THE 'HOW AND WHY' OF NUCLEON TRANSFER REACTIONS WITH RADIOACTIVE BEAMS*

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(Received February 2, 2001)

The intensity of secondary radioactive beams has now reached the stage where it is sufficient to perform nucleon transfer reactions with them. Traditionally proven spectroscopic tools such as (p, d) and (d, p) reactions must be performed using inverse kinematics, which introduces characteristic experimental constraints. In particular, it is not possible to achieve resolutions better than 200 keV typically, using just the detection of the outgoing charged particles, and gamma-ray detection is required to improve the achievable resolution for bound states. The kinematics are insensitive to the details of individual reactions, so it is possible to construct a dedicated detection system with wide applicability, for example the TIARA array being constructed in the UK for use at GANIL.

PACS numbers: 25.40.Hs, 25.45.–z, 25.60.Je, 29.30.Ep

1. Introduction

The complexity of experiments that can be attempted using nuclear beams is a function of the beam intensity that can be obtained. Using stable beams, with increasing complexity in the apparatus, the selection of very rare reaction products has been pursued and it has been shown at this conference how recoil decay tagging has taken this to new levels. For example, studies in the neutron deficient Pb–Po–Rn region can access nuclei produced with cross sections of order $\sigma = 0.1$ μb, using typical experimental parameters and beams of order 6×10^{10} particles/sec (pps), *i.e.* 10 pnA. With a radioactive beam of an interesting isotope, quite removed from stability, it is realistic to suppose that a beam of order 10^{-6} of this intensity could be produced at present. Thus, reactions of order 0.1 b can be considered, and if a highly efficient detection system were developed then it could be

* Presented at the XXXV Zakopane School of Physics "Trends in Nuclear Physics", Zakopane, Poland, September 5–13, 2000.

supposed that a cross section of 10 mb would be accessible. These are the typical magnitudes of nucleon transfer cross sections, so it can be seen that these reactions — a proven means for studying single-particle structure and hence shell structure — become accessible at beam intensities in excess of 1–10 $\times 10^4$ pps.

Proven traditional spectroscopic tools such as (p,d) and (d,p) reactions, when applied to radioactive beams, must be performed using inverse kinematics. This introduces certain experimental constraints that are characteristic of the reaction geometry and vary only a little between different specific reactions. In general, it is not possible to achieve resolutions better than typically 200 keV for the excitation energy of the final particles, using just the detection of the outgoing charged particles. Gamma-ray detection, which is possible for excited bound states, is required to improve the achievable resolution. Fortunately, the kinematics are very similar for a wide range of beam masses and incident energies, and this allows the construction of a dedicated detection system with wide applicability. One such system is the UK's TIARA array, which is presently under construction.

The need to develop new spectroscopic techniques to deal with the extremely low intensity of radioactive beams, compared to traditional stable beams, has led to the study of nucleon removal via a knockout mechanism which was not widely used previously. This has successfully been applied to very exotic beams with extremely low intensities, of the order of 1 particle/min or less, and is an interesting complement to the study of transfer reactions.

2. Transfer reactions and knockout reactions

Nucleon transfer reactions have a long and venerable history in nuclear reaction studies [1, 2]. They have well-defined two-body initial and final states, and the interaction involves the transfer of a nucleon between the target and projectile nuclei. The probability for this process involves kinematic matching conditions as well as structural overlap contributions. The transfer probability falls with increasing beam velocity, and an energy regime of 10–20 MeV/u has proven most useful for the study of transfer, free of compound nuclear effects. Differential cross sections are typically of the order of 10 mb sr^{-1}.

In the most commonly applied theories (*e.g.* DWBA), the transferred nucleon is represented as being in a single particle orbital before and after the transfer, or at least in a state that can be represented as a linear combination of such wavefunctions. This is a reasonable approximation, so long as the various configurations involved are not strongly coupled together, as they could be for example by a strong collective rotational or vibrational excitation. Experimental measurements give the angular momentum transferred by the nucleon and measure the overlap of the actual nuclear states with specified single-particle states, characterised by a spectroscopic factor.

