ENERGY GAP EFFECT IN THE SHELL MODEL WITH RANDOM TWO-BODY INTERACTIONS*

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We investigate the nature of the low-lying spectra of many-body systems with random two-body interactions. Our study shows that the higher J-ground states are more orderly and develop the larger energy gaps than the ones in the $J = 0$-sector.

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It is a well known fact that the outcome of shell-model calculations depends in a very sensitive manner on a detailed form of the nucleon–nucleon interaction used. This in particular applies to specific locations of the corresponding eigenvalues. Typically, a small change of values of some nuclear Hamiltonian parameters leads to a sizable redistribution of its eigenvalues. Thus, as far as such fine structure effects are concerned, this property, analogous to classical chaos, sets a significant limitation on the predictive power of the corresponding methodology. At the same time, however, certain more global characteristics like the eigenvalue density distribution or the spectral fluctuations are largely generic, *i.e.*, they do not depend on the detailed form of the Hamiltonian. The above facts imply the success of the Random Matrix Theory (RMT) in modeling such more global characteristics. The Gaussian Orthogonal Ensemble (GOE) version of the RMT respects some global symmetries of realistic nuclear systems and is frequently used in modeling the strongly interacting Fermi systems.

The GOE does not however account for the fact that nuclear interactions are predominantly two-body in nature. In this respect the more precise and adequate frame is prescribed by the Two-Body Random Ensemble (TBRE) [1]. Its recent refinement termed Random Quasiparticle Ensemble (RQE) [2] seems most appropriate since it properly respects the relation

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between the particle–particle and particle–hole interaction channels, as expressed by the Pandya relation. Deviations from pure GOE of the results based on such ensembles are of great interest because they may identify some system specific properties that do not depend on details of the interaction. They may also shed light on the general mechanism generating the related effects.

From this perspective, in the present study we address the question of a possible emergence of the pairing-like effects in the two-body random ensembles of random matrices. One principal directly observable characteristics of such effects is the appearance of energy gaps at the edges of the spectrum. The model to be quantitatively explored here in this connection consists, similarly as in Ref. [2], of six identical particles (all single particle energies are set to zero) operating in the *sd*-shell. From the nuclear spectroscopy point of view this corresponds to the ^{22}O nucleus. The statistics is collected from one thousand of RQE samples of two-body matrix elements.

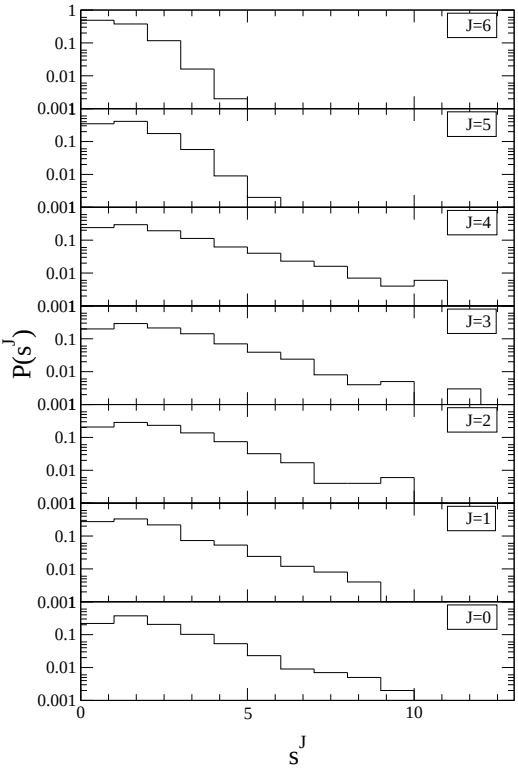

Fig. 1. Distributions of ground state energy gaps for successive J's as defined by the Eq. (1).

The resulting central result of our related investigations is shown in Fig. 1. Here the distribution of the ground state (E_1^J) energy gaps

$$s^J = \frac{\left(E_2^J - E_1^J\right)}{D^J}, \tag{1}$$

for different J-sectors is shown. In this expression D^J denotes the average global level spacing among the remaining states, characteristic for a given J:

$$D^J = \frac{\langle E_{M_J}^J - E_1^J \rangle}{M_J}. \tag{2}$$

It seems quite natural to see that there is a significant probability of nonzero ground state energy gaps relative to D^J. This can easily be traced back to the different distributions of states corresponding to the RQE relative to GOE (Gaussian-like versus semicircular). However, and this is very interesting, there is a nonzero probability for appearance of even very large (~ 10) gaps. What however is even more amazing is that the large ground state energy gaps are more probable in the higher J-sectors ($J = 2$–4) than in the $J = 0$-sector.

That this indicates more orderly ground states in the $J = 2$–4 sectors turns out consistent with their localization length which can be quantified in terms of the information entropy

$$K_l^J = -\sum_{\alpha=1}^{M_J} \left|a_{l,\alpha}^J\right|^2 \ln \left|a_{l,\alpha}^J\right|^2 \tag{3}$$

of an eigenstate labeled by l from the J-sector. The coefficients $a_{l,\alpha}^J$ denote the eigenvector components in the basis $|\alpha|$. Each K_l^J is normalised to its GOE limit [3]

$$K_{\mathrm{GOE}}^J = \psi\left(\frac{M_J}{2} + 1\right) - \psi\left(\frac{3}{2}\right), \tag{4}$$

where ψ is the digamma function. Within our model the so-calculated and RQE ensemble averaged quantity for all the states versus their corresponding energies E_l^J is illustrated in Fig. 2.

