INDUCED PAIRING INTERACTION IN NUCLEI*

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The induced pairing interaction arising from the exchange of low-lying collective surface vibrations among nucleons moving in time reversal state close to the Fermi energy is found to lead to values of the pairing gap which constitute a large fraction of those experimentally observed.

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1. Introduction

In previous work [1] it has been shown that Cooper pair formation in nuclei can benefit from the induced interaction arising from the exchange of low-lying collective surface vibrations between the members of the pair. Detailed calculations carried out for a number of isotopic chains ($^{A}_{20}$Ca,$^{A}_{22}$Ti,$^{A}_{50}$Sn) confirms that this mechanism gives rise to pairing gaps which account for $\approx$50% of the experimental values. In my talk and after having briefly recalled the formalism used to calculate the induced interaction, I shall discuss in some detail the case of the Cadmium isotopes.

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2. The induced interaction matrix element

To calculate the pairing effects arising from the induced interaction, we solved the BCS gap equation

$$\Delta_\nu = -\sum_{\nu'>0} \frac{2j_{\nu'}+1}{2} \frac{\Delta_{\nu'}}{2E_{\nu'}} v_{\nu\nu'} \,, \tag{1}$$

coupled to the BCS number equation

$$N = 2\sum_{\nu>0} V_\nu^2 \,. \tag{2}$$

The solution of these equations provides both the Fermi energy and the state dependent pairing gap Δ_ν associated with the single-particle levels of quantum number ν. The BCS equations are well defined once the single-particle energies ϵ_ν, the number of nucleons N, and the matrix elements of the induced interaction are specified.

The matrix elements of the induced interaction are given by (*cf.* inset Fig. 1, and also Ref. [1]) by the relation

$$v_{\nu\nu'} = \sum_{J,n} \frac{2}{2j_{\nu'}+1} \frac{2(M^n_{\nu,J\nu'})^2}{E_o - E_{\rm int}} \,. \tag{3}$$

The quantity $M^n_{\nu,J\nu'}$ is the particle-vibration matrix element coupling the single-particle states ν,ν' to the n-th vibrational state of multipolarity J and energy $\hbar\omega_J(n)$. In the calculations, phonons with multipolarities $2 \le J \le 5$ and energy $\hbar\omega_J \le 30$ MeV have been used. The quantity E_0 is the pairing energy, while $E_{\rm int} = \mathrm{e}_\nu + \mathrm{e}_{\nu'} + \hbar\omega_J(n)$ is the energy of the intermediate state of the induced interaction graph (*cf.* inset in Fig. 1), while e_ν is the single-particle energy measured with respect the Fermi energy.

3. The Cadmium isotopic chain

We shall focus our attention on the Cadmium isotopes, starting with ^{110}Cd. In Table I, I report the energy levels lying near the Fermi energy obtained solving the Schrödinger equation for an average Saxon–Woods field containing a spin–orbit term. The values of the depth, radius and diffusivity of the field were taken from Ref. [2]. The induced interaction matrix elements (in MeV) between the levels of Table I are shown in Fig. 1.

The pairing, matrix elements used to describe the nuclear superfluid state has been parametrized, as a rule, in terms of a constant G. Typical values of $25/A$ MeV are used to reproduce the values of the odd-even mass difference. In the case of Cd, $G \approx 0.22$ MeV. This value can be compared with

TABLE I

Single particle levels in ^{110}Cd around Fermi energy ($\varepsilon_{\rm F} = -9.03$ MeV).

nlj	$1\ g_{9/2}$	$2\ d_{5/2}$	$1\ g_{7/2}$	$3\ s_{1/2}$	$2\ d_{3/2}$	$1\ h_{11/2}$
ε_k (MeV)	-15.6	-10.38	-9.43	-8.34	-7.71	-6.99

the average value of the induced pairing matrix elements $v_{\nu\nu'} \approx 0.16$ MeV (*cf.* Fig. 1). From this figure it is seen that these matrix elements display large fluctuations, fluctuations which are rather simple to understand in term of the properties of the particle-vibrations matrix elements $M^n_{\nu,J\nu'}$. For instance, the fact that the $g_{9/2}$ level has larger matrix elements with spin up states than with spin down states can be understood remembering that low-lying surface collective vibrations are isoscalar modes. We can recognize this spin-flip effect also in the case of the $d_{5/2}$ level. The $g_{9/2}$ and the $h_{11/2}$ have globally smaller matrix elements because they lie far away from the Fermi energy and so they have larger energy denominators. Moreover, several couplings are forbidden due to angular momentum conservation. For example $s_{1/2}$ and $g_{9/2}$ can exchange only 4^+ phonos while $s_{1/2}$ and $h_{11/2}$ only 5^- phonons.