Knockout reactions measure the probability of nucleon removal from the projectile by observing the surviving nuclei [3,4]. These reactions are studied in an energy regime where the interaction with the target can be taken to be extremely peripheral, namely 100–200 MeV/u. The nucleon is removed by the interaction between the tail of its wave function, where it extends beyond the core of the projectile nucleus, and the target nucleus. It is then obvious that this mechanism is an excellent probe of halo nuclei and other weakly bound nuclei, such as those found near the neutron drip line. The cross sections in these cases are of order 100 mb sr^{-1}. Most likely, the applicability of knockout reactions will eventually be demonstrated over a wide range of masses and binding energies, though for more bound nuclei the cross sections will be much closer to those for transfer. In knockout studies, the ground state of the projectile can again be characterised in terms of single particle structure using a spectroscopic factor. The angular momentum of the removed nucleon can be determined directly from the width of the longitudinal momentum distribution of the surviving core, which carries the imprint of the sudden removal.

It is clear that transfer reactions can be used to identify single particle levels, and the fragmentation of single particle strength, near closed shells such as ^{132}Sn for example. Another application that is less widely appreciated is to the study of deformed nuclei. Single nucleon transfer on an even-even deformed target can populate states in a rotational band built on a particular Nilsson orbital. The wave function of the orbital can be written as an expansion in terms of the spherical orbitals in the same major shell, which have a well defined angular momentum. When the transferred nucleon populates a state in the rotational band, it carries in a spin j which must match the spin of the state, and it selects just the component of the expansion that has spin j. For each state in the band, this corresponds to a different component in the expansion, and the result is a 'fingerprint' pattern across the states in the band, which is characteristic of the Nilsson orbital [5, 6]. The ability to identify the Nilsson orbitals helps to determine the deformation. In principle the knockout studies can also be adapted to study 'fingerprint' patterns from deformed nuclei.

3. Inverse kinematics

When dealing with radioactive beams, transfer reactions such as (p,d), (d,p) and (d,^{3}He) need to be studied using hydrogen targets. In general, the beam particle is many times heavier than the target, and the kinematics are massively inverse. Transfer reactions initiated by heavy ions (as the target nucleus) lead to problems if the target-like recoil is to be observed except in special cases (since, except for the lightest heavy ions, the recoil will usually be too low in energy to escape from the target) and will not be discussed here.

A key result of the highly inverse kinematics is that the energy-angle systematics of the target-like particles (the d, p or ^{3}He in the above examples) are very similar for all reactions of a given type. That is, the mass or energy of the incident beam, and to a large extent the Q-value, all have relatively little effect. The over-riding factor is the change in mass of the target-like particle: from 1 to 2, 2 to 1, or 2 to 3 in the examples.

To see this, it is perhaps easiest to consider the velocity addition diagrams for the two-body kinematics. Even though these are not relativistically accurate, they give a good qualitative indication of the results and are even rather good quantitatively in the energy regime of interest. For elastic scattering (see Fig. 1) the target-like particles emerge close to 90 degrees in the laboratory frame, for small centre of mass scattering angles. Their energies also start at zero and rise rapidly with increasing c.m. scattering angle. For the beam-like particle, the velocity in the c.m. frame is very much lower (by the ratio of the target to the beam mass) and the deflection angle is also very small. These results are clearly independent of the precise mass or velocity of the incident beam particle, so long as it is heavy compared to the target.

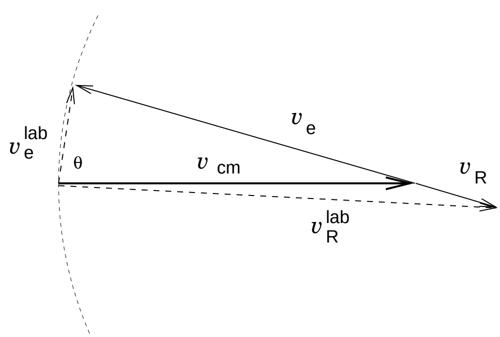

Fig. 1. For elastic scattering, the velocity of the c.m. in the laboratory is equal and opposite to the initial velocity of the target in the c.m. frame, and small c.m. scattering angles give laboratory angles near 90 degrees.

