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RELATIVISTIC MEAN FIELD ANTINUCLEON–NUCLEUS POTENTIAL*

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The real part of antinucleon–nucleus potential is determined in the framework of the Relativistic Mean Field model using the charge conjugation. The solution of the Dirac equation for antiproton is given in case of ^{208}Pb. The bound state antiprotonic wave functions are compared to those obtained from the pure point charge Coulomb potential. The spectrum of $\bar{p}$ is shown.

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The antinucleon–nucleus ($\bar{N}N$) potentials ($V_{\bar{N}A}$) are meaningful in studies of nuclear reactions involving antiparticles, *e.g.*, the $\bar{p}$ anihilation on the atomic nuclei [1]. The $\bar{N}A$ interaction consists of the Coulomb potential and at the intermediate distances some kind of annihilation mechanism. In case of pure $\bar{N}N$ interaction the annihilation cross section is rather large and in order to simulate this one usually introduces an *ad hoc* optical potential.

In the calculations of the anihilation halo factors (*e.g.*, [2]) the $\bar{N}A$ potential is given phenomenologically or is assumed as the Coulomb potential of the nucleus in question [3]. The resulting $\bar{p}$-wave function does not contain any informations on the finiteness of the nucleus. Additionally, only the circular states with $l = n - 1$ are assumed to contribute to the anihilation process. As it is shown here the circular $\bar{p}$-states up to $n = 20$ can be deeply bound and do not contribute to the annihilation process. The finite dimensions of the nucleus and the deformation of the nuclear system results in a completely new picture of the $\bar{p}$-wave functions and different binding energies as compared to the Coulomb case.

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In the paper we construct the real part of the $\bar{N}A$ potential in framework of relativistic mean field model (RMF) starting from the properties of the C-parity symmetry of $\bar{N}N$ interactions. As in case of the bar $\bar{N}N$ interaction [4,5] one can apply the C- or G-parity [6] to the potentials generated by RMF model or the WS potential.

The paper is organized as follows. First we define the RMF model as well as scalar and vector potentials. Next the C-parity properties of the nucleon–nucleus potentials and the $\bar{N}A$ potentials are shown. Finally an examples of the antiprotonic spectra and the eigenfunctions of $\bar{p}$ in WS potential of ^{208}Pb are shown. The dissimilarities with the point charge Coulomb spectra are pointed out.

The nuclear interactions in RMF model [7,8] are governed by the one meson exchange potentials. The fields have the properties of the fundamental scalar σ, vector ω and ρ mesons.

The Lagrangian of the RMF model reads

$$\mathcal{L}(N,\sigma,\omega,\rho) = \mathcal{L}_\psi + \mathcal{L}_\sigma + \mathcal{L}_\rho + \mathcal{L}_\omega + \mathcal{L}_{N\sigma} + \mathcal{L}_{N\rho} + \mathcal{L}_{N\omega} + \mathcal{L}_\gamma\,, \tag{1}$$

where

$$\begin{aligned}
\mathcal{L}_\sigma &= -\frac{1}{2}m_\sigma^2\sigma^2 - \left(\frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4\right) + \frac{1}{2}g_\sigma\bar{\psi}\psi\sigma\,, &(2)\\
\mathcal{L}_\omega &= \frac{1}{2}m_\omega\omega_\mu\omega^\mu - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}g_\omega\omega_\mu\gamma^\mu\bar{\psi}\psi\,, &(3)\\
\mathcal{L}_\rho &= \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu - \frac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\mu\nu} + \frac{1}{2}g_\rho\vec{\rho}_\mu\gamma^\mu\cdot\vec{\tau}\bar{\psi}\psi\,, &(4)\\
\mathcal{L}_\gamma &= -\frac{1}{4}A^{\mu\nu}A_{\mu\nu} - e\bar{\psi}\,\gamma^\mu\,\frac{(1-\tau_3)}{2}\,\psi\,A_\mu\,. &(5)
\end{aligned}$$

