Vol. 32 (2001) *ACTA PHYSICA POLONICA B* No 3

SHELL EFFECTS IN BUBBLE NUCLEI, ATOMIC CLUSTERS, AND INHOMOGENEOUS NEUTRON MATTER*

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(Received November 2, 2000)

We analyze the character of the shell effects/Casimir energy in inhomogeneous fermion systems. We estimate magnitude of the shell effects and discuss their dependence on a number of physical parameters (geometry, fermion density, temperature).

PACS numbers: 21.10.Dr, 21.65.+f, 97.60.Jd, 67.55.Lf

The problem of relative arrangement of impurities inside a many fermion system is determined solely by the quantum effects, as the volume, surface or curvature terms in the liquid drop expansion of the total energy are not affected when impurities are moved around (if Coulomb effects are irrelevant). The properties of the quantum systems that contain impurities of various shapes have been studied in the case of the Bose–Einstein condensate [1]. There, due to the fact that all the particles have the same single particle wave function, the behavior of the impurity is rather obvious and one can easily show that it will be expelled from the condensate [1]. In the case of fermion systems the many-body wave function has a complex character due to the Pauli principle and the answer is not obvious. The additional energy associated with the relative arrangement of impurities can be termed shell correction energy. In fact, there is no well established terminology for the energy corrections we are considering here, even though the problem has

* Presented at the XXXV Zakopane School of Physics "Trends in Nuclear Physics", Zakopane, Poland, September 5–13, 2000.

been addressed before to some extent by other authors. In the case of finite systems, the energy difference between the true binding energy and the liquid drop energy of a given system is typically referred to as shell correction energy. In field theory a somewhat similar energy appears, due to various fluctuation induced effects and it is referred to as the Casimir energy [2]:

$$E_{\rm Casimir} = \int_{-\infty}^{\infty} d\varepsilon\varepsilon[g(\varepsilon, \mathbf{l}) - g_0(\varepsilon)] \,, \tag{1}$$

where $g_0(\varepsilon)$ is the density of states per unit volume for the fields in the absence of any objects, $g(\varepsilon, \mathbf{l})$ is the density of states per unit volume in the presence of some inhomogeneities and $\mathbf{l}$ is an ensemble of geometrical parameters describing these objects and their relative geometrical arrangement. Thus the Casimir energy can be thought of as a measure of the fluctuations induced in the energy spectrum in the presence of various "obstacles". There are a number of situations when such inhomogeneities can be formed. In the nuclear physics it was suggested a long time ago that very heavy nuclei will tend to develop a hole inside in order to minimize the Coulomb energy [3–5] and similar objects can be created in the case of atomic clusters [6]. The creation of voids is also predicted to happen in the nuclear matter at subnuclear densities. Apparently, an agreement has been reached in literature concerning the existence of the following chain of phase transitions as the density increases: nuclei $\to$ rods $\to$ plates $\to$ tubes $\to$ bubbles $\to$ uniform nuclear matter [7]. In these cases, fermions reside in a rather unusual mean-field, which is much deeper for bound than for unbound nucleons. Since the amplitude of the wave function in the semiclassical limit is proportional to the inverse square root of the local momentum, the single particle wave functions for the unbound states will have a small amplitude over the deep well. Hence the deep well will act almost like a hard wall (in most situations). For the same reasons the halo nuclei [8] can be thought of as some kind of bubbles as well. Let us consider first the problem of positioning of a spherical bubble inside a finite spherical Fermi system. The total energy can be expressed in the form:

$$E(N) = E_{\rm LD}(N) + E_{\rm shell}(N) = e_v N + e_s N^{2/3} + e_c N^{1/3} + E_{\rm shell}(N), \tag{2}$$

where $E_{\rm LD}$ is the smooth liquid drop part of the total energy and $E_{\rm shell}$ is the quantum shell correction contribution to the total energy. As we mentioned above, once the bubble is formed its displacement will not affect either the volume, surface or curvature terms in the liquid drop expansion. Hence, classically moving bubble off-center costs no energy, if Coulomb energy is left aside for the moment. The closer investigation of the shell correction energy shows that it depends strongly on the position of the bubble. The most pronounced shell effects are predicted for the spherically symmetric system (bubble in the center). In this case the system is classically integrable and thus its quantum-mechanical counterpart will have large gaps in the spectra resulting in a large amplitude of the shell correction energy. The rapid fluctuation of the shell energy as a function of the fermion density indicate that for some numbers of fermions it would be energetically more favorable to expel the bubble off-center. For a finite eccentricity the system is chaotic and the more the bubble is shifted from the center the larger part of phase-space is occupied by chaotic trajectories. Nevertheless, the shell effects are still strong. One can show that the shortest periodic orbit determines the gross structure of the shell energy when the bubble is close to the surface [9,10]. The problem of two or more objects immersed in an infinite Fermi system has a similar character. In an infinite system the presence of impurities results in an appearance of resonances, which contribute to the shell correction energy. In order to better appreciate the nature of the problem, let us consider the situation when two identical spherical bubbles have been formed in an otherwise homogeneous Fermi system. We shall ignore here the possible Coulomb interaction, as its main contribution is to the smooth part of the total energy of the system. In the semiclassical approach, which is justified for the "sizeable" bubbles (*i.e.* when the Fermi wavelength is small comparing to the size of the bubble), the shell correction energy is determined by the periodic orbits in the system. In the case of two spherical bubbles there exist only one such trajectory (with repetitions) which gives rise to the interaction energy between bubbles. It is the hyperbolic orbit lying in the line connecting the bubbles centers and characterized by the Lyapunov exponent: $\lambda = 2\ln\left[1+\frac{d}{R}+\sqrt{\frac{d}{R}\left(\frac{d}{R}+2\right)}\right]$, where R is the radius of the bubble and d is the distance between their centers. The interaction energy between the two bubbles due to the existence of this periodic orbit reads:

$$E_{\rm shell} = \frac{\hbar^2 k_{\rm F}^2}{2m}\frac{1}{4\pi(k_{\rm F}d)^2}\sum_{n=1}^{\infty}\frac{[2nk_{\rm F}d\cos(2nk_{\rm F}d)-\sin(2nk_{\rm F}d)]}{n^3\sinh^2(n\lambda/2)}, \qquad (3)$$

where $k_{\rm F}$ denotes the Fermi momentum and m the mass. Similar arguments can be presented for other shapes, *e.g.* cylinders or plates. One finds that at large separations the interaction energy oscillates as well but it decays as $\propto 1/d^{5/2}$ in case of cylinders and as $\propto 1/d^2$ in the case of plates. Hence it is clearly seen that there are some preferable arrangements in the systems of two impurities. The interaction between them depends mostly on their shapes and the geometry of the mutual arrangement. Our results show that shell effects associated with the appearance of inhomogeneities in the neutron matter at subnuclear densities may strongly influence the phase transition pattern in the neutron star crust since the shell effects for various "nuclear lattices" are of the same order as the differences of the total energy between phases [11]. We suspect that there are a lot of other effects, which might be relevant. We did not consider periodic orbits bouncing between three or more objects. An orbit bouncing between two bodies leads to a pairwise interaction. Orbits bouncing between three or more bodies would lead to many body interactions, which however turn out to be of lesser importance [12]. We have also considered only perfectly smooth objects. If one allows for some degree of corrugation of these surfaces, many more periodic orbits are likely to appear and that would lead to even more complicated interactions and more complicated interference patterns. If temperature has a simple to predict qualitative effect on the shell correction energy, the role of pairing effects is not that transparent and deserves further investigation.

We appreciate the financial support of DOE and of the Polish State Committee for Scientific Research (KBN) under Contract No. 2 P03B 040 14.

REFERENCES

[1] S.A. Chin, H.A. Forbert, *Phys. Lett.* **A272**, 402 (2000) and `cond-mat/9810269`; *Physica* **B284-288**, 13 (2000) and `cond-mat/9901028`.

[2] M. Kardar, R. Golestanian, *Rev. Mod. Phys.* **71**, 1233 (1999) and references therein.

[3] H.A. Wilson, *Phys. Rev.* **69**, 538 (1946); J.A. Wheeler, (unpublished); P.J. Siemens, H.A. Bethe, *Phys. Rev. Lett.* **18**, 704 (1967); C.Y. Wong, *Ann. Phys.* (N.Y.) **77**, 279 (1973).

[4] K. Pomorski, K. Dietrich, *Nucl. Phys.* **A627**, 175 (1997); K. Dietrich, K. Pomorski, *Phys. Rev. Lett.* **80**, 37 (1998); J. Dechargé *et al.*, *Phys. Lett.* **B451**, 275 (1999).

[5] M. Bender, private communication.

[6] K. Pomorski, K. Dietrich, *Eur. Phys. J.* **D4**, 353 (1998).

[7] C.J. Pethick, D.G. Ravenhall, *Ann. Rev. Nucl. Part. Sci.* **45**, 429 (1995) and references therein.

[8] S.M. Austin, G.F. Bertsch, *Scientific American*, **272**, 62 (1995).

[9] Y.Yu, A. Bulgac, P. Magierski, *Phys. Rev. Lett.* **84**, 412 (2000).

[10] A. Bulgac *et al.*, in *Proc. Intern. Workshop on Collective Excitations in Fermi and Bose Systems*, eds. C.A. Bertulani and M.S. Hussein, World Scientific, Singapore 1999, pp 44-61 and `nucl-th/9811028`; A. Bulgac, P. Magierski, `astro-ph/0007423`, `nucl-th/0009026`.

[11] A. Bulgac, P. Magierski, *Nucl. Phys.* **A** (in press).

[12] A. Bulgac, P. Magierski, A. Wirzba, (unpublished).