THE EFFECTIVE CHIRAL MEAN–FIELD THEORY FOR SUPERHEAVY NUCLEI* **

Andrzej Staszczak

Institute of Physics, M. Curie-Skłodowska University
pl. M. Curie-Skłodowskiej 1, 20-031 Lublin, Poland

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The effective chiral mean-field model of Furnstahl, Serot and Tang (FST) is proposed to examine stability of superheavy nuclei. The FST model is an example of a nuclear effective field theory where the hadronic Lagrangian is constructed according to the symmetries of quantum chromodynamics and the "naturalness" condition.

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1. Introduction

The search for SuperHeavy Elements (SHE's) for more than 30 years has been an area of active experimental and theoretical investigation. The latest experimental successes in the synthesis of SHE's (for review see, *e.g.* [1]) cause this part of nuclear physics alluring.

Most of the theoretical calculations on ground-state and decay properties of SHE's have been performed with Microscopic–Macroscopic (MM) models, where a global tendency of the nuclear energy is obtained from the different macroscopic models and the local (quantum) fluctuations by use of a Strutinsky's method [2].

From the MM models it is expected that a spherical doubly-magic nucleus next to ^{208}Pb should exist for $Z = 114$ and $N = 184$. The enhancement in nuclear stability is also expected near the deformed shells at $Z = 108$ and $N = 162$, what is consistent with recent experimental results.

Masses of SHE's have been also calculated in the Fermion Dynamical Symmetry Model (FDSM) [3], where SU(2) and SU(3) dynamical symmetries connected with pairing and quadrupole interactions, relatively are taken

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into account. In the FDSM model the spherical double-magic nucleus is shifted downward in neutron number in comparison with the MM prediction and found at $Z = 114$ and $N = 164$.

At the more fundamental level, the ground-state properties of SHE's have been investigated in self-consistent models starting with an underlying nucleon–nucleon interaction. In [4] the nonrelativistic Skyrme–Hartree–Fock (SHF) model [5] with different types of effective interactions has been used and compared with the MM results. The Hartree–Fock–Bogoliubov (HFB) approach with the Gogny force has been used to study the ground-state properties of about one hundred nuclei with $104 \leq Z \leq 128$ in [6].

The Relativistic Hartree–Bogoliubov (RHB) model in the spherical limit [7] and the axially deformed Relativistic Mean–Field (RMF) model [8] have been employed for SHE's in [9]. The RMF model has been also preformed for SHE's in [10–12] and more recently in [13, 14], where the shell structure of SHE's within the SHF and RMF models has been compared for different parametrization. Authors have concluded that all used Skyrme models predict the strongest shell effect at $Z = 124, 126$ and $N = 184$. On the other hand, in RMF approaches the strongest shell stabilization appears for $Z = 124$ and $N = 172$.

The uncertainty in predictions of the stability for SHE's encourages to employ new more fundamental theoretical nuclear models. Quantum chromodynamics (QCD) is nowadays the established theory of the strong interactions. On the other hand, at low-energy where non-perturbative effects of QCD dominate, the relevant degrees of freedom are not quarks and gluons but hadrons. These two facts one has to take into account in constructing the modern nuclear theory.

It is generally believed that an important clue toward understanding nuclear phenomena in the context of QCD is the chiral symmetry and the concept of the effective field theory (EFT) see, *e.g.* [15]. Almost perfect chiral $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$ symmetry of QCD is associated with the fact that the up and down quarks are very light. This symmetry is spontaneously broken to its vectorial subgroup $\mathrm{SU}(2)_{\mathrm{V}}$ with the appearance of Goldstone bosons, which are the pseudoscalar mesons (pions).

The EFT technique is based on a famous "theorem" by Weinberg [16]: *When we calculate a physical amplitude from Feynman diagrams using the most general Lagrangian that involves the relevant degrees of freedom and satisfies the assumed symmetries of the underlying high-energy theory, we are simply constructing the most general amplitude that is consistent with general principles of relativity, quantum mechanics, and the assumed symmetries.*

2. Effective chiral Lagrangian for nuclei

One of the latest attempts at formulating EFT for finite nuclei and nuclear mater is the generalization of Walecka quantum hadrodynamics (QHD) [17, 18] by Furnstahl, Serot and Tang (FST) [19–21]. The effective chiral Lagrangian of FST is expanded in powers of fields and their derivatives, with terms organized by applying Georgi's "naive dimensional analysis" [22–24] and "naturalness" condition.

