SHELL CORRECTIONS OF SPHERICAL NUCLEI CALCULATED BY HARTREE–FOCK PROCEDURE WITH THE GOGNY FORCE* **

M. Kleban†, B. Nerlo-Pomorska, K. Pomorski

Institut of Physics, The Maria Curie-Skłodowska University
pl. M. Curie-Skłodowskiej 1, 20-031 Lublin, Poland

J.F. Berger and J. Dechargé

Commisariat a L'Energie Atomique, Service de Physique Nucléaire
B.P. 12, 91680 Bruyères-le-Châtel, France

(Received November 2, 2000)

The selfconsistent Hartree–Fock calculation with the Gogny effective interaction D1S was performed for the spherical nuclei Ca, Sr, Sn, Sm. The shell effects were extracted by Strutinsky procedure and then the macroscopic energy evaluated by the liquid drop formula. Its parameters and isospin dependent formulae for radii were found.

PACS numbers: 21.24.Dr, 21.30.Fe, 21.60.Jz

1. Introduction

The selfconsistent Hartree–Fock–Bogolubov (HFB) [1] method with the two body effective nucleon–nucleon forces is up to now the best tool to investigate the nuclear properties. The masses, separation energies, radii and fission lifetimes should be reproduced in such a calculation when phenomenological parameters are well established. The D1S [2] Gogny force is very effective in describing of the spherical and deformed nuclei in the ground states.

As the results of HFB+Gogny method reproduce the properties of nuclei it is interesting to investigate how they correspond to the simpler models like formulas of liquid drop energy or nuclear radii. It was our aim to obtain their

* Presented at the XXXV Zakopane School of Physics "Trends in Nuclear Physics", Zakopane, Poland, September 5–13, 2000.

** The work was partially sponsored by the Polish State Committee for Scientific Research (KBN) No. 2P 03B 115 19.

† e-mail: m.kleban@tytan.umcs.lublin.pl

parameters from HFB+Gogny results and compare them with the parameters evaluated within the relativistic mean field theory [2] and experimental data [3,4].

2. Theory

We have chosen the representative spherical nuclei Ca–Sm to get the systematics of their potential energies E and radii. We have performed the Hartree–Fock (HF) calculation without pairing with the Gogny D1S [2] force. Then, we have extracted the shell effects from the HF energies by Strutinsky [5] averaging method. We have taken the selfconsistent single particle levels e_ν and smoothed them ($\tilde{e}_\nu$) by Gaussian function looking carefully for the proper Gaussian width γ giving the plateau for Strutinski shell corrections

$$E_{\rm shell} = \sum_\nu e_\nu - \sum_\nu \tilde{e}_\nu \,. \tag{1}$$

Then we extracted the shell corrections from the total energies getting the pure macroscopic energy

$$E_{\rm macr} = E_{\rm HF} - E_{\rm shell} \,. \tag{2}$$

This energy served us to get the "Gogny" parameters of the liquid drop model of an nucleus with Z protons, N neutrons and mass number A

$$E_{\rm macr} = a_{\rm vol} A - a_{\rm surf} A^{2/3} - a_{\rm sym} \frac{(N-Z)^2}{2A} - a_{\rm coul} A^{-1/3}\ Z^2 \tag{3}$$

as $a_{\rm vol} = 15.91 {\rm MeV}, a_{\rm surf} = 18.04 {\rm MeV}, a_{\rm sym} = 49.82 {\rm MeV}, a_{\rm coul} = 0.73 {\rm MeV}$.

In Fig. 1 the shell corrections $E_{\rm shell}$ (1) of Ca, Sr, Sn, Sm isotopes in dependence on neutron number N are drawn. The dashed lines denote neutron and proton shell corrections, solid lines — total shell correction. The removing macroscopic energy (2) is fitted by formula (3).

We have also gathered the root mean square radii (rmsr) of proton, neutron, charge and mass distributions of the spherical nuclei in order to approximate them by the isospin formula like for relativistic mean field theory in [3].

The new formulae for radius constants r_0, where

$$\sqrt{\langle r^2 \rangle} = \sqrt{\frac{3}{5}} r_0\ A^{1/3} \tag{4}$$

are for neutrons

$$r_0^n = 1.17 \left(1 + 0.12 \frac{N-Z}{A} + \frac{3.46}{A} \right) {\rm fm} \,, \tag{5}$$