Now, considering transfer reactions, the key factor is the change in mass for the target-like particle, since this dramatically affects its c.m. velocity. In the c.m. frame, the kinetic energy of the heavy (beam-like) particle is smaller than that of the light particle by a factor of order $1/m_{\text{beam}}$ and is negligible for $m_{\text{beam}} \gg m_{\text{ejectile}}$. Considering ($p$,$d$) as an example, and ignoring Q-value effects for the moment, the kinetic energy of the d in the *c.m. frame* is thus about the same as that of the p in elastic scattering. However, its mass is doubled and therefore since $K \approx p^2/2m$ its momentum is increased by $\sqrt{2}$, and its velocity $v = p/m$ in the c.m. frame is multiplied by a factor of $\sqrt{2}/2$. In the general case where the mass of the target M_{T} and the ejectile M_e are in the ratio $f = M_{\mathrm{T}}/M_e$ then the factor is $\approx \sqrt{f}$.

The consequences for the velocity diagrams are illustrated in Fig. 2. If the masses of the projectile and beam-like 'recoil' are denoted as M_{P} and M_{R} respectively, then [7]

$$\frac{v_e}{v_{\mathrm{c.m.}}} = \left(q\ f\ \frac{M_{\mathrm{R}}}{M_{\mathrm{P}}} \right)^{1/2} \approx \sqrt{qf} \ \text{ if } \ M_{\mathrm{R}} \approx M_{\mathrm{P}}\,,$$

where $v_{\mathrm{c.m.}}$ is the velocity of the c.m. *in the laboratory frame* and $q = 1 + Q_{\mathrm{tot}}/E_{\mathrm{c.m.}}$, with $Q_{\mathrm{tot}} = (Q_{\mathrm{g.s.}} - E_x)$ being the Q-value for a state at energy E_x in the recoil, and $E_{\mathrm{c.m.}}$ the collision energy in the c.m. frame. Typically q differs from unity by less than 10%, getting closer as the beam energy per nucleon (E/A) is increased: $q \approx 1 + Q_{\mathrm{tot}}/(E/A)_{\mathrm{beam}}$.

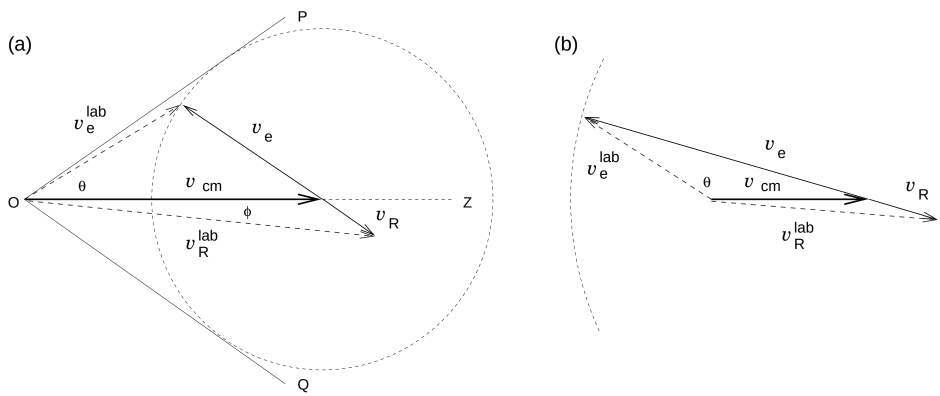

Fig. 2. As for the previous figure, except for the situation when transfer changes the mass of the light particle (and recoil): (a) for reactions such as (p,d) or $(d,{}^3\mathrm{He})$ where the target-like mass increases, (b) for reactions such as (d,p) where the mass is decreased.