The relevant degrees of freedom are nucleons $N(x) = \begin{pmatrix} p(x) \\ n(x) \end{pmatrix}$, pions $\pi(x) \equiv \pi(x)\,\frac{1}{2}\tau$ (with τ^a being Pauli matrices), and the low-lying non-Goldstone bosons: isovector–vector ρ meson $\rho_\mu(x) \equiv \rho_\mu(x)\,\frac{1}{2}\tau$, isoscalar–vector meson ω represented by a vector field $V_\mu(x)$, and an effective isoscalar–scalar field $\phi(x)$ to simulate two-pion exchange (σ meson). Chiral symmetry $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$ is realized nonlineary [25, 26] and $U(x) \equiv \xi^2(x) = \exp\left(2i\pi(x)/f_\pi\right) \in \mathrm{SU(2)}$ is presumed to describe the pion field, where $f_\pi \approx 93$MeV is the pion-decay constant. We can also define an axial vector field $a_\mu \equiv -\frac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger) = a_\mu^\dagger$, a polar vector field $v_\mu \equiv -\frac{i}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger) = v_\mu^\dagger$, and the covariant tensors $v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu + i[v_\mu, v_\nu] = -i[a_\mu, a_\nu]$, $\rho_{\mu\nu} = D_\mu\rho_\nu - D_\nu\rho_\mu + ig[\rho_\mu, \rho_\nu]$, where $D_\mu\rho_\nu = \partial_\mu\rho_\nu + i[v_\mu, \rho_\nu]$ is the chirally covariant derivative of the rho field.

The effective Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_\mathrm{N} + \mathcal{L}_\mathrm{M} + \mathcal{L}_\mathrm{EM}\,, \tag{2.1}$$

where each term is truncated by considering the various values of $\nu = d + \frac{n}{2} + b$, where d is the number of derivatives, n the number of nucleon fields and b the number of non-Goldstone boson fields in the interaction term. Through $\nu = 4$, the part of the effective Lagrangian involving nucleons may be written as

$$\begin{aligned} \mathcal{L}_\mathrm{N}(x) \;=\; & \overline{N}\Big(i\gamma^\mu(\partial_\mu + iv_\mu + ig_\rho\rho_\mu + ig_\mathrm{v}V_\mu) + g_\mathrm{A}\gamma^\mu\gamma_5 a_\mu - M + g_\mathrm{s}\phi\Big)N \\ & -\frac{f_\rho g_\rho}{4M}\overline{N}\rho_{\mu\nu}\sigma^{\mu\nu}N - \frac{f_\mathrm{v} g_\mathrm{v}}{4M}\overline{N}V_{\mu\nu}\sigma^{\mu\nu}N - \frac{\kappa_\pi}{M}\overline{N}v_{\mu\nu}\sigma^{\mu\nu}N\,, \end{aligned} \tag{2.2}$$

where $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$, $V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$, $g_\mathrm{A} \approx 1.26$ is the axial coupling constant, $M = 939$MeV is the nucleon mass, $g_i, f_i, (i = \rho, \mathrm{v})$ are vector and so-called tensor couplings for ρ and ω mesons, g_s is a Yukawa coupling for the effective scalar field ϕ, and $\kappa_\pi = \frac{f_\rho}{4}$ is the coupling for higher-order πN interaction. The mesonic part of the Lagrangian up to order $\nu = 4$ is

