Vol. 32 (2001) ACTA PHYSICA POLONICA B No 3

SHELL MODEL INTERACTION AROUND ^{208}Pb*

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(Received January 25, 2001)

The shell model residual interaction around ^{208}Pb is studied. Matrix elements of a realistic interaction, calculated from the interaction between free nucleons, are compared with experiment. The calculated interaction has been improved by adjustments to experimental data, to better describe nuclei around ^{208}Pb. Some systematic trends of the differences between calculated and empirical interaction have been found. Specifically, the interaction between particles and holes in ^{208}Pb is treated and that between two proton holes, based on new data for ^{206}Hg.

PACS numbers: 21.60.Cs, 27.80.+w

1. Introduction

Much progress has been recently achieved in understanding nuclear structure by means of the shell model. Many new experimental data have been measured around ^{132}Sn. Some properties of the nuclei around ^{100}Sn have been found, and also ^{78}Ni might soon be within reach of experimental exploration [1]. Many states around ^{132}Sn have been successfully calculated with interactions taken from the ^{208}Pb region [2]. These nuclei are doubly magic and therefore particularly interesting for shell model studies. The main aim is, for these cases of nuclei far from stability, to explore if there are some marked differences in their structure from that of nuclei close to stability. Nuclei around ^{208}Pb can serve as a reference in the valley of stability.

Also advances in computational techniques and computing power allow now shell model calculations for nuclei with many active particles, and the

* Presented at the XXXV Zakopane School of Physics "Trends in Nuclear Physics", Zakopane, Poland, September 5–13, 2000.

number of configurations, that can be included in the calculations, has been substantially increased. Therefore many more nuclei and nuclear properties have now become accessible to detailed shell model calculations.

Of course knowledge of the interaction is a prerequisite for any shell model calculation. It can and has been best studied around ^{208}Pb. Any matrix element of the residual interaction, that can be newly determined from experiment, might be used directly to explain nuclear properties, particularly if configuration mixing is negligible and the structure of the states is relatively simple. However the residual interaction between the particles around ^{208}Pb comprises some ten thousand matrix elements for the Kuo–Herling space [3], while a few hundred can be directly measured. Therefore the residual interaction has to be taken from theory, but then checked by comparing with the experimentally known matrix elements, with the goal of a better interaction. Perhaps one can then even improve the calculations of the realistic interaction for the whole region of the nuclear chart.

This comparison of the interaction around ^{208}Pb between experiment and theory is the main theme of this contribution. For the theoretical part, realistic interactions are taken, following the original work of Kuo and Brown [4]. They have developped the method, to calculate the interaction between nucleons inside a nucleus, the shell model residual interaction, from the measured interaction between free nucleons. Kuo and Herling [3] calculated then the interaction between particles, the orbitals above the shell closure in ^{208}Pb, and that between holes, the orbitals below. This Kuo–Herling interaction is still the basis for most shell model studies around ^{208}Pb. But now such calculations of realistic interactions can and have been done without previously necessary computational simplifications [5]. Warburton and Brown [6] examined and adjusted the Kuo–Herling interaction for particles. Rydstroem *et al.*, [7] did the same for the interaction between two proton holes and that between a neutron hole and a proton hole. McGrory and Kuo [8] improved the interaction between two neutron holes. So far no calculations had been done for the interaction between particles and holes. Now Brown and Rejmund [9] calculated these two body matrix elements from the H7B free nucleon potential [10]. In Sec. 3.2 this interaction is compared with experiment and adjusted. Then the combination of these 4 interactions [6–9] can be used, to cover the whole space from $^{132}_{50}$Sn$_{82}$ as core to $Z = 126$ and $N = 184$, the so called Kuo–Herling space.

The solution of the Schroedinger equation in the shell model gives the energies of the levels and the wave functions. The energies might be directly compared with experiment. Other observables, as quadrupole moments or M1 transition rates, have to be calculated as the expectation values of effective operators from the wave functions. In the same way, as the interaction energy between particles inside the nucleus is modified from that between free particles, these operators are also modified. Therefore these effective operators have to be calculated together with the effective interaction and measurements are necessary, to check the theory. The expectation values of the operators can then be calculated for some trial wave function and compared with the corresponding measured observables, in order to check or determine the wave function [11].

