrm{e}^{ik_x} & 0 & \cdots & 0 & \mathrm{e}^{-ik_x} \\ \mathrm{e}^{-ik_x} & M_1 & \mathrm{e}^{ik_x} & 0 & \cdots & 0 \\ 0 & \mathrm{e}^{-ik_x} & M_2 & \mathrm{e}^{ik_x} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \vdots & 0 & \mathrm{e}^{-ik_x} & M_{q-2} & \mathrm{e}^{ik_x} \\ \mathrm{e}^{ik_x} & 0 & \vdots & 0 & \mathrm{e}^{-ik_x} & M_{q-1} \end{pmatrix},$$

$$M_n = 2\,\cos\left(k_y + ng\right), \quad g = |\boldsymbol{g}| = \frac{2\pi p}{q},$$

$$\hat{\boldsymbol{\Delta}}_{\boldsymbol{k}} = \begin{pmatrix} \Delta_{\boldsymbol{k}} & 0 & \cdots & 0 \\ 0 & \Delta_{\boldsymbol{k}-\boldsymbol{g}} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \Delta_{\boldsymbol{k}-(q-1)\boldsymbol{g}} \end{pmatrix}.$$

2. Density of states

In the case of noninteracting electrons ($V = 0$) the spin-up and spin-down segments decouple and the eigenproblem of the Hamiltonian (3) reduces to the original Harper's equation [4] in a form derived by Hasegawa *et al.* [5] The solutions constitute the famous "Hofstadter butterfly" [1] presented in Fig. 1. The calculations were carried out for a finite-size 60×60 square lattice with periodic boundary conditions.

When turning on the pairing interaction the subbands become broader than in the noninteracting case. We have numerically solved the eigenproblem of the Hamiltonian (3) for s-wave ($\Delta_{\boldsymbol{k}} = \mathrm{const}$) and d-wave ($\Delta_{\boldsymbol{k}} = \Delta\left(\cos k_x - \cos k_y\right)$) symmetry of the energy gap. The results are presented in Figs. 2 and 3.

Due to the fact that $\hat{\boldsymbol{T}}_{-\boldsymbol{k}} \neq \hat{\boldsymbol{T}}_{\boldsymbol{k}}$ the subband containing the Fermi level ($\varepsilon_{\mathrm{F}} = 0$ for half filled band) is split into two subbands only for certain values of the magnetic field. Therefore there is a finite density of states at the Fermi level, even in the case of s-wave pairing.

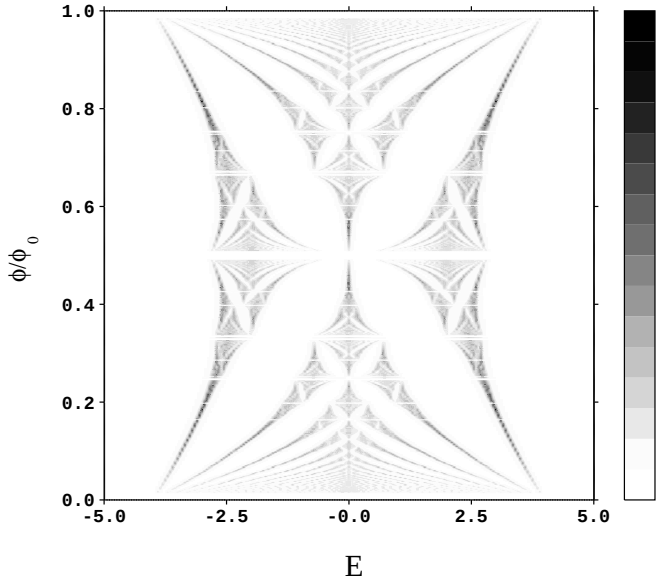

Fig. 1. Density of states for noninteracting electrons in a perpendicular magnetic field. The horizontal variable is the quasiparticle energy, and the vertical variable is the flux through a lattice cell divided by the flux quantum.

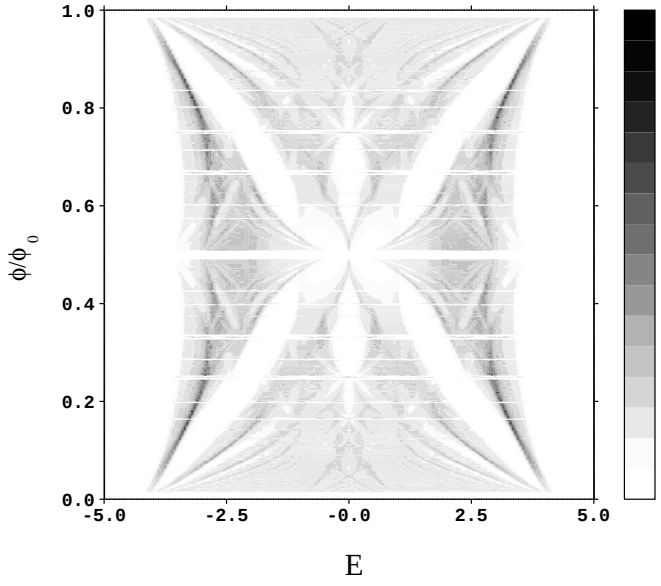

Fig. 2. Density of states for s-wave state. $\Delta = 1$ was used.