$$\begin{aligned}\mathcal{L}_{\mathrm{M}}(x) \;=\; & \frac{1}{2}\left(1+\alpha_1\frac{g_{\mathrm{s}}\phi}{M}\right)\partial_\mu\phi\partial^\mu\phi+\frac{f_\pi^2}{4}\,\mathrm{tr}\,(\partial_\mu U\partial^\mu U^\dagger) \\ & +\frac{f_\pi^2}{4}\,m_\pi^2\mathrm{tr}\,(U+U^\dagger-2) \\ & -\frac{1}{2}\mathrm{tr}\,(\rho_{\mu\nu}\rho^{\mu\nu})-\frac{1}{4}\left(1+\alpha_2\frac{g_{\mathrm{s}}\phi}{M}\right)V_{\mu\nu}V^{\mu\nu}-g_{\rho\pi\pi}\frac{2f_\pi^2}{m_\rho^2}\mathrm{tr}\,(\rho_{\mu\nu}v^{\mu\nu}) \\ & +\frac{1}{2}\left(1+\eta_1\frac{g_{\mathrm{s}}\phi}{M}+\frac{\eta_2}{2}\frac{g_{\mathrm{s}}^2\phi^2}{M^2}\right)m_{\mathrm{v}}^2V_\mu V^\mu+\frac{1}{4!}\zeta_0 g_{\mathrm{v}}^2(V_\mu V^\mu)^2 \\ & +\left(1+\eta_\rho\frac{g_{\mathrm{s}}\phi}{M}\right)m_\rho^2\mathrm{tr}\,(\rho_\mu\rho^\mu)-m_{\mathrm{s}}^2\phi^2\left(\frac{1}{2}+\frac{\kappa_3}{3!}\,\frac{g_{\mathrm{s}}\phi}{M}+\frac{\kappa_4}{4!}\,\frac{g_{\mathrm{s}}^2\phi^2}{M^2}\right),\end{aligned} \tag{2.3}$$

where $m_{\mathrm{v}} = 782\,\mathrm{MeV}, m_\rho = 770\,\mathrm{MeV}, m_{\mathrm{s}}$ are ω, ρ and σ mesons masses, $g_{\rho\pi\pi}$ is the $\rho\pi\pi$ coupling. The electromagnetic interactions are described by

$$\begin{aligned}\mathcal{L}_{\mathrm{EM}}(x) \;=\; & -\frac{1}{4}F^{\mu\upsilon}F_{\mu\nu}-e\overline{N}\gamma^\mu\frac{1}{2}(1+\tau_3)NA_\mu \\ & -\frac{e}{4M}F_{\mu\nu}\overline{N}\lambda\sigma^{\mu\nu}N-\frac{e}{2M^2}\overline{N}\gamma_\mu(\beta_{\mathrm{s}}+\beta_{\mathrm{v}}\tau_3)N\partial_\nu F^{\mu\nu} \\ & -2ef_\pi^2A^\mu\mathrm{tr}\,(v_\mu\tau_3)-\frac{e}{2g_\gamma}F_{\mu\nu}\Big[\,\mathrm{tr}\,(\tau_3\rho^{\mu\nu})+\frac{1}{3}\,V^{\mu\nu}\Big],\end{aligned} \tag{2.4}$$

where A_μ is the electromagnetic field, $F_{\mu\nu}$ is the electromagnetic field tensor, $g_\gamma = 5.01$, and $\lambda \equiv \frac{1}{2}\lambda_{\mathrm{p}}(1+\tau_3)+\frac{1}{2}\lambda_{\mathrm{n}}(1-\tau_3)$, with $\lambda_{\mathrm{p}} = 1.793$ and $\lambda_{\mathrm{n}} = -1.913$ the anomalous magnetic moments of the proton and the neutron, respectively.

The effective chiral Lagrangian Eq. (2.1) at a given order contains certain parameters that are not constrained by the symmetries, the so-called low-energy constants (LECs). Apart from β_{s}, β_{v}, and f_ρ, which are fixed from the free-space charge radii of the nucleon, the remaining thirteen LECs g_{s}, g_{v}, g_ρ, η_1, η_2, η_ρ, κ_3, κ_4, ζ_0, m_{s}, f_{v}, α_1, and α_2 have to be determined from experimental data. The LECs are defined applying the “naive dimensional analysis” so that they are assumed to be of order unity (“natural”).

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