On the experimental side the ^{208}Pb region, and in particular ^{208}Pb itself, has been studied in much detail [12–15]. Because ^{208}Pb and a few neighbouring nuclei, as ^{207}Pb and ^{209}Bi, are stable, transfer reactions and inelastic scattering of electrons, protons, neutrons and heavy ions can be used. But also experiments with radioactive beams and on radioactive targets have been performed, as ^{210}Bi* (t,α) ^{209}Pb [16]. Lately also detailed γ-spectroscopy has been performed with transfer reactions and in coincidence with charged particles [13] and with deep inelastic reactions [17–19]. Often the new experimental data add previously missing information, and facilitate to fully exploit the older data. For instance γ-decay data combined with the results from transfer reactions might fix the spin of a state and by this make the measured spectroscopic factor really meaningful. Little is known about the neutron rich nuclei close to ^{208}Pb, as the two valence neutron nucleus ^{210}Pb [20], and even less about proton holes. Therefore the new experimental information [21] on ^{206}Hg, the nucleus with two proton holes is evaluated below.

2. ^{206}Hg the interaction between proton holes

Only the groundstate, the 2^+ level and the 5^- isomer had been known in ^{206}Hg [22], and the interaction between proton holes has been adjusted [7] to fit these energies. Now, in an experiment with gammasphere at the Atlas accelerator, excited states in ^{206}Hg have been populated by deep inelastic reactions of a 1360 MeV ^{208}Pb beam with a thick ^{238}U target and their γ-decay measured [21]. The level scheme, as derived from these results, is shown in Fig. 1 and compared with shell model calculations using the "original" interaction [7]. Spins have been assigned from this comparison with theory. The 10^+ isomer and the 8^+ and 7^- levels are so characteristic for the expected yrast-states formed by two proton holes, that these assignments are certain.

The strong 2344 keV line into the $\pi h^{-2}_{11/2}$ 10^+ state resembles very much the other octupole excitations, that have been observed in neighbouring nuclei on top of two particle or hole high spin states [23]. The energy of this transition is calculated to be shifted by -244 keV from the energy of the octupole excitation in ^{208}Pb at 2615 keV by the coupling between the octupole vibration and the two holes, compared with the observed shift of

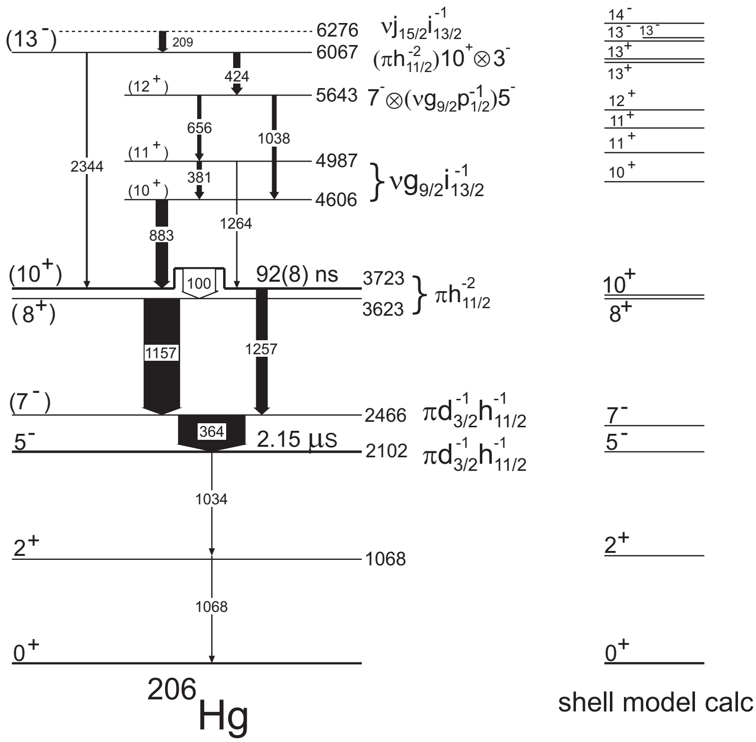

Fig. 1. Experimental and calculated level scheme of ^{206}Hg. The dominant configurations of the states are indicated.

