DIRAC EQUATION FOR THE NUCLEAR MEAN-FIELD WITH A WOODS–SAXON POTENTIAL*

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We present some properties of the solutions to the Dirac equation with Woods–Saxon potentials. The results obtained for spherical nuclei are compared to those of the Relativistic Mean Field Theory. A possibility of very significant improvements in the description of single-particle properties is illustrated, and the corresponding choice of the parametrization is presented.

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1. Introduction

Relativistic Mean Field (RMF) approach has rapidly become a rather well-established theory. It combines the advantages of a fully microscopic treatment of the nucleus together with those of a relativistic approach (see [1] for review). A wide variety of topics in nuclear structure has been studied by means of the RMF methods (see *e.g.* [2] and references therein).

However, it is often stressed that the RMF remains a phenomenological tool, due to the fact that some approximations, which are rather difficult to control in a sufficient detail, are made: indeed, some constants are adjusted to reproduce the global properties of the nuclei, and thus may differ from their experimental values known from the elementary particle physics, Fock terms are neglected, as well as the contribution of possible additional mesons. The theory in its present state is therefore not really completed.

On the other hand, it has been suggested already some time ago in Ref. [4] that the results of the RMF can be approximated while the formalism (and the numerical calculations) can be simplified when parametrizing the nuclear potentials with *e.g.* Woods–Saxon functions.

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In this short report, we are going to present some of the results obtained by such an approach, which turns out to be extremely promising for a correct description of the single-particle properties in nuclei.

2. Brief overview of the formalism

Let us start with the stationnary Dirac equation for the nucleons:

$$\left\{ c\,\vec{\alpha}\cdot\hat{\vec{p}} + V(\vec{r}) + \beta\left[\, m_0c^2 + S(\vec{r})\,\right]\right\}\psi_n = \varepsilon_n\psi_n\,, \tag{1}$$

where $\{\vec{\alpha},\beta\}$ are the usual Dirac 4×4 matrices, m_0 is the rest mass of the nucleon, ψ_n are the eigenfunctions and ε_n the eigen-energies. Potentials $V(\vec{r})$ and $S(\vec{r})$ are called, respectively, the vector and scalar potentials and, in the RMF theory they are calculated selfconsistently from their Klein–Gordon equations of propagation.

One then introduces the so-called effective-mass via:

$$m^*(\vec{r}) = m_0c^2 + \frac{1}{2}\left[S(\vec{r}) - V(\vec{r})\right]\,. \tag{2}$$

It is possible to show that around the Fermi level, the Dirac equation (1) may be expanded as a functional of $\varepsilon/2m^*(\vec{r})$ (ε being the single-particle energy), within an error of less than 1%. This leads to the Schrödinger-like equation for the "big component" of the Dirac bi-spinor:

$$\left\{\frac{1}{2m^*(\vec{r})}\hat{\vec{p}}^2 + V_{\rm cen}(\vec{r}) + V_p(\vec{r}) + V_{\rm SO}(\vec{r})\right\}\xi_n = \varepsilon_n\xi_n\,, \tag{3}$$

where $V_{\rm cen}(\vec{r})$ is the central- and $V_{\rm SO}(\vec{r})$ the spin-orbit potential:

$$V_{\rm SO}(\vec{r}) = \frac{2}{[2m^*(\vec{r})]^2}\left\{\vec{\nabla}(V-S)\wedge\vec{p}\right\}\cdot\vec{s}\,, \tag{4}$$

and $V_p(\vec{r})$ is a potential that is linear in terms of the $\hat{\vec{p}}$ operator [5].

Since all potentials depend only on $V-S$ or $V+S$, it is possible to parametrize these two expressions instead of V and S separately. The parameters of the corresponding Woods–Saxon functions must be fitted to the experimental data. We performed such a fit for eight doubly magic spherical nuclei, from ^{40}Ca to ^{208}Pb, and the results (for lead) are presented in Fig. 1.

In our fit, we also required that the r.m.s. radii of each nucleus are as close as possible to the experimental values. For most of the 8 doubly magic nuclei, the discrepancy between the theoretical calculation and the experimental value is less than 0.02 fm.

