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APPROXIMATE TREATMENT OF TIME DEPENDENT RESONATING HARTREE–FOCK THEORY APPLICATION TO LIPKIN MODEL*

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The Time Dependent Resonating Hartree–Fock (TD Res HF) theory is expected to work better than the usual TDHF one if we describe large-amplitude collective motions in soft nuclei with large quantum fluctuations. To make an essential feature of the TD Res HF theory clear, we apply it to the exactly solvable Lipkin model. We adopt an adiabatic perturbative approach to a solution of the TD Res HF equation and derive inertia parameter of kinetic energy and potential energy of the collective motion.

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1. Introduction and basic assumptions

In a series of papers Fukutome and one of the present author (S.N.) have proposed theories called the time dependent resonating Hartree–Fock [1] (TD Res HF) (referred to as I) and the time dependent resonating Hartree–Bogoliubov [2] (TD Res HB) theory to describe collective excitations in normal fermion systems and/or superconducting fermion systems with large quantum fluctuations, respectively. In the TD Res HF and the TD Res HB theories as well as the Res HF [3] and the Res HB [4] theories, each ground-state wave function is approximated by superposition of non-orthogonal Slater determinants (S-dets) or HB wave functions with different correlation structures. Resonance of S-dets or HB wave functions takes place if a mean-field energy functional has multiple local minima with near energies. The theories lead us to approximations called the resonating HF random phase

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approximation [1] (Res HF RPA) and the resonating HB random phase approximation [2] (Res HB RPA) if a time dependent small fluctuation is taken into account around the stationary Res HF/HB ground-state solution.

To clarify an essential feature of the Res HF approximation and to show its advantage over the usual HF theory, we have applied that approximation to the exactly solvable Lipkin model [5,6] ([6] is referred to as II) by making use of a new orbital optimization algorithm [7]. We have assumed that a Res HF wave function is superposed by only two S-dets which give corresponding two local energy minima of monopole *deformation*. The self-consistent Res HF calculation in II produces an excellent ground-state correlation energy [6, 8]. The first application of the Res HB theory had been performed to a problem of describing resonance of prolate and oblate shapes coexisting in nuclei by using a schematic model with pairing correlations [9]. We also have given a relativistic Res HF theoretical description of exotic phenomena in nuclei standing on the relativistic mean-field spirit [10]. Recently, the Res HF theory has been successfully applied [11, 12] for description of mass spectra and associated properties of the π and σ mesons. The Res HF RPA was applied to the Lipkin model using the stationary solution in II [13].

The TD Res HF theory is expected to work better than the usual TDHF theories if we apply them to describe large-amplitude collective motions in soft nuclei with large quantum fluctuations. The TD Res HF wave function is given by superposition of non-orthogonal S-dets. The Res HF wave function, the overlap integral S and the interstate density matrix W between two S-dets together with the normalization condition are expressed as

$$\begin{aligned}
|\Psi\rangle &= |u\rangle c = \sum_g |u_g\rangle c_g\,, \quad \langle\Psi| = d^*\langle v| = \sum_f c_f^*\langle u_f|\,,\\
S &= \langle u|v\rangle = \det z\,, \qquad z = v^\dagger u\,,\\
W &= uz^{-1}v^\dagger\,, \quad W^2 = W\,, \quad W^\ddagger = W\,,\\
\langle\Psi|\Psi\rangle &= d^*Sc = \sum_{f,g} c_f^* S_{fg} c_g = 1\,.
\end{aligned} \tag{1}$$

To make the essential features of the TD Res HF theory clear we apply it to the exactly solvable Lipkin model with two N-fold degenerate levels [5]. For simplicity, we consider the case of two resonating HF energy levels. We adopt an adiabatic perturbative approach, *i.e.*, the ATDHF approximation [14], to seek for a solution of the TD Res HF equation. First under the ATDHF approximation, we consider the first-order part of the coupled TD Res HF equations of motion for TDHF density matrices, though the significant lowest order of the approximation arises from the second-order. Because the ATD Res HF equations can be separated into two types of equation consisting of only the time-even part and the time-odd one [14]. The mixing coefficients are determined by the normaliztion condition. We solve approximately the first-order TD Res HF equations. Then we can derive a kinetic energy of second order and obtain a mass term of collective motion. Thus the ATD Res HF theory leads us to new expressions for inertia parameter and potential energy of the collective motion.