-271 keV. This calculation is based on the measured octupole coupling of one $h_{11/2}$ proton hole in ^{207}Tl. It is very reliable, as the experience with several similar cases, particularly the octupole excitation on top of $\nu i^{-2}_{13/2}$ 12^+ state in ^{206}Pb proves [23]. A second 13^- level of the main configuration $\nu j_{15/2}\, i^{-1}_{13/2}$ is calculated at 6180 keV just 113 keV higher. Mixing between these two levels could explain the 10% difference between the observed and calculated octupole coupling.

With the assignment of 13^- to the 6067 keV level from these considerations, the parallel γ-branch to the 10^+ level through 3 intermediate levels agrees with theoretical expectations. Calculations clearly predict positive parity for these three states and very much favour 10^+ for the first level above the 10^+ isomer, as any other levels are 300 keV higher. If one in addition excludes M2 multipolarity, then 12^+ is certain for the 5643 keV level and 11^+ highly favoured for the 4987 keV state. Because 3 crossover transitions have been found above the 10^+ isomer and yrast states are strongly favoured in this type of experiment, the possible spins of the four levels are already restricted. The density of states with appropriate spins is low enough, that shell model calculations determine the spins of the new levels rather reliably, as indicated in the figure. The 10^+ and 11^+ states correspond to the particle-hole excitations in ^{208}Pb with the same spin, while the two proton holes of ^{206}Hg couple to 0^+. The 12^+ level is mainly the two proton hole 7^- state coupled to the lowest 5^- state of ^{208}Pb. This structure is however not necessarily very pure, as for spin 11^+, 12^+ and 13^- the next level lies within 250 keV.

The interaction between proton holes for the calculations of the states has been taken from Ref. [7]. They took the Kuo–Herling interaction [3] and adjusted the diagonal matrix elements of the main configuration (see Fig. 1) to reproduce the experimental energies, namely by 206 keV and 18 keV for the 2^+ and 5^- states. Adjustments of 106,3 and 66 keV are needed for the new 7^-, 8^+ and 10^+ states. As all corrections are positive, likely a general shift of +80 keV (the average of the 5 values) for all diagonal interaction elements between proton holes would improve the interaction.

The admixture of $h_{11/2}\mathrm{d}_{5/2}$ to the main component $h_{11/2}\, d_{3/2}$ in the 7^- state can be estimated from the measured $B(\mathrm{E3}, 10^+ \to 7^-) = 0.25(4)$ W.u., as the E3 transition can only proceed to this configuration. $B(\mathrm{E3}, h_{11/2} \to d_{5/2}) = 25$ W.u. can be reliably estimated [23] from the analogous measured $B(\mathrm{E3}, j_{15/2} \to g_{9/2}) = 26(4)$ W.u. in ^{209}Pb, and then $B(\mathrm{E3}, h^2_{11/2} 10^+ \to h_{11/2} d_{5/2} 7^-) = 31$ W.u. calculated.

The admixed amplitude squared is 0.008(3) from this, while the calculated is 0.017, or twice as large. In a similar way the $B\,(\mathrm{E3}, 5^- \to 2^+)$ is mainly determined by the transition from the main $h_{11/2}\, s_{1/2}$ component to the small admixture of $d_{5/2}\; s_{1/2}$, that is calculated as 10% for the 2^+ state. The measured $B(\mathrm{E3})$ gives only 1%. But a closer inspection shows, that there are other contributions that might interfere destructively and then a reasonable reduction, around a factor 2, of the $d_{5/2}\, s_{1/2}$ probability can achieve agreement with experiment. Rydstroem *et al.*, [7] already reduced all mixing matrix elements for 0^+ to reproduce the ground state energy of ^{206}Hg. So the three pieces of experimental information favor a reduction of the nondiagonal elements of the interaction.