Fermion fields fulfill the Dirac equation

$$\{-i\vec{\alpha}\cdot\vec{\nabla} + V_v(\vec{r}) + \beta(m + V_s(\vec{r}))\}\psi_i = \varepsilon_i\psi_i\,, \tag{6}$$

where the attractive scalar potential and the effective nucleon mass are given by

$$V_s(\vec{r}) = g_\sigma\sigma(\vec{r})\,,\quad m^*(\vec{r}) = m + V_s(\vec{r})\,. \tag{7}$$

The repulsive vector potential

$$V_v(\vec{r}) = g_\omega\omega_0(\vec{r})h + g_\rho\tau_3\rho_0(\vec{r}) + e\frac{1+\tau_3}{2}A_0(\vec{r})\,. \tag{8}$$

Both the vector and the scalar potential are the linear functions of the mesonic fields. The scalar potential and the effective mass m^* depend on the scalar σ field only.

The mesonic fields are described by the Klein–Gordon equations.

$$-\Delta + m_X^2 X(\vec{r}) = \mathcal{S}_X(\vec{r})\,, \tag{9}$$

where

$$\mathcal{S}_X(\vec{r}) = \begin{cases} -g_\sigma\rho_s(\vec{r}) - g_2\sigma^2(\vec{r}) - g_3\sigma^3(\vec{r}) & \text{if } X=\sigma\,, \\ g_\omega\rho_v(\vec{r}) & \text{if } X=\omega_0\,, \\ g_\rho\rho_3(\vec{r}) & \text{if } X=\rho_0\,, \\ e\rho_c(\vec{r}) & \text{if } X=A_0\,. \end{cases}$$

The densities entering the right hand sides of the equations are

$$\begin{aligned} \rho_s(\vec{r}) &= \sum_{i=1}^{A}\bar{\psi}_i\psi_i\,, & \rho_v(\vec{r}) &= \sum_{i=1}^{A}\psi_i^\dagger\psi_i\,, \\ \rho_3(\vec{r}) &= \sum_{p=1}^{Z}\psi_p^\dagger\psi_p - \sum_{n=1}^{N}\psi_n^\dagger\psi_n\,, & \rho_c(\vec{r}) &= \sum_{p=1}^{Z}\psi_p^\dagger\psi_p\,. \end{aligned}$$

The solution of the equations of motion leads to the nucleon–nucleus potentials: $V(\sigma)$, $V(\omega)$, $V(\rho)$ entering V_v in Eq. (8).

Introducing the two component wave function $\psi^T = (f,g)$, leads to the Dirac equation of the form

$$\begin{pmatrix} m+V_s+V_v & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -m-V_s+V_v \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix} = (m+\varepsilon)\begin{pmatrix} f \\ g \end{pmatrix}. \tag{10}$$

Eliminating the lower component g and introducing a new effective mass $m_{\rm eff} = m-(V_v-V_s)/2$ after expanding the mass term in the small parameter $\varepsilon/m_{\rm eff}$ one obtains

$$\left\{\vec{\sigma}\cdot\vec{p}\frac{1}{2m_{\rm eff}}\vec{\sigma}\cdot\vec{p} + V + S\right\} f = \varepsilon f\,. \tag{11}$$

The simple algebra leads to the Schrödinger like equation

$$\left\{\vec{p}\frac{1}{2m_{\rm eff}(\vec{r})}\vec{p} + V^{\rm centr} + \frac{1}{(2m_{\rm eff})^2}\left(\nabla V^{\rm so}\right)(\vec{p}\times\vec{\sigma})\right\} f_i = \varepsilon_i f_i\,, \tag{12}$$

where the central $V^{\rm centr}$ and the spin–orbit $V^{\rm so}$ potentials are, *viz.*

$$V^{\rm centr} = V_v + V_s\,, \qquad V^{\rm so} = \frac{m}{m_{\rm eff}}\left(V_v - V_s\right)\,. \tag{13}$$

Strong interactions are invariant under C operation. If a meson is exchanged between two nucleons, and thus contribute to nuclear forces, it can also be exchanged between a nucleon and an antinucleon (*e.g.*, in QED, since $C^\dagger A_\mu C = -A_\mu$, repulsive $e^- e^-$ Coulomb interaction becomes attractive in case of $e^- e^+$).