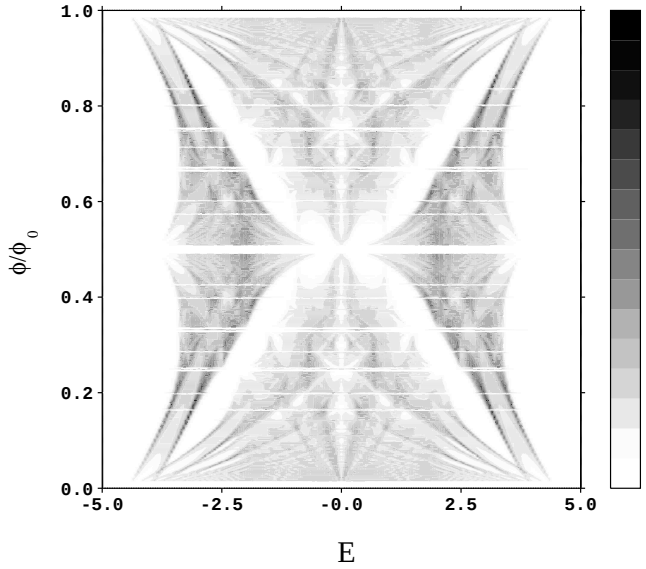

Fig. 3. Density of states for d-wave state.

3. BCS equations

We have also carried out a selfconsistent cluster calculations for the order parameter in the presence of magnetic field. The fractal structure of the "Hofstadter butterfly" results in a very irregular dependence of Δ on the external magnetic field, presented in Fig. 4. Moreover, when the de-

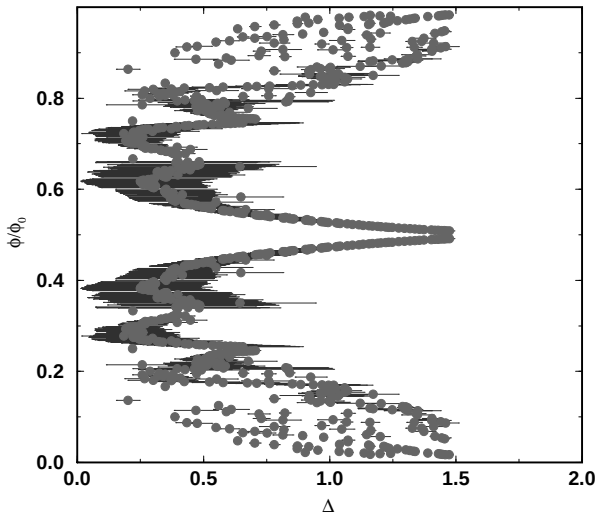

Fig. 4. Zero temperature s-wave energy gap Δ as a function of magnetic field.

nominator q ($\phi/\phi_0 = p/q$) is incommensurable with the size of the cluster additional, finite size errors appears (marked with horizontal bars). In spite of these irregularities one can note that the maxima of $\Delta(\phi)$ appear at even-denominator values of q in p/q, *e.g.*, for $\phi = \frac{1}{2}\phi_0,\ \frac{1}{4}\phi_0,\ \frac{1}{6}\phi_0, \ldots$. Also for small odd values of q the maxima appear, but they are much less noticeable. The oscillatory behavior can be explained within a framework of a composite fermion picture, but it is out of the scope of this paper.

The Hofstadter spectrum corresponding to the selfconsistently calculated Δ is presented in Fig. 5.

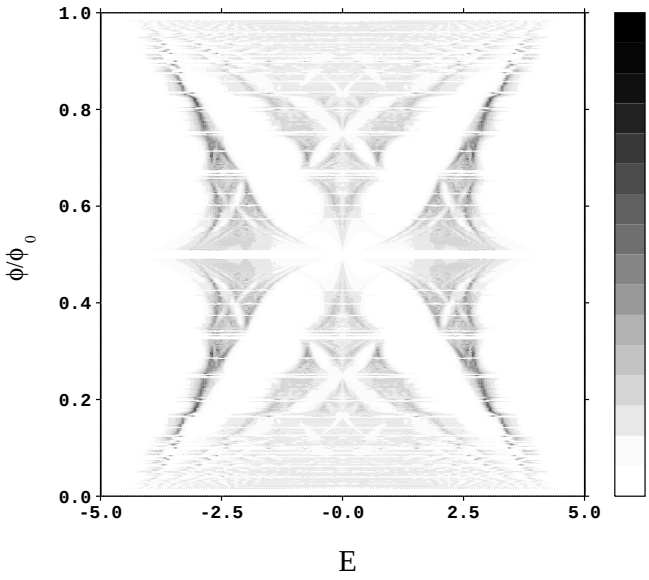

Fig. 5. Density of states for s-wave state (Δ is calculated selfconsistently for $V = 1$).

4. Summary

The proposed generalization of Harper's equations allows to study the impact of the external magnetic field on superconductivity with different symmetries. We have found oscillatory behavior of the energy gap as a function of the external field. Here, we have presented results for the zero temperature case but relevant calculations were carried out also for finite temperature. The values of the critical temperature (not presented here) are the same as the results obtained from the use of much more CPU time-consuming real-space cluster calculations [6, 7].

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