The energies of the core excited 10^+ and 11^+ levels are lowered relative to ^{208}Pb, but not enough. Perhaps the interaction between neutrons and proton holes, that is little known experimentally should be more attractive. The 12^+ level is calculated too high by about as much as its one component, the $\pi h^{-1}_{11/2}\, d^{-1}_{3/2}\; 7^-$ state.

3. The neutron neutron-hole and proton proton-hole interaction 208Pb

3.1. Pandya transform

One neutron or proton is raised from the occupied orbitals below the shell closure to an empty orbital above, to form the excited states of ^{208}Pb. Therefore the level scheme of ^{208}Pb provides information on the diagonal matrix elements of the interaction between neutrons and neutron holes and protons and proton holes. Also configuration mixing between these one particle-one hole states can be determined from experimental data,and informationon the nondiagonal elements of the interaction deduced. Rejmund *et al.*,have in this way derived many matrix elements of the interaction [11]. In the following these empirical interaction elelements and also the measured level scheme are compared with calculations, in an attempt to find some general trends.

Usually shell model calculations are performed in the particle–particle representation, as for instance by the OXBASH program, and we are used to think in this way. But the excited one-particle one-hole states of ^{208}Pb are directly related to the particle-hole interaction. The Pandya transformation [24] connects the matrix elements of both representations:

$$E_I(j_1j_4^{-1};j_3j_2^{-1}) = -(-1)^{j_1+j_2+j_3+j_4}\sum_J(2J+1)W(j_1j_2j_4j_3;JI)E_J(j_1j_2;j_3j_4).$$

Some features of the Pandya transformation have to be recognized.

(i) It relates diagonal and nondiagonal elements.

(ii) To calculate any particle-hole element, the particle–particle elements for all spins belonging to the orbitals involved are needed; the same holds for the reverse transformation. Because the experimental information is nearly always incomplete, one can only transform from the theoretical particle–particle elements to particle-hole.

(iii) The Pandya transform does not describe the matrix elements, that mix one-particle one-hole and two-particle two-hole states. Therefore only states of rather pure one-particle one-hole structure can be treated here.

(iv) It is often surprising, which states are connected by this transformation. For instance the interaction between ν-particle $g_{9/2}$ and π-hole $h_{11/2}$ in low lying states of ^{208}Tl gives that between ν-particle $g_{9/2}$ and π-particle $h_{11/2}$ that would be manifest in low lying states of ^{194}Tb, that is 35 neutrons above stable ^{159}Tb. Or, as has been found by chance, the mixing element $E_{13+}(\nu j_{15/2}\,\pi h_{11/2};\,\nu i_{13/2}\,\pi i_{13/2})$ influences the 3$^{\rm rd}$ 2^- state in ^{208}Pb strongly.

3.2. Comparison of calculated and empirical interactions

It was found in Ref. [11], that proton–proton hole and neutron–neutron hole elements are very similar, if 300 keV are added to the empirical π–π^{-1} interaction (diagonal elements). Breaking the 0^+ proton pair in the ground state of ^{208}Pb gains the Coulomb pairing energy, and a constant amount of -300 keV, independent of orbitals and spin, resembles the experimental energies well. This is one finding of a general feature, and allows to treat neutron and proton interactions together in the following.

The interaction between particles and holes depends primarily on simple geometry. The success of the schematic surface delta interaction reflects this for instance. Therefore the diagonal interaction elements are presented as a function of the classical angle α between the spins of particle and hole. As the particles and holes are concentrated in the plane perpendicular to their spin, α determines largely the overlap of the wave functions. The overlap is large for $\alpha \simeq 0$ or 180 deg and minimal for 90 deg and the interaction energy should reflect this, if the interaction is of short range. In practical terms, the various interaction elements can be presented together as a function of the classical angle, independent of the detailed quantum numbers.