The C properties of the RMF mesonic fields are the following $C^\dagger \sigma C = +\sigma$, $C^\dagger \omega C = -\omega$, and $C^\dagger \rho C = -\rho$. We shall use this in constructing the antinucleon–nucleus potentials. Assuming an even-even system of nucleons one has the following C-rule (see *e.g.*, [9]):

$$V(N, A) \stackrel{C}{\Longrightarrow} V(\bar{N}, A)\,. \tag{14}$$

In the RMF model the $V(N, A)$ potentials are given in Eqs. (7),(8). The linearity of the vector potential in the meson fields is of great concern for the application of the C-rule. Under the C operation the vector potential changes sign.

In case of the Schrödinger equation based on the Woods–Saxon potential one has

$$V_{p|n}^{\rm centr} = V_{s,p|n} + V_{v,p|n}\,, \quad V_{p|n}^{\rm so} = V_{s,p|n} - V_{v,p|n}\,. \tag{15}$$

where $V^{\rm centr}$ and $V^{\rm so}$ are the central and spin–orbit potentials. Let the effective mass be constant. Assuming a "mesonic content" of the Woods–Saxon potentials similar to RMF model, using Eq. (13) and the C-rule one can calculate both scalar and vector potentials for antinucleons, *viz.*

$$V_{s,\bar{p}|\bar{n}} = +\frac{1}{2}\left[V_{p|n}^{\rm centr} - V_{p|n}^{\rm so}\right]\,, \qquad V_{v,\bar{p}|\bar{n}} = -\frac{1}{2}\left[V_{p|n}^{\rm centr} + V_{p|n}^{\rm so}\right]\,. \tag{16}$$

Eq. (13) then gives

$$V_{\bar{p}|\bar{n}}^{\rm centr} = V_{s,\bar{p}|\bar{n}} + V_{v,\bar{p}|\bar{n}} = -V^{\rm so}\,, \qquad V_{\bar{p}|\bar{n}}^{so} = V_{v,\bar{p}|\bar{n}} - V_{s,\bar{p}|\bar{n}} = -V^{\rm centr}\,. \tag{17}$$

The strengths of both central and spin–orbit potential are interchanged. In NA interaction the magnitude of the central part is approximately equal to -50 MeV while the spin–orbit potential is roughly 800 MeV. In case of the $\bar{N}A$ potential the strengths are $V_{\bar{p}|\bar{n}}^{\rm centr} \approx -800$ MeV and $V_{\bar{p}|\bar{n}}^{\rm so} \approx +50$ MeV. The central part of the $\bar{N}A$ potential becomes very strong whereas the spin–orbit term is weak. This will cause the spin degeneracy in the antiparticle spectra. The sign of the spin orbit term is opposite to the spin–orbit term of NA system and the spectrum shows the sequence of levels characteristic for the atomic physics.

It was shown [10–12] that the average RMF potentials are equivalent to some Woods–Saxon potentials.

The Dirac $\bar{p}$-spectrum of ^{208}Pb based on WS potentials [13] is shown in Fig. 1. To solve the Dirac equation the procedure `RADIAL` [14] has been used. The energy levels are plotted *vs* κ – the eigenvalue of $\hat{K} = -\beta(L \cdot \sigma + 1)$. The spectrum differs significantly from the Coulomb one.

