PERTURBATIONS OF PLANAR INTERFACES IN GINZBURG–LANDAU MODELS

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Certain dissipative Ginzburg–Landau models predict existence of planar interfaces moving with constant velocity. In most cases the interface solutions are hard to obtain because pertinent evolution equations are nonlinear. We present a systematic perturbative expansion which allows us to compute effects of small terms added to the free energy functional of a soluble model. As an example, we take the exactly soluble model with single order parameter φ and the potential $V_0(\varphi) = A\varphi^2 + B\varphi^3 + \varphi^4$, and we perturb it by adding $V_1(\phi) = \frac{1}{2}\varepsilon_1\varphi^2\partial_i\varphi\partial_i\varphi + \frac{1}{5}\varepsilon_2\varphi^5 + \frac{1}{6}\varepsilon_3\varphi^6$. We discuss the corresponding changes of the velocity of the planar interface.

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1. Introduction

Properties and evolution of interfaces is a very interesting topic in soft condensed matter physics, see, *e.g.*, [1] for a recent review. There are two main lines of theoretical research in that direction. The first one, computer simulations, provides a relation to microscopic, molecular dynamics level [2,3]. In the second approach one is satisfied with a coarse-grained description by effective Ginzburg–Landau models.

In the framework of Ginzburg–Landau models the interfaces are described by particular solutions of pertinent nonlinear evolution equations. The nonlinearity is crucial for existence of these solutions, but it is also a serious obstacle in obtaining them in an explicit form. In most cases only numerical solutions are available. A well-known exception is a model which appears in the context of reaction-diffusion chemical processes [4], and also is related to Landau–deGennes effective model for nematic liquid crystals [5]. It involves a single, non-conserved, real-valued order parameter $\varphi(x^k, t)$, and

the free energy functional has the form

$$F_0 = \int d^3x \left(\frac{1}{2} \nabla\varphi\nabla\varphi + V_0(\varphi) \right), \tag{1}$$

$$V_0(\varphi) = A\varphi^2 + B\varphi^3 + \varphi^4, \tag{2}$$

where $\nabla = \left(\frac{\partial}{\partial x^k}\right)$, $\alpha = 1, 2, 3$, x^k are Cartesian coordinates in the space. For simplicity we have rescaled x^k, φ and F_0 so that they are dimensionless and the coefficient in front of the quartic term in $V_0(\varphi)$ is equal to 1. The constants A, B are positive, and A is restricted to the interval $0 < A < \frac{9B^2}{32}$. Then $V(\varphi)$ has two minima and a maximum between them. In this model exact formulas for profile and velocity of a planar interface are known, see, *e.g.*, [4].

Exactly soluble models provide natural starting points for perturbative expansions. If the expression (1) for the free energy is changed a little bit, one may hope that the corresponding change of the planar interface can be calculated with the help of a perturbative expansion around the known solution. In this way we would obtain analytical results for velocity and profile of planar interfaces in a class of Ginzburg–Landau models. Moreover, this would open the way for investigations of evolution of curved interfaces in that class of models because one could use the expansion in curvatures [6] for which the knowledge of planar interfaces is the only prerequisite.

In the present paper we explore the idea of the perturbative expansion for the planar interfaces. We show that indeed such perturbative scheme can be constructed. Corrections to the profile are obtained in it as solutions of ordinary, linear differential equations, while corrections to the velocity follow from integrability conditions which are due to existence of zero-modes. This type of integrability conditions is well-known in statistical physics [7]. As an example we consider perturbations of the form

$$\delta F = \int d^3x V_1 \,,$$

where

$$V_1 = \tfrac{1}{2}\varepsilon_1\varphi^2\nabla\varphi\nabla\varphi + \tfrac{1}{5}\varepsilon_2\varphi^5 + \tfrac{1}{6}\varepsilon_3\varphi^6. \tag{3}$$

Values of the perturbation parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are such that $F_0 + \delta F$ is bounded from below. The ε_1 correction to the gradient term introduces dependence on φ of the elastic constant $K = 1 + \varepsilon_1\varphi^2$. The ε_2 and ε_3 terms are corrections to the potential $V_0(\varphi)$. The perturbations (3) are vanishing in the disordered phase ($\varphi = 0$), while in the ordered phase ($\varphi \neq 0$) they may give finite contributions.

Our paper is organised as follows. In Section 2 we construct the perturbative expansion. In Section 3 we apply it to the model (1), (2) with the

particular perturbation δF given by formula (3). Summary and remarks are presented in Section 4. In the Appendix we show that the perturbatively calculated velocity of the planar interface coincides with the one calculated from an exact formula, provided that the perturbation is not too strong and that the perturbative series is convergent.