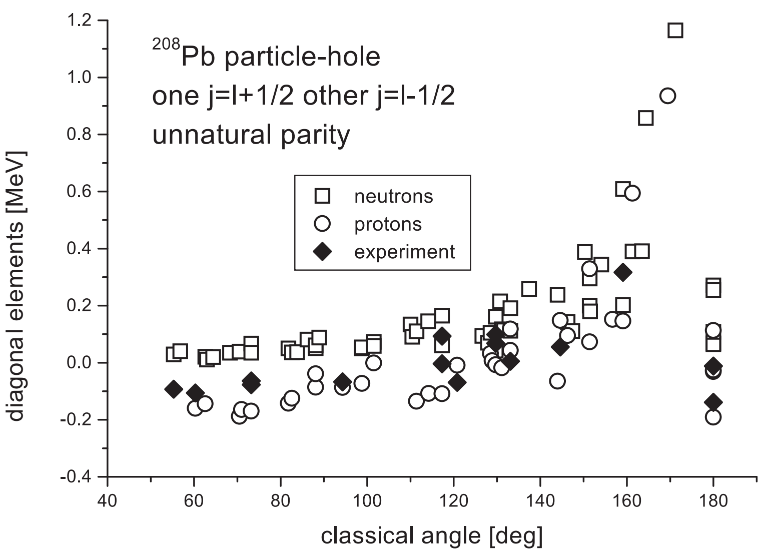

Fig. 2. Comparison of experimental and calculated particle-hole matrix elements of the residual interaction in ^{208}Pb. The parity of the states is unnatural $\pi = -(-1)^I$; if $j = l+1/2$ for the particle, $j = l-1/2$ for the hole or *vice versa*. The experimental elements include those for neutrons and protons, the latter are shifted by $+300$ keV, see text.

It was found, that the interaction is characteristically different depending on two parameters, *(i)* natural ($\pi = (-1)^I$) or unnatural parity of the states, and *(ii)* particle and hole have both either $j = l + 1/2$ or $j = l - 1/2$ or they differ in this respect. Fig. 2 compares for one of these four cases, the empirical elements with the realistic neutron and proton elements. Theoretical neutron and proton elements and the empirical elements exhibit the same trend. The steep rise around 160 deg is experimentally resembled by the $(\pi h_{9/2} h^{-1}_{11/2}; 1^+)$ level, and the two 0^- states show the sharp drop at exactly 180 deg. The calculations of the realistic interaction included the Coulomb energy for protons; in the figure the proton elements are lower by around 150 keV than the neutron elements. It has to be remembered, that the empirical proton elements, as shown, have been adjusted by +300 keV, in order to agree with the neutron elements.

TABLE I

Adjustments of the empirical interaction.

	eq nat	*eq un*	*not nat*	*not un* *
neutron	−107	−57	−202	−127
proton	+62	+91	−49	+25
proton–neutron	+169	+148	+153	+152
All diagonal proton elements −300 keV				
All nondiagonal 3-elements have been multiplied by 0.895				
For *not nat* and the highest spin				
all configurations				+150 keV
neutron $i_{11/2}\, i_{13/2}$ 12^+				+280 keV
proton $h_{9/2}\, h_{11/2}$ 10^+				+170 keV
For neutron $g_{9/2}\, i_{13/2}$ all spins				−70 keV
For neutron $j_{15/2}\, i_{13/2}$ all spins				−40 keV
For neutron $j_{15/2}\, i_{13/2}$ 14^-				+95 keV
to reproduce the lowest 2^+ 4^+ 6^+				
neutron $g_{9/2}\, i_{13/2}$	2^+ − 480 keV	4^+ − 480 keV	6^+ − 440 keV	

* All stated adjustments, that are appropriate for a state, have to be added.
eq: particle and hole are both $j = l + 1/2$ or both $j = l - 1/2$.
not: particle and hole differ in $j = l + 1/2$ or $j = l - 1/2$
nat: natural parity, $\pi = (-1)^I$
un: unnatural parity, $\pi = -(-1)^I$

