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ROTATING DIRAC MEAN-FIELD AND PARTICLE-NUMBER PROJECTION CALCULATIONS FOR SUPERDEFORMED NUCLEI*

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In this contribution we first recall briefly the basic formulae related to the Dirac mean-field used in conjunction with the Bogolyubov formalism and particle-number projection. We present some results obtained for calculations of the moments of inertia for Super Deformed (SD) bands in the $A \sim 150$ mass-region, and compare them to the experimental ones. It is shown that this formalism is able to get rid of one of the drawbacks observed in standard Woods–Saxon or Hartree–Fock mean-field calculations, namely the systematic overestimation of the calculated moments of inertia.

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1. Introduction

The question of (near) non-existence of pairing in the superdeformed bands of the rare-earth nuclei has been often posed and very often it has been assumed that the pairing is negligible. Some authors present the arguments that instead a non-negligible influence of pairing in these nuclei should still be present. The results presented in this article shed some light on this problem in nuclear structure physics of superdeformed nuclei.

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2. Theoretical method

In this section we briefly outline the theoretical method used for the calculation of the SD bands investigated. First, the linearized version of the Dirac mean-field equation is solved in order to obtain the fermionic single-particle eigenstates. Then the residual monopole-pairing interaction is introduced and the Bogolyubov Cranking (HFBC) equations solved by taking into account explicit particle-number projection[1].

It can be shown that under realistic conditions valid for the nuclear cases investigated here, the standard Dirac equation for a nucleon

$$\left\{ c\vec{\alpha}\cdot\vec{p} + \hat{V}(\vec{r}) + \beta\left[m_0c^2 + \hat{S}(\vec{r})\right]\right\}\psi_n = E_n\psi_n \tag{1}$$

reduces to the following energy-linearized equation for the grand component ξ of the Dirac field

$$\left\{(c\vec{\sigma}\cdot\vec{p})\frac{1}{2m^\star(\vec{r})}(c\vec{\sigma}\cdot\vec{p}) + \hat{S}(\vec{r}) + \hat{V}(\vec{r})\right\}\xi_n = \epsilon_n\xi_n\,, \tag{2}$$

which is solved by diagonalization in a Cartesian harmonic oscillator basis. In the latter equation $m^\star$ stands for the reduced mass and is defined as $m^\star(\vec{r}) = m_0c^2 + \frac{1}{2}\left[\hat{S}(\vec{r}) - \hat{V}(\vec{r})\right]$, where $\hat{S}$ and $\hat{V}$ represent the scalar and vector potentials respectively, as described *e.g.* in Ref. [1]. In the linearized Dirac equation (2) the eigenvalues are counted with respect to m_0c^2, *i.e.* $E_n = \epsilon_n + m_0c^2$. The solutions to Eq. (2) serve as input to pairing calculations. For a review of the standard cranking Bogolyubov type formalism the reader is referred to *e.g.* Ref. [2]. In the calculations, particle-number projection is introduced explicitly by making use of the procedure described in Ref. [3]. We recall that the particle-number projection operator can be defined as

$$\hat{Q}_N = \frac{1}{2\pi}\int_0^{2\pi} d\phi\; e^{i\phi(N-\hat{N})}\,. \tag{3}$$

The standard monopole-pairing Hamiltonian is defined as

$$\hat{H} = \sum_i \varepsilon_i c_i^\dagger c_i - G\sum_{i>j>} c_i^\dagger c_{\tilde{i}}^\dagger c_{\tilde{j}} c_j\,, \tag{4}$$

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[1] Strictly speaking, we do not solve the full Hartree–Fock–Bogolyubov problem, yet we use the symbol HFBC (Hartree–Fock–Bogolyubov) similarly to many other authors, *i.e.* we solve selfconsistently the pairing channel in the Hamiltonian.

where the indices i represent the eigenstates of the one-body part of the Hamiltonian, and $\tilde{i}$ their time-reversed conjugates. If $|\mathrm{HFB}\rangle$ represents the HFB solution, the corresponding number-projected state can be obtained as

$$|N\rangle = \frac{1}{\sqrt{N_0}}\,\hat{Q}_N\,|\mathrm{HFB}\rangle\,, \tag{5}$$

and one can show (see Ref. [3]) that the expectation value of the monopole pairing Hamiltonian in this state is given by the expression

$$\langle N|\hat{H}|N\rangle = 2\sum_{i>} \varepsilon_i v_i^2 \frac{N_i}{N_0} - G\left(\sum_{i>j>} u_i v_i u_j v_j \frac{N_{ij}}{N_0} - \sum_{i>} v_i^4 \frac{N_{ii}}{N_0}\right). \tag{6}$$

In the last expression the coefficients u_i and v_i are those describing the standard BCS-type wave-function. The various coefficients N_i and N_{ij} are given in Ref. [3]. We use a slightly generalized version of this formalism so that the cranking term can be treated as well.