2. General structure of the perturbative expansion

We will consider Ginzburg–Landau models in which time evolution of the order parameter φ is governed by equation of dissipative type, namely

$$\frac{\partial\varphi}{\partial t} = \Delta\varphi - V'(\varphi, \nabla\varphi)\,, \tag{4}$$

where $V = V_0 + \varepsilon V_1$, t denotes a rescaled dimensionless time, $\Delta = \partial_{x^k}\partial_{x^k}$ is the three-dimensional Laplacian, and

$$V' = \frac{\partial V}{\partial\varphi} - \nabla\left(\frac{\partial V}{\partial(\nabla\varphi)}\right).$$

We have assumed that the perturbation V_1 does not depend on second and higher derivatives of φ. The r.h.s. of Eq. (4) is equal to the variational derivative $\delta F/\delta\varphi$ multiplied by -1.

The interface is identified with a solution of the evolution equation which interpolates between minima φ_+, φ_- of the potential V. Potential V_0 given by formula (2) has two minima, and the perturbation V_1 is by assumption weak enough, so that also V has just two minima which smoothly merge with the minima of V_0 when V_1 is switched off. In the present paper we concentrate on planar interfaces. Due to translational invariance of Eq. (4), it is sufficient to consider the planar interfaces parallel to the plane (x^1, x^2) — then φ is a function of x^3 and t. We require that in the limits $x^3 \to \pm\infty$,

$$\varphi(x^3, t) \to \varphi_\pm\,, \qquad \partial_{x^3}\varphi \to 0\,. \tag{5}$$

Let us multiply Eq. (4), in which now $\nabla\varphi \to \partial_{x^3}\varphi(x^3, t)$, by $\partial_{x^3}\varphi$ and integrate it over x^3 in the interval $(-\infty, +\infty)$. With the help of integration by parts, and using a formula for the full derivative of V with respect to x^3, namely

$$\frac{dV}{dx^3} = \frac{\partial V}{\partial\varphi}\partial_{x^3}\varphi + \frac{\partial V}{\partial(\partial_{x^3}\varphi)}\partial^2_{x^3}\varphi\,,$$

we obtain the following relation

$$\int_{-\infty}^{+\infty} dx^3\,\partial_t\varphi\partial_{x^3}\varphi = V(\varphi_-) - V(\varphi_+)\,. \tag{6}$$

It implies that φ has to be time-dependent if the minima $\varphi_\pm$ are nondegenerate. The simplest way to ensure that the l.h.s. of formula (6) is constant in time and nonvanishing consists in assuming that φ depends only on the combination $x^3 - vt$,

$$\varphi = \varphi(x^3 - vt), \tag{7}$$

where v is a constant equal to the velocity of the interface. This form of dependence on time is also consistent with boundary conditions (5). For such φ, relation (6) can be written in the form

$$v = \frac{V(\varphi_+) - V(\varphi_-)}{\int_{-\infty}^{+\infty} dz(\partial_z \varphi)^2}, \tag{8}$$

where $z = x^3 - vt$. Equation (4) is reduced to

$$\partial_z^2 \varphi + v\partial_z \varphi - V'(\varphi, \partial_z \varphi) = 0, \tag{9}$$

where φ is a function of z only, and

$$\varphi(z) \to \varphi_\pm \tag{10}$$

when $z \to \pm\infty$, respectively. The asymptotic values $\varphi_\pm$ are determined from the equations

$$V'(\varphi_\pm) = 0. \tag{11}$$

From a mathematical viewpoint (8) and (9) should be regarded as a set of equations for v and $\varphi(z)$, with (10) as the boundary conditions for $\varphi(z)$.

The perturbative Ansatz for $\varphi(z)$ has the form

$$\varphi(z) = \varphi_0(z) + \varepsilon\varphi_1(z) + \varepsilon^2\varphi_2(z) + \cdots. \tag{12}$$

Here $\varphi_0(z)$ is the initial, unperturbed interface. It obeys the following equation

$$\partial_z^2 \varphi_0 + v_0 \partial_z \varphi_0 - V_0'(\varphi_0) = 0, \tag{13}$$

with the boundary conditions at $z \to \pm\infty$

$$\varphi_0(z) \to a_\pm,$$

where $a_\pm$ are the minima of V_0. The velocity v_0 of the unperturbed interface is given by the formula analogous to (8), namely

$$v_0 = \frac{V_0(a_+) - V_0(a_-)}{\int_{-\infty}^{+\infty} dz(\partial_z \varphi_0)^2}. \tag{14}$$

Formula (8) suggests that the velocity of the interface has the following perturbative expansion

$$v = v_0 + \varepsilon v_1 + \varepsilon^2 v_2 + \cdots . \tag{15}$$

Let us stress that in equations (8), (9) and in expansion (12) the independent variable is just z, and not x^3 and t. Therefore, when solving Eqs. (8), (9), the variable z is not treated as $x^3 - vt$, and, therefore, we do not expand z in ε, contrary to what formula (15) might suggest. Only after $\varphi(z)$ and v are determined we may substitute $z = x^3 - vt$ in order to relate the solution of the set of equations (8), (9) with the interface solution of the original evolution equation (4).