Fig. 2 shows that a uniform shift of the calculated neutron elements by −130 keV gives on average already good agreement with the empirical elements. The proton elements have to be shifted up by around +25 keV. Also in the other three cases the main difference between calculated and empirical elements is simply a constant, independent of any detailed structure. The adjustments of the interaction are summarized in Table I. The difference of the correction between neutrons and protons is about 150 keV in all 4 cases, in agreement with the assumption of just one overall Coulomb energy. Fig. 3 compares the known empirical elements with the corresponding realistic ones. The agreement is in general quite good. But the individual elements deviate often by around 100 keV in a seemingly random fashion. Many elements are also only around 100 keV, or in other words the deviations can amount to 100%. The errors of the empirical elements are nearly always below 50 keV [11]. Particularly the two 0^- elements indicate, that better agreement could be achieved with an adjustment that varies with the angle. But there are too few data points close to 0 or 180 deg to determine

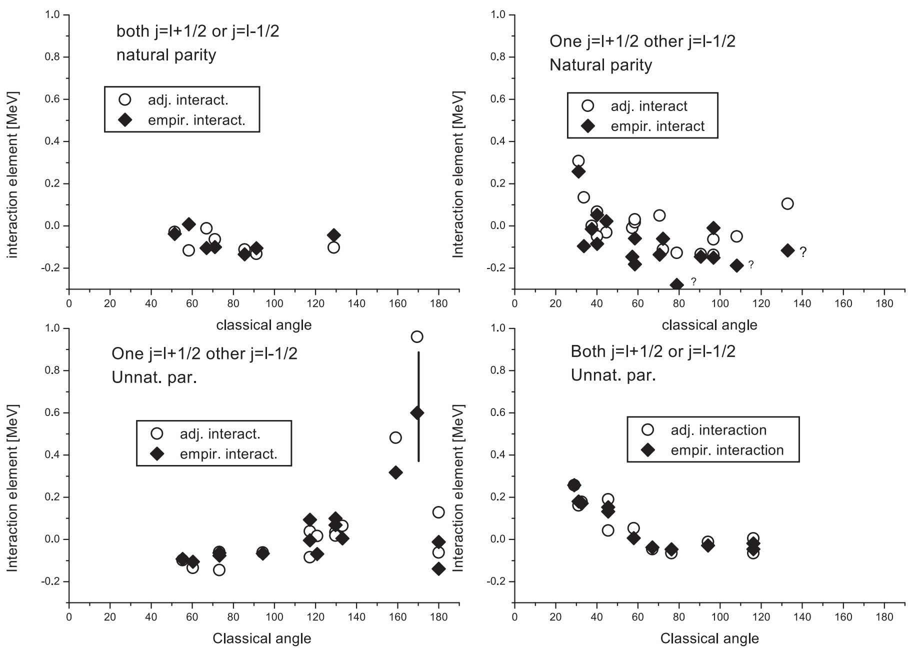

Fig. 3. Comparison of the adjusted with the empirical particle-hole interaction for ^{208}Pb. The proton elements are shifted by +300 keV, to make them comparable with the neutron elements. The values marked by ? belong to the 2^+ 4^+ and 6^+ states of the $\pi h_{11/2}^{-1} \pi h_{9/2}$ configuration.

further parameters. A clear finding is, that the states of highest spin of all configurations with different alignment of spin and orbital angular momentum ($j = l^{+}_{-}1/2$) are calculated too low by about 150 keV. Moreover for the spin orbit partners ($\nu i_{11/2} i^{-1}_{13/2}; 12^+$) and ($\pi h_{9/2} h^{-1}_{11/2}; 10^+$) an additional difference of 280 and 170 keV, respectively, is found.