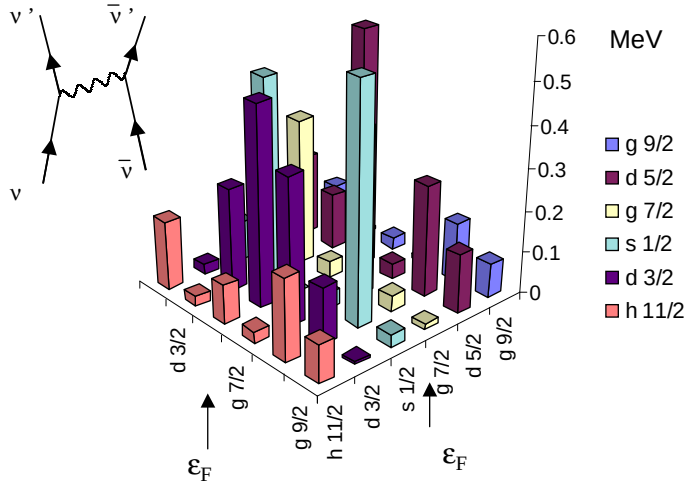

Fig. 1. Induced pairing interaction matrix elements in MeV between states near the Fermi energy in ^{110}Cd. In the inset is schematically shown a typical scattering process between pairs of nucleons arising from the induced pairing interaction.

The state dependent pairing gap calculated making use of the induced interaction matrix elements are shown in Fig. 2. From this figure it is clear that induced interaction plays an important role in defining the properties of single particle levels lying near the Fermi energy.

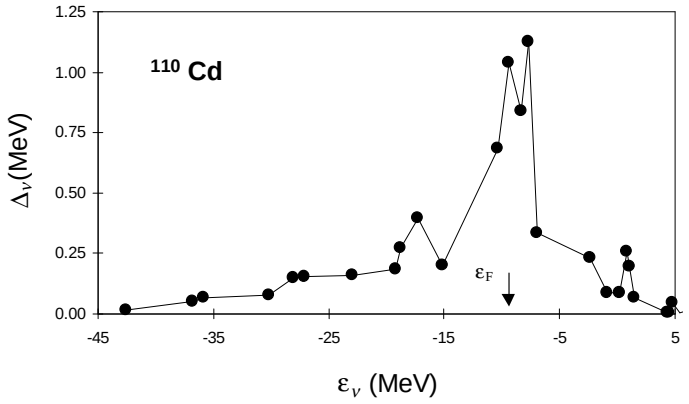

Fig. 2. Graphical representation of the solution of the BCS gap equation: on the abscissas the single-particle energies are displayed while on the ordinates the state dependent pairing gap Δ_ν is displayed.

We can compare the average value of Δ_ν of levels lying around the Fermi energy with the odd–even mass difference (OEMD). The former is 0.93 MeV, the latter 1.27 MeV. The induced pairing interaction in the case of ^{110}Cd thus accounts for ≈70% of the experimental pairing gap. As can be seen from Table II, similar results are obtained in the case of other Cd-isotopes.

TABLE II

Average value of Δ_ν around Fermi energy (Theor.) for five isotopes of the Cadmium isotopic chain compared with the OEMD (Exp.).

	^{106}Cd	^{108}Cd	^{110}Cd	^{112}Cd	^{114}Cd
Exp. (MeV)	1.21	1.20	1.27	1.21	1.25
Theor. (MeV)	0.91	0.85	0.93	0.90	1.03

4. Conclusion

From the results presented in this talk, together with those reported in reference [3] where a successful description of ^{11}Li was given in term of pairing induced correlations, one can conclude that a consistent description of the structure of nuclei close to the ground state requires the explicit treatment of the coupling of nucleons to vibrations and of the associated self-energy and core-polarization process.

REFERENCES

[1] F. Barranco, R.A. Broglia, G. Gori, E. Vigezzi, P.F. Bortignon, J. Terasaki, *Phys. Rev. Lett.* **83**, 2147 (1999).

[2] A. Bohr, B.R. Mottelson, *Nuclear Structure*, Vol. I, Benjamin, Reading, Massachussets 1969, p. 239.

[3] F. Barranco, R.A. Broglia, G. Coló, E. Vigezzi, `nucl-th/0005073`, (to be published).