Inserting the expansions (12), (15) in Eq. (9) and equating to zero the l.h.s. of (9) order by order in ε, we obtain the infinite chain of equations for the corrections $\varphi_k, k \geq 1$. These equations have the form

$$\hat{L}\varphi_k \;=\; f_k\,, \tag{16}$$

where

$$\hat{L} \;=\; \partial_z^2 + v_0\;\partial_z - V_0''(\varphi_0)\,, \tag{17}$$

and f_k in general depend on $v_1, \ldots, v_k$ and $\varphi_1, \ldots, \varphi_{k-1}$. Simple calculations give

$$f_1 \;=\; -v_1\,\partial_z\varphi_0 + V_1'(\varphi_0, \partial_z\varphi_0)\,, \tag{18}$$

and for $k \geq 2$

$$f_k \;=\; -v_k\partial_z\varphi_0 - \sum_{j=1}^{k-1} v_j\;\partial_z\varphi_{k-j} + h_k(z)\,, \tag{19}$$

where $h_k(z)$ denotes the sum of all terms of the k-th order in ε obtained by inserting formula (12) in $V_0'(\varphi) + \varepsilon V_1'(\varphi, \partial_z\varphi)$ with exception of the term $V_0''(\varphi_0)\varphi_k$ which has been included into the l.h.s. of Eq. (16).

It is a well-known fact that inhomogeneous linear equations not always have solutions — certain integrability conditions have to be satisfied. In the case of Eq. (16) such conditions appear because for the operator $\hat{L}$ there exists so called left zero-mode, that is a normalizable function $\psi_l(z)$ such that the following identity holds

$$\int_{-\infty}^{+\infty} dz\psi_l(z)\hat{L}\varphi_k = 0\,. \tag{20}$$

If φ_k obeys Eq. (16) then

$$\int_{-\infty}^{+\infty} dz \psi_l(z) f_k = 0 \,. \tag{21}$$

In our perturbative scheme the integrability conditions (21) serve as equations which determine the perturbative contributions v_k to the velocity of the interface. Using formulas (18), (19) and conditions (21) we find that

$$v_1 = N^{-1} \int_{-\infty}^{+\infty} dz V_1'(\varphi_0, \partial_z \varphi_0) \psi_l(z) \,, \tag{22}$$

and for $k \geq 2$

$$v_k = N^{-1} \int_{-\infty}^{+\infty} dz \left(h_k(z) - \sum_{j=1}^{k-1} v_j \ \partial_z \varphi_{k-j} \right) \psi_l(z) \,, \tag{23}$$

where

$$N = \int_{-\infty}^{+\infty} dz \psi_r(z) \psi_l(z) \,.$$

The functions ψ_l, ψ_r will be given shortly. In the Appendix we check that these recursive formulas and the expansion (15) give the velocity v which coincides with the one obtained by expanding in ε the r.h.s. of formula (8), provided that ε is not too large.

The left zero-mode ψ_l can be found in the following way. Differentiation of the both sides of Eq. (13) with respect to z gives the identity

$$\hat{L} \psi_r = 0 \,, \tag{24}$$

where

$$\psi_r = \partial_z \varphi_0(z) \,. \tag{25}$$

The function ψ_r is called the right zero-mode. ψ_l has the form

$$\psi_l(z) = \exp(v_0 z) \psi_r(z) \,. \tag{26}$$

Because derivation of identity (20) involves integration by parts, we have to discuss behaviour of ψ_l and ψ_r for $z \to \pm\infty$. Notice that Eq. (24) implies that for $z \to \pm\infty$

$$\psi_r(z) \cong \exp\left(-\frac{v_0}{2} z\right) \exp\left(\mp \sqrt{\frac{{v_0}^2}{4} + V_0''(a_\pm)} \ \ z \right),$$

respectively. Therefore, $\psi_l(z)$ exponentially vanishes for $z \to \pm\infty$, provided that

$$V_0''(a_\pm) > 0\,.$$

For V_0 given by formula (2) this condition is satisfied if $0 < A < \frac{9B^2}{32}$