The nondiagonal elements cannot be presented as a function of just one parameter. Therefore a straightforward comparison between theory and experiment as for the diagonal elements is not possible. The energy of the collective 3^- state however is very sensitive to configuration mixing. A reduction of 10% for all nondiagonal 3^- elements gets its energy right. A reduction of all nondiagonal elements also for other spins gives however no improvement. These findings might be helpful to improve the calculations of the interaction.

All adjustments of the realistic interaction are summarized in Table I. In addition to the adjustments mentioned above, the elements of the configuration $\nu g_{9/2}\, i^{-1}_{13/2}$ for 2^+, 4^+ and 6^+ have been lowered by around 500 keV, in order to reproduce the energies of the yrast 2^+, 4^+ and 6^+ levels. Very likely admixtures of two particle–two hole states are the real cause for the lowering of these states, but this change will anyway improve calculations for neighbouring nuclei. Fig. 4 compares the positive and negative parity levels, as calculated with this interaction, with experiment. The results are quite satisfactory. A one to one correspondence of calculated and experimental levels is evident. The second experimental 2^+ and 4^+ levels are likely two-

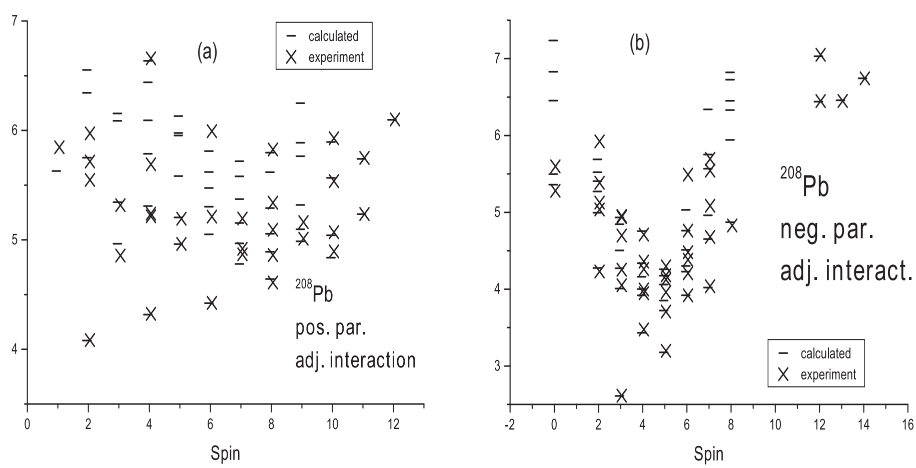

Fig. 4. The level scheme of ^{208}Pb for (a) positive and (b) negative parity. The energies calculated with the adjusted interaction (horizontal bars) are compared with experimental energies (x).

particle two-hole states and therefore without theoretical counterparts. The rms error for 54 levels is 66 keV and the average linear deviation -21 keV. Without the adjustments of the interaction the corresponding numbers are 189 and 128 keV, respectively.

4. Conclusions

Much should yet be measured around ^{208}Pb, mainly on the neutron rich and proton deficient side. Nevertheless a wealth of experimental data on states composed of two particles or holes exist, from which the shell model residual interaction can be directly determined [11]. For the particle-hole interaction in ^{208}Pb the realistic interaction reproduces the systematic features of the empirical interaction well. This is shown as a function of the classical angle in Figs. 2, 3. A wide range of angles is covered here, as the spins of the states range from 0^- to 14^- and the angular momenta of particles and holes from 1/2 to 15/2. This angular dependence reflects by and large the dependence of the interaction between the particles on the distance between them, that is explored in this way. Adjustments of the calculated interaction resulted in an improved set of matrix elements, that should reproduce any states around ^{208}Pb that include core excitations well. Moreover the trends, that have been found, might give hints to improve the calculations for the interaction in the nucleus from that between free nucleons.

Many people cooperated in this work. I thank particularly M. Rejmund, B. Fornal and R. Broda. Thanks are also due to all colleagues at the H. Niewodniczański Institute of Nuclear Physics, Kraków, for a very pleasant time there, and the Foundation for Polish Science (FNP) for financing this stay.

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