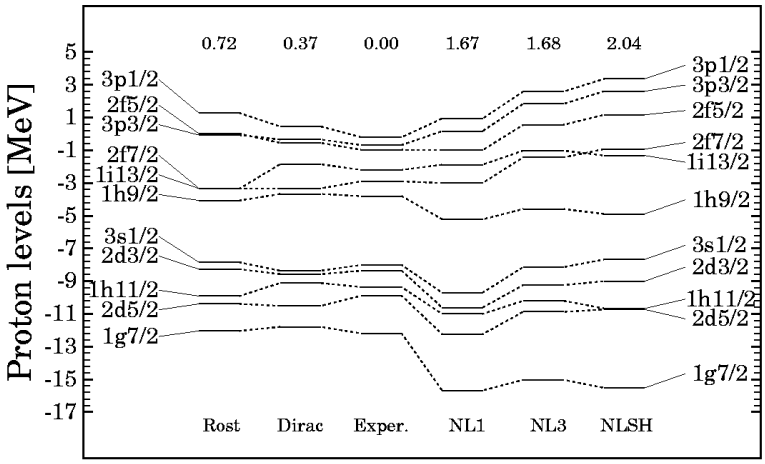

Fig. 1. Proton single-particle levels for $^{208}_{82}$Pb calculated by various methods. On the right-hand side of the experimental levels are the results of the RMF theory for 3 different sets of parameters. On the very left are the results obtained with the parametrization of Rost for the non-relativistic Woods–Saxon approach [3]. The column marked "Dirac" corresponds to our calculations. The numbers at the top of the figure are the root-mean-square deviations between theory and experiment.

3. Discussion of the parametrization

This model is based on a simple parametrization of the nuclear mean-field potentials. It has already been stressed in section 2 that all potentials depend only on $V - S$ or $V + S$, and thus that there are in principle only two sets of 6 parameters for each kind of nucleons. However, we allowed the effective-mass to have a different parametrization from that of the spin-orbit potential, which therefore leads to a total of 9 parameters for each type of nucleons. We show these parameters in Table I. We expect that these

TABLE I

Parameters of the potentials for the neutrons and the protons in the region of $^{208}_{82}$Pb

	$V_{\rm cen}$			$V_{\rm SO}$			$V_{\rm eff}$		
	V_0	$r_{\rm cen}$	$a_{\rm cen}$	$\lambda_{\rm SO}$	$r_{\rm SO}$	$a_{\rm SO}$	$\lambda_{\rm eff}$	$r_{\rm eff}$	$a_{\rm eff}$
Neutrons	64.5	1.12	1.14	17.3	1.10	0.63	13.6	0.76	0.36
Protons	74.4	1.19	1.13	16.2	1.14	0.53	13.2	0.80	0.34

extra degrees of freedom may improve the results by possibly correcting the effects of the assumptions made in the RMF theory which are not completely under control. A comparison between the "natural" (6 parameters) and the "extended" (9 parameters) parametrization is shown in Fig. 2.

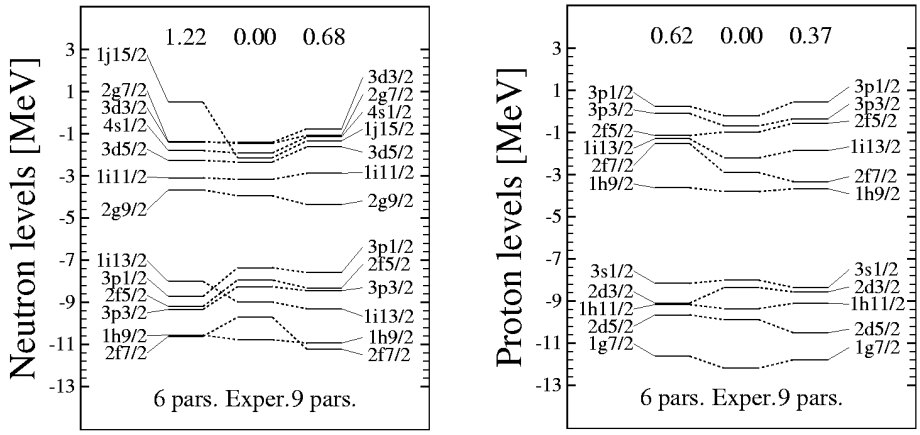

Fig. 2. Neutron (left) and proton (right) single-particle levels for $^{208}_{82}$Pb. The figure illustrates the difference between the results obtained with the "natural" parametrization, with 6 parameters, on the left-hand side of each figure, and those of the extended parametrization, with 9 parameters, on the right-hand side.

From the parameters obtained for each doubly-magic nucleus, it is possible to extract a systematic dependence on the isospin and on the nuclear mass. Such a relation allows to get the optimal set of parameters for any nucleus in the nuclear chart. We could therefore extend our calculations to deformed nuclei. So far, band-head spins and parities have been investigated, and some preliminary results have been obtained for the moments of inertia of nuclei in mass $A \sim 150$ (*cf.* Molique, these proceedings p. 1107).

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