As anticipated, it is not $J = 0$ whose lowest eigenstate comes out most localised, *i.e.*, most regular. The lowest states for several higher J values deviate much more from GOE. This in particular applies to $J = 2$ and, especially, to $J = 4$. This thus indicates more favorable conditions for the emergence of energy gaps for larger J than for $J = 0$.

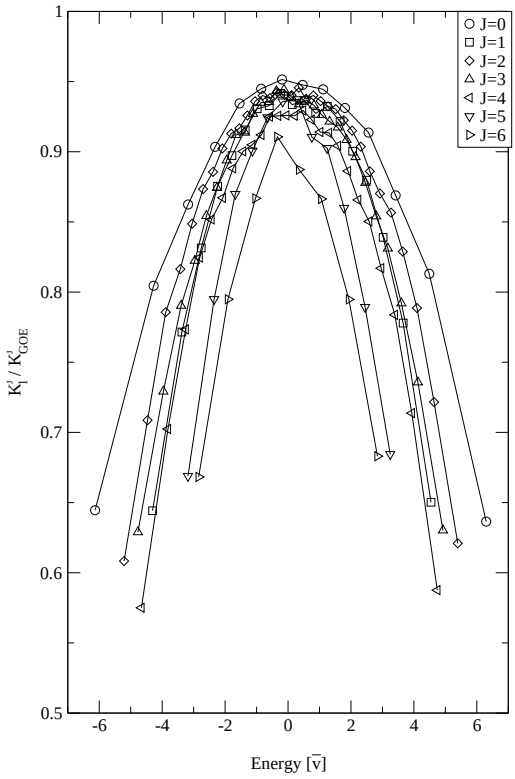

Fig. 2. The information entropy normalised to its GOE limit ($K_l^J/K_{\rm GOE}^J$) for all the states l from various J-sectors (all positive parity) versus the energies (E_l^J) of those states. Circles correspond to $J=0$, squares to $J=1$, diamonds to $J=2$, upwards oriented triangles to $J=3$, left oriented triangles to $J=4$, downwards oriented triangles to $J=5$ and right oriented triangles to $J=6$. Lines are drawn to guide the eye.

The above numerical results can also be understood on a more formal level. The point is that in statistical ensembles of matrices the crucial factor determining the structure of eigenspectrum is the probability distribution $P_V(v)$ of matrix elements [4]. Especially relevant are the tails of such distributions since they prescribe the probability of appearance of the large matrix elements. From the point of view of the mechanism producing the energy gaps they are most effective in generating a local reduction of dimensionality responsible for such effects. In the shell-model basis $|\alpha\rangle$ the interaction matrix elements $v^J_{\alpha,\alpha'}$ of good total angular momentum J can be expressed as follows [5]:

$$v^J_{\alpha,\alpha'} = \sum_{J'} \sum_{ii'} c^{J\alpha\alpha'}_{J'ii'}\, g^{J'}_{ii'}\,. \tag{5}$$

The summation runs over all combinations of the two-particle states $|i\rangle$ coupled to the angular momentum J' and connected by the two-body interaction g. $g^{J'}_{ii'}$ denote the radial parts of the corresponding two-body matrix elements and are Gaussian distributed by construction. The factors $c^{J\alpha\alpha'}_{J'ii'}$ represent elements of the angular momentum recoupling geometry. The complicated quasi-random coupling of individual spins is believed [6] to result in the so-called geometric chaoticity [7] which for a global estimate allows to assume that $c^{J\alpha\alpha'}_{J'ii'}$ are also Gaussian distributed. An explicit verification shows [8] that this assumption does indeed apply for all J-sectors except for $J = 0$. In this second case a uniform distribution over a certain finite interval is more appropriate.

Under the above assumptions one can analytically derive [8] the distributions of the off-diagonal matrix elements in both cases. For $J \neq 0$ one obtains

$$P_V(v) = \frac{|v|^{(N-1)/2} K_{(N-1)/2}(|v|)}{2^{(N-1)/2}\Gamma(\frac{N}{2})\sqrt{\pi}}\,, \tag{6}$$

where K stands for the modified Bessel function. Asymptotically, for large v, this leads to

$$P_V(v) \sim \exp(-|v|)\,|v|^{N/2-1}\,. \tag{7}$$

The $J = 0$ case results in

$$P_V(v) = \frac{1}{\sqrt{2\pi}} \int\limits_0^\infty \left[\frac{erf(c_0\omega/\sqrt{2})}{c_0\omega}\right]^N \cos(\omega v) d\omega\,, \tag{8}$$

which for large v behaves like

$$P_V(v) \sim \exp(-v^2). \tag{9}$$

These expressions indicate that the distribution of off-diagonal matrix elements in the $J = 0$ sector resembles more a Gaussian, and the tails of this distribution drop down faster relative to the $J \neq 0$-sectors, where the large v tails drop down is consistent with an exponential asymptotics of Eq. (6). At the same time the $J \neq 0$ sectors are dominated by very small matrix elements to a larger degree than $J = 0$. The probability of appearance of a large off-diagonal matrix element which in magnitude overwhelms the remaining ones is thus greater for $J \neq 0$ than for $J = 0$. Such an effective reduction of the rank in the former case is expected to result in a stronger tendency to localization as compared to GOE [4]. These considerations thus provide a formal explanation of the above numerical results. From this perspective a predominance of $J = 0$ ground states in RQE, as claimed in Ref. [2], turns out to be the result of putting on together states with different characteristic

energy scales from different J-sectors. Finally, it seems appropriate to point out a potential relevance of of the above results to the high-temperature superconductivity where the $J \neq 0$ pairing-like effects may offer a likely mechanism.

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