Thus, for a reaction such as (p,d) the light ejectiles are confined (*cf.* Fig. 2) to within a cone of half-angle POZ given by $\theta_{\max} \approx \sin^{-1}\sqrt{f}$ where $f = 1/2$ for (p,d) and $f = 2/3$ for (d,t). This gives about 50° in each case, but the extra focussing for (p,d) can be significant experimentally. Application of the cosine and sine rules shows that, for a (d,p) reaction, the scattering angles $\theta_{\mathrm{c.m.}} < 30°$ are focussed to laboratory angles backward of about 110°. Note that the beam mass and bombarding energy have no effect on these results, within the $q = 1$ approximation.

These general results are illustrated in Fig. 3 using two examples with substantially different projectile masses and velocities. The figures are labelled with the c.m. scattering angles (according to the traditional 'normal kinematics' convention, where the light particle is the projectile) from 0 to 30 degrees, where the differential cross section is the greatest. The fig-

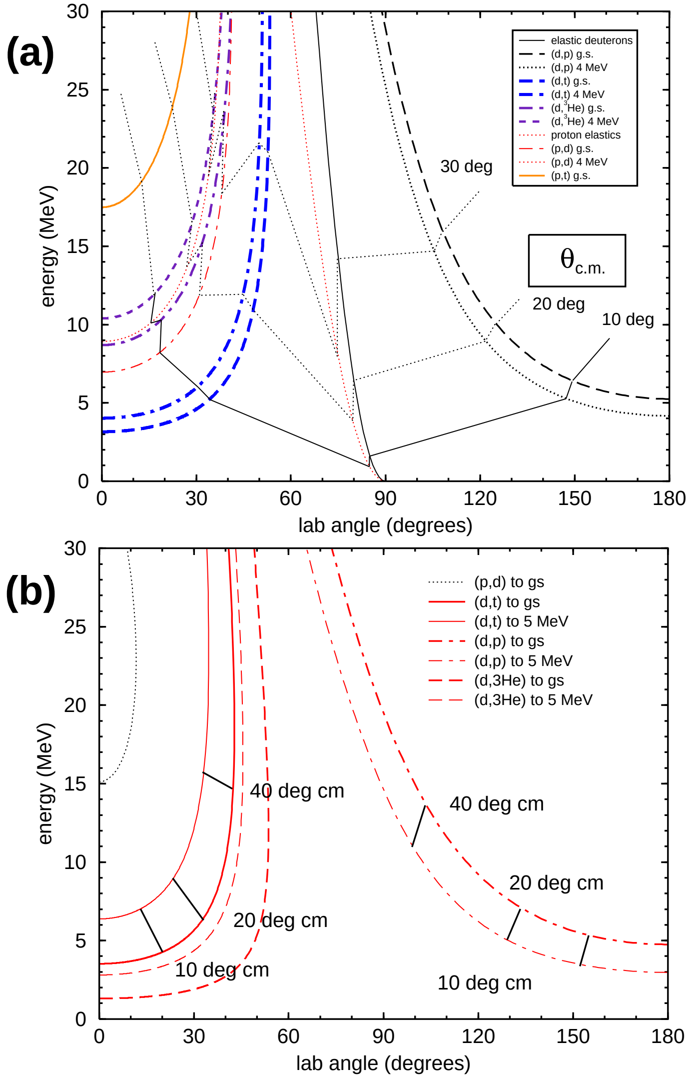

Fig. 3. Typical energy-angle systematics for transfer reactions in inverse kinematics: (a) for ^{16}C at 35 MeV/u, and (b) for ^{74}Kr at 12.16 MeV/u, each on proton and deuteron targets.

ures confirm that the yield for particle removal from the projectile, for elastic scattering, and for particle addition are concentrated at forward angles, near 90 degrees, and at backward angles respectively. An important additional result is that the energies of the scattered light particles are similar for the various reactions, which means that an all-purpose charged particle array can be considered for transfer studies with radioactive beams. It is also apparent that, if the Z of the beam-like particle is measured, then the energy and angle of the light particle largely serve to identify its Z and A.