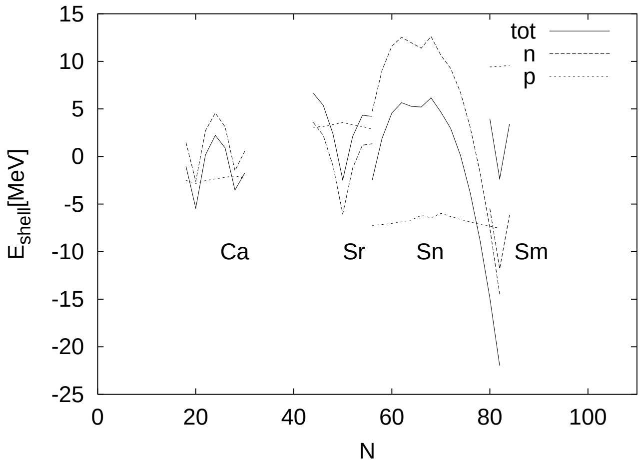

Fig. 1. Gogny shell corrections of protons, neutrons (dashed lines) and total (solid lines) for Ca, Sr, Sn and Sm isotopes in dependence on N.

for protons

$$r_0^p = 1.21 \left(1 - 0.19\frac{N-Z}{A} + \frac{2.15}{A}\right)\mathrm{fm}\,, \tag{6}$$

for charge distribution

$$r_0^{\mathrm{ch}} = 1.23 \left(1 - 0.21\frac{N-Z}{A} + \frac{2.40}{A}\right)\mathrm{fm}\,, \tag{7}$$

for total radius constant

$$r_0^{\mathrm{tot}} = 1.19 \left(1 + 0.01\frac{N-Z}{A} + \frac{2.91}{A}\right)\mathrm{fm}\,. \tag{8}$$

We have also fitted the ratio of neutron to proton radius to get rid of the possible deformation influence on the radii

$$\sqrt{\frac{\langle r^2\rangle^p}{\langle r^2\rangle^n}} = \frac{r_0^p}{r_0^n} = 1.03 \left(1 - 0.29\frac{N-Z}{A} - \frac{1.06}{A}\right)\mathrm{fm}\,. \tag{9}$$

This formula allows to estimate the neutron distribution radii from the charge ones

$$r_n = \frac{(\langle r^2\rangle^{\mathrm{ch}} - 0.64)^{1/2}}{1.03\left(1 - 0.29\frac{N-Z}{A} - \frac{1.06}{A}\right)}\ \mathrm{fm}\,, \tag{10}$$

similarly to the relativistic mean field theory (RMFT) estimation [2].

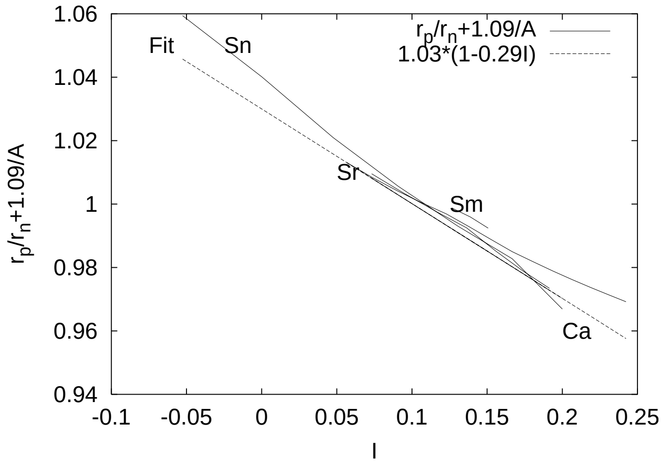

Fig. 2. Isospin dependence of the proton to neutron Gogny radii ratios for Ca, Sr, Sn and Sm isotopes (solid lines) fitted by formula 9 (dashed line).

In Fig. 2 the isospin dependence of protons to neutrons rmsr ratio shifted up by $1.09/A$ in comparison with the formula 9 (dashed line) is shown.

3. Conclusions

The following conclusions can be drawn from our calculations.

1. The Gogny D1S force gives the proper macroscopic energy in agreement with the liquid drop formula.
2. The isospin dependence of proton and neutron distributions radii constants is linear for heavier nuclei, the lighter ones demand κ/A factor.
3. The ratio of theoretical r_p/r_n radii can be used to foresee the neutron radii of all the nuclei, which have experimentally known charge rmsr.

REFERENCES

[1] J. Dechargé, D. Gogny, *Phys. Rev.* **C21**, 1568 (1980).
[2] J. F. Berger, M. Girod, D. Gogny, *Nucl. Phys.* **A438**, 236 (1984).
[3] M. Warda, B. Nerlo–Pomorska, K. Pomorski, *Nucl. Phys.* **A635**, 484 (1998).
[4] H. de Vries, C. W. de Jager, C. de Vries, *At. Data Nucl. Data Tables* **36**, 495 (1987).
[5] V. M. Strutinsky, *Yad. Fiz.* **36**, 614 (1966).