3. Results and conclusions

In this section we would like to illustrate the method by studying the case of the SD ground-state band of the nucleus ^{152}Dy. In Figs. 1 and 2 are plotted the calculated dynamical and kinematical moments of inertia in the case of no pairing and compared to the cases when the pairing correlations are taken into account. Fig. 1 corresponds to the results obtained with the Woods–Saxon mean-field with the universal parameters, whereas Fig. 2 displays the results corresponding to the Dirac mean-field. In the figures, the different values of GF stand for different pairing strength factors that are common for neutrons and protons, and that multiply the neutron (G_n) and proton (G_p) strength parameters defined in Ref. [4] as:

$$\begin{cases} G_n = [18.95 - 0.078(N-Z)]/A \\ G_p = [17.90 + 0.176(N-Z)]/A\,. \end{cases} \tag{7}$$

This is done in order to study the influence of the pairing correlations on the calculated moments.

As we can see from the figures, the results obtained with the Dirac mean-field seem to reproduce markedly better the observed moments, both dynamical and kinemetical. In both mean-field calculations one notices that the effect of pairing on the kinematical moments is to decrease the corresponding values as compared with the no-pairing case, and this effect seems to be in accordance with the generally observed behavior. On the contrary, one can see that the effect of pairing on the dynamical moments of inertia

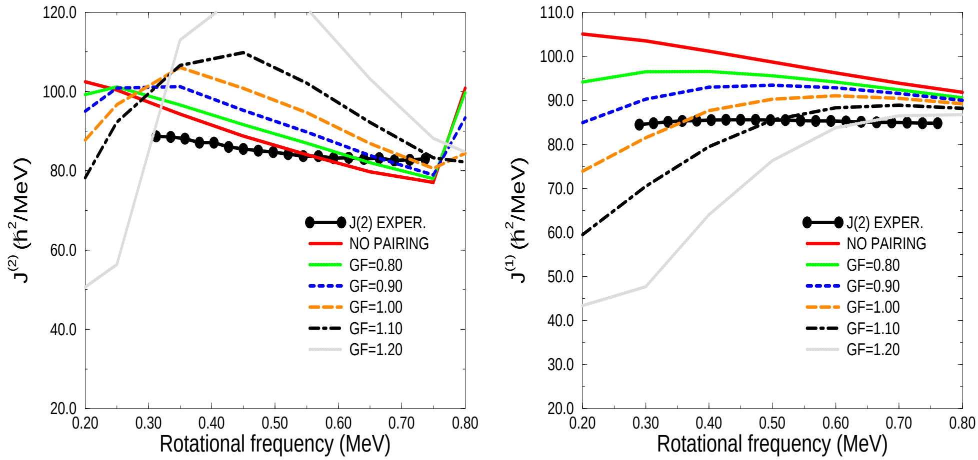

Fig. 1. Calculated dynamical (left) and kinematical (right) moments of inertia obtained with the Woods–Saxon mean-field with the so-called "universal parameters", for the superdeformed ground state band in the nucleus ^{152}Dy. The dots correspond to the experimental values.

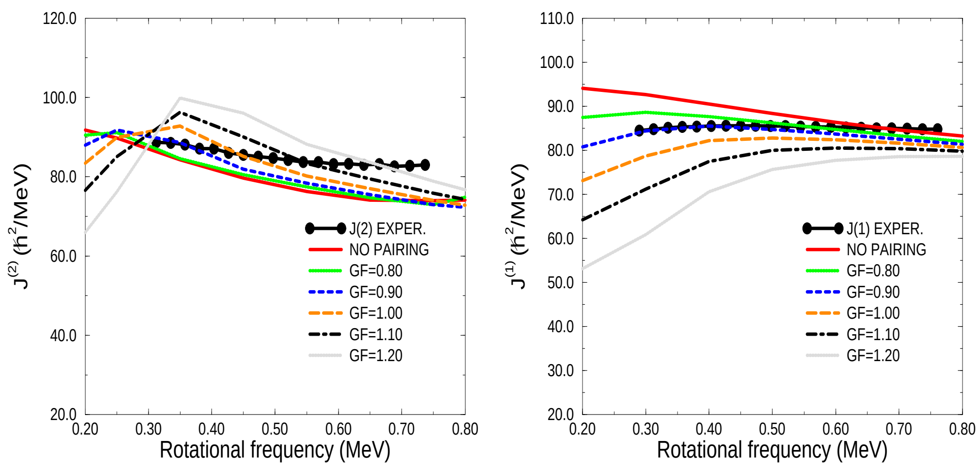

Fig. 2. The same as in Fig. 1, but for the Dirac mean-field.

is just the opposite: an increase of the calculated values is observed when the pairing is used. It is quite obvious from the figures that the results obtained with the Dirac mean-field are much more in accordance with the experimental ones. The too large moments of inertia calculated with the standard Woods–Saxon mean-field also appear if Hartree–Fock calculations with Skyrme interactions are performed, as it is shown in detail in a recent article by Aouad *et al.* (*cf.* Ref. [5]). In this article it is suggested that one can compensate somehow for this effect by introducing a scaling factor to adjust the calculated results in order to get a better agreement with the experimental data. However, this prescription seems to be rather artificial and therefore not very satisfactory. If the Dirac mean-field is used instead, such a scaling procedure does not seem to be necessary.

The observation that the slopes of the dynamical moments are not well reproduced in either case may originate from the fact that as the cranking frequency increases the deformation of the mean-field is kept constant. The details are under investigation and the corresponding results will be published elsewhere.

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