An interesting further feature of the inverse kinematics is the effect of the jacobian which relates the differential cross section measured in the laboratory frame to that in the c.m. frame (see, for example, Ref. [8]). The nature of the kinematic focussing of angles can be inferred from the labelling of the c.m. angles in the kinematics of Fig. 3. In some angular ranges, the experimental angular resolution becomes critical. A particularly interesting result is the defocussing that occurs for (d,p) at the small scattering angles in the 'light ion' convention (near 180 degrees in the laboratory frame). This is illustrated in Fig. 4 which refers to a proposed study of Sr isotopes [9].

In Fig. 4 the dashed line would be the mirror image of the solid line, around 90 degrees, if it were not for the jacobian effects. Due to the kinematic focussing, the region of largest differential cross section is no longer the region of smallest scattering angles. However, it is still true in general that the smallest scattering angles are the ones best modelled by the reaction

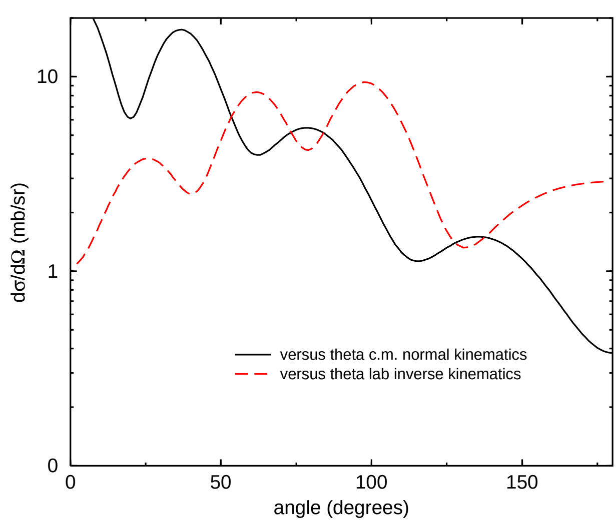

Fig. 4. Calculation of the differential cross section for (d,p) on ^{95}Sr, using zero-range DWBA, plotted as (a) solid line, $\mathrm{d}\sigma/\mathrm{d}\Omega$ *vs* $\theta_{\mathrm{c.m.}}$ in the traditional *light ion* convention, (b) dashed, $\mathrm{d}\sigma/\mathrm{d}\Omega$ *vs* θ_{lab} in the laboratory frame, see text.

codes and therefore most important to measure. The difference compared to traditional measurements is that it becomes relatively easy to extend the experiment to include larger scattering angles as well.

4. Summary of experimental constraints

The experimental factors affecting the energy resolution that can be obtained in practice have been discussed by Winfield *et al* [10] and previous authors [11, 12]. The factors taken into account included the energy and angular resolution of the beam, as well as those of the detectors.

To summarise, the best resolution attainable for excitation energy is of order 200 keV, and this requires targets as thin as 0.5 mg/cm^2. (If plastic polymer targets are used, rather than say solid hydrogen, then the partial thickness of hydrogen is even less). In the case of light beams ($A < 20$) good results can be obtained by recording the angle and energy of just the beam-like particle precisely (though the coincident detection of the light particle from the target can remove background reactions on target contaminants very effectively). More generally, the light particle needs to be recorded. The various options have been enumerated previously [13, 14] and are discussed in more detail below:

1. *Rely on detecting the beam-like ejectile in a spectrometer:* This has the advantage that the particles are kinematically focussed into a small angular range that can be spanned by a high resolution magnetic spectrometer. If the beam mass is too high, however, the focussing is so great that the required angular resolution becomes prohibitive. A disadvantage is that any spread in the beam energy is translated directly into excitation energy resolution. This must be overcome either by using a dispersion-matched spectrometer (which implies a spatially dispersed beam spot at the target), or else by tagging the energies of individual beam particles somehow. Even then, the resolution is intrinsically limited by the gamma-decay of the detected particles in flight, which spreads the image at the focal plane.

2. *Rely on detecting the target-like ejectile in a Si detector:* In this case, the particles are spread over a significantly larger angular range, but it is possible to envisage covering this range with suitable high resolution detectors such as Si strips. Any spread in the beam energy has little effect on the resolution, so no energy tagging is needed and a focussed beam spot can be employed. However, the target thickness becomes an important constraint, due to the differential energy loss suffered by ejectiles produced at different depths, and gives the limit cited above. This method is of course the only possible choice to study the

production of unbound states. It can be combined with the detection of recoils or breakup particles near zero degrees, to give improved channel selection.

3. *Detect decay gamma-rays in addition to particles:* The limitations on resolution that are imposed by the target thickness can be avoided when gamma-ray energy information is used to give precise excitation energies. This method is clearly limited to bound excited states, but that includes many cases of interest. Taking into account the low intensities of radioactive beams, an exceptionally high gamma-ray efficiency is demanded, say of order 25% or better. This is achievable with modern arrays, but a closely packed geometry is implied, and the Doppler broadening introduced by the angular acceptance of the gamma-ray detectors is a potential problem. This also can be solved, using a segmented germanium detector to measure the point of gamma-ray interaction. For example, with the EXOGAM array [15] is its closest geometry, a typical Doppler-limited resolution of 20 keV is attainable. Then, the target thickness requirements can be relaxed, and the new limitation becomes the multiple angular scattering of the ejectiles. This must not obscure the angular distribution from the nuclear reaction, which is required to identify the transferred angular momentum. Typically, this allows almost an order of magnitude increase in the target thickness, so overall there is an increase in counting rate of a factor of ~ 2 compared to option 2 above.

Whilst the third option is very attractive, the experimental challenges should not be overlooked. Of course, gamma-decays will in general occur in cascades and with branching ratios. It will be necessary to measure this information in order to extract angular distributions for individual excited states. Some (usually small) corrections will need to be applied for gamma-ray angular distribution effects. The efficiency of the gamma-ray detectors will also need to be known accurately, especially in order to extract the results for the ground state. The ground state distribution must be obtained from a particle singles measurement, by subtracting suitably scaled numbers of gamma-ray coincident counts. The feasibility of such techniques has been demonstrated in the knockout studies mentioned earlier, *e.g.* Ref. [3], which followed just such a procedure.

5. Experimental results

Several experimental studies of transfer reactions, to measure spectroscopic information, have now been reported. The reader is referred to the original articles for details. The first example is the experiment by Rehm an coworkers [16], who measured reaction probabilities of astrophysical interest using the reaction ^{56}Ni(d,p)^{57}Ni in inverse kinematics. The beam of ^{56}Ni was obtained using radioactive source material and a tandem accelerator, and was of relatively high optical quality. The protons were detected in an array of silicon strip detectors at backward angles, as can be understood from the above discussion of the kinematics (Method 2). The second example is a study of the structure of the ground state of the halo nucleus ^{11}Be, using the reaction ^{11}Be(p,d)^{10}Be in inverse kinematics [17, 18]. The ^{10}Be reaction products were detected using a dispersion matched magnetic spectrometer (Method 1). The coincident detection of deuterons was vital in the elimination of background events arising from the carbon in the extended $(CH_2)_n$ polymer target. A third example is a study of the low-lying levels of the unbound nucleus ^{10}Li using the reaction ^{11}Be(d,^{3}He)^{10}Li in inverse kinematics [14, 19]. The ^{3}He products were recorded using an array of silicon strip detectors (Method 2), and the Li ions originating from ^{10}Li breakup were recorded in a scintillator telescope spanning the forward angles. An indication of how gamma-ray coincidences (Method 3) can be expected to improve transfer measurements with radioactive beams is given by the exploratory work of Ref. [20].