2. Inertia parameter and potential energy of collective motion

In this talk, starting from the two TD isometric matrices u_1 and u_2,

$$\begin{aligned} u_{1(2)} &= \begin{bmatrix} \cos\frac{\theta_{1(2)}}{2} e^{-i\frac{1}{2}(\psi_{1(2)}+\varphi_{1(2)})} 1_N \\ \sin\frac{\theta_{1(2)}}{2} e^{i\frac{1}{2}(\psi_{1(2)}-\varphi_{1(2)})} 1_N \end{bmatrix}, \\ u^\dagger_{1(2)} u_{1(2)} &= 1_N, \quad z_{12} = u^\dagger_1 u_2, \end{aligned} \tag{2}$$

we have applied the ATD Res HF theory to the exactly solvable Lipkin model and derived new expressions for the collective mass parameter $\mathcal{M}(\theta)$ and the collective potential energy $\mathcal{V}(\theta)$.

Defining the collective mass parameter through $\mathcal{K} = \frac{1}{2}\mathcal{M}(\theta)\dot{\theta}^2$, we get

$$\begin{aligned} \frac{N}{2\varepsilon\mathcal{M}(\theta)} &= A^2(\theta)\left\{1 + N\frac{\sin^2\theta}{\cos^2\theta}(\cos\theta)^N - \frac{1}{\cos^2\theta}(\cos\theta)^{2N}\right\}^{-2} \\ &\times\left\{\cos\theta + \chi(1+\sin^2\theta) - \frac{\cos\theta - \chi(\cos^2\theta - \sin^2\theta)}{\cos^4\theta}(\cos\theta)^N\right\}^{-1} \\ &\times\{1 + (\cos\theta)^N\}, \end{aligned} \tag{3}$$

where

$$\begin{aligned} A(\theta) &= \cos\theta + \chi\sin^2\theta + \chi \\ &+ \left\{(N-2)\frac{\sin^2\theta}{\cos\theta} + \chi\left(1 + \frac{1}{\cos^2\theta} + \sin^2\theta + \frac{1}{2}N\frac{\sin^4\theta}{\cos^2\theta}\right)\right\}(\cos\theta)^N \\ &- \frac{1}{\cos\theta}\left(1 - \chi\frac{1}{\cos\theta}\right)(\cos\theta)^{2N}, \quad \chi = \frac{(N-1)V}{\varepsilon}, \end{aligned} \tag{4}$$

which reduces to the well-known result for the collective mass parameter in the limit of the single S-det case [15]. The potential energy of the collective motion is given as

$$\mathcal{V} = -\tfrac{1}{2}\varepsilon N(\cos\theta + \tfrac{1}{2}\chi\sin^2\theta)\left\{1 + \tfrac{1}{\cos^2\theta}(\cos\theta)^N\right\}\tfrac{1}{1+(\cos\theta)^N}\,. \tag{5}$$

In the above ε and V are kinetic energy and interaction strength of the Lipkin-model Hamiltonian, respectively.

The quantity related to the inverse of the collective mass parameter $\frac{N}{2\varepsilon\mathcal{M}(\theta)}$ is plotted against the *deformation* parameter θ for $N = 10$. As numerical calculation shows, in the ATD HF case this quantity has the unusual feature of not being positive definite for values of $\chi < 1$. On the contrary, the ATD Res HF case causes no difficulty in the physical interpretation the collective mass parameter. However, in this case, there happens another difficulty connected with a singularity in the very near regions of $\theta = 0$ and $\theta = \pm\pi$. This singularity occurs due to the existence of the term

$$\left\{1 + N\frac{\sin^2\theta}{\cos^2\theta}(\cos\theta)^N - \frac{1}{\cos^2\theta}(\cos\theta)^{2N}\right\}^{-2},$$

as is seen from Eq. (3). Furthermore, the double minima appears though in the ATD HF case it does not. As for the deficiency of such singular behaviours, we may expect to remove it and to have no serious problem if we drop the restrictions $\theta_2 = \theta_1 = \theta$ and consequently $c_1 = c_2 = c$.

The collective potential energy is also plotted against the *deformation* parameter θ for $N = 10$. For $\chi < 1$ the collective potential energies for both the cases take almost same values. But for $\chi > 1$ the collective potential energy in the ATD Res HF case is higher than that in the ATDHF case. It should be noticed that the behaviour of the former becomes shallower than that of the latter.

It is a very important and interesting problem to solve an eigenvalue equation

$$\left\{-\frac{1}{2}\frac{\partial}{\partial\theta}\frac{1}{\mathcal{M}(\theta)}\frac{\partial}{\partial\theta} + \mathcal{V}(\theta)\right\}\Psi = E\Psi, \tag{6}$$

and to compare the low-lying eigenvalues with the exact ones. We will attack the problem for a more complicated case $\theta_2 \neq \theta_1$ keeping $\psi_2 - \psi_2 = \pi$ and solve the Schrödinger equation for the corresponding collective Hamiltonian.

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