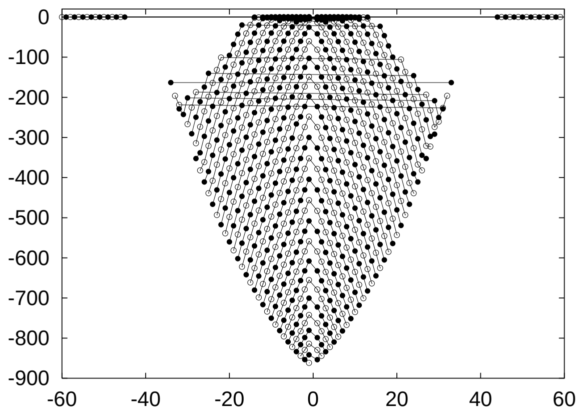

Fig. 1. $\bar{p}$ spectrum in Woods–Saxon potential in ^{208}Pb nucleus as a function of the quantum number κ.

The eigenfunctions corresponding to $(n,\kappa)=(20,1)$ $\bar{p}$-state in WS+Coulomb potential and the point nucleus Coulomb potential are displayed in Fig. 2 and Fig. 3 respectively. Both (f,g) components of the Dirac spinor are shown. The single particle energies are $e_{20,1}^{\mathrm{WS}} = -41.804$ and $e_{20,1}^{\mathrm{Coul}} = -0.428$ MeV.

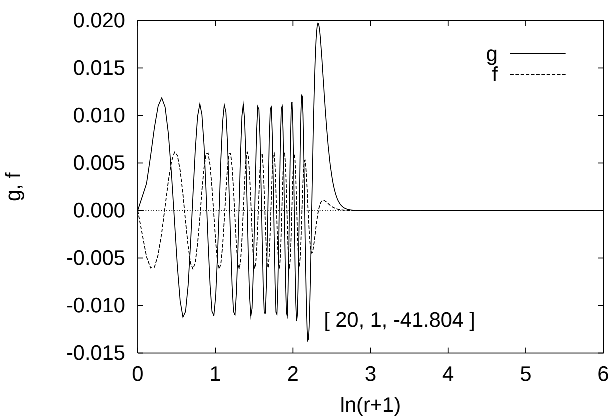

Fig. 2. The upper and lower components of the Dirac wave function (f,g) of the state $n=20$, $\kappa=1$ in ^{208}Pb and the WS $\bar{p}A$-potential. The radial distance r is in fm units. The energy of the state is $e_{20,1} = -41.804$ MeV.

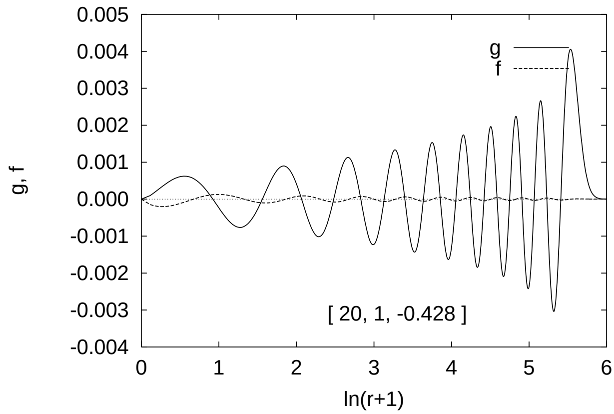

Fig. 3. The same as Fig. 2 but for the point nucleus Coulomb potential. The corresponding energy $e_{20,1} = -0.428$ MeV.

In the present work we generated the single antiparticle scalar and vector potentials in the framework of RMF model and the phenomenological Woods–Saxon model. The antinucleon potentials were obtained from the corresponding particle potentials according to charge conjugation rule. As an example, the Dirac antiproton wave functions (f, g) and the single particle eigenenergies were calculated by solving the $\bar{p}$-eigenvalue problem for ^{208}Pb.

The scheme is easily applicable in case of the deformed nuclei.

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