The ^{11}Be(p,d)^{10}Be is an interesting example, because the interpretation of the experimental data has led to new theoretical ideas: the coupling between weakly bound nuclear states and the continuum has been extended to include breakup of complex nuclei such as ^{11}Be [21]. The data also highlight the difficulties in extracting spectroscopic data for nuclei in which the single particle orbitals are strongly coupled [18], a situation that was mentioned in section 2. It should be noted, however, that more generally the interpretation can be expected to be more straightforward. The ^{11}Be experiment was aimed at quantitative measurements of spectroscopic factors at the 10% level. A more typical experiment would be seeking to locate states with a largely single particle character and the level of accuracy required in the absolute spectroscopic factors will be relatively modest.

6. Outlook

The MUST array [22] developed in France is an example of a high solid angle silicon strip system that is suitable for this type of experiment, and it has been used successfully to study inelastic scattering in inverse kinematics and for the ^{10}Li experiment described in Section 5. This array is particularly well adapted to studies of inelastic scattering and of reactions leading to unbound nuclei, using Method 2.

The TIARA array [23] has recently commenced development in the UK, and is planned to be installed at GANIL for experiments using both secondary fragmentation beams and reaccelerated beams from the SPIRAL facility. The preliminary design for the silicon array at the heart of TIARA is shown in Fig. 5. This is designed to be mounted inside of the EXOGAM gamma-ray array [15] in its most compact geometry with the germanium detectors approximately 50 mm from the target. The paramount criterion was to fit a high resolution, $\sim 4\pi$ array within this small space. The adopted solution includes an octagonal barrel of silicon detectors with resistive strips oriented parallel to the beam direction. One face of the octagonal barrel is omitted, to allow the target to be inserted via a vacuum interlock, and oriented at a suitable angle on its frame. The most forward and backward reaction angles are instrumented using annular non-resistive strip detectors that are placed further from the reaction region and can be shielded from the EXOGAM detectors. The vacuum vessel away from the interaction region is large enough to allow additional beam tracking detectors. At angles close to zero degrees, the beam-like particles emerge and are detected using either a pixellated particle telescope or, ideally, a magnetic spectrometer that can discard the actual un-reacted beam particles. The TIARA design has been made to allow the coupling of the array to the VAMOS spectrometer [15]

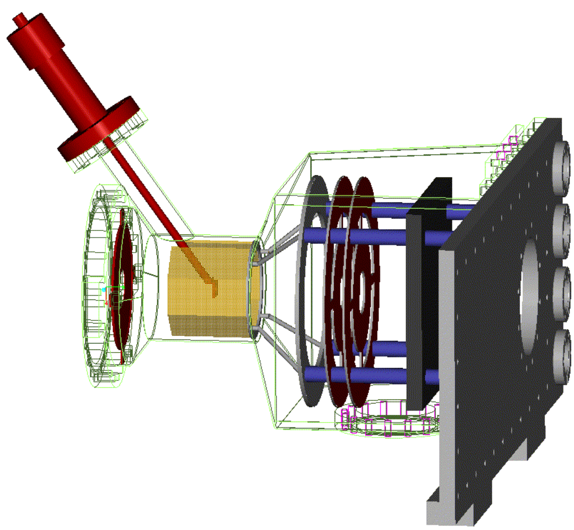

Fig. 5. Preliminary design for TIARA (Transfer and Inelastic All-angle Reaction Array) which is being developed in the UK, specifically to use particle-gamma coincidences to study transfer reactions induced by radioactive beams.

which is designed with large angular and momentum acceptances, specifically for radioactive beam experiments, and is being built at GANIL. Fig. 5 shows the back plate of the TIARA assembly, which mounts directly on to the front of the entrance quadrupole of VAMOS.

Transfer reactions induced by radioactive beams have already been established as a useful spectroscopic tool. The first generation of specialised transfer arrays, of which TIARA is an example, is under construction. Their use is poised to escalate with the imminent availability, at several facilities worldwide, of a wide range of reaccelerated radioactive beams at Coulomb barrier energies. This will reveal new information about shell structure near exotic magic numbers, and also new information about low-lying levels in deformed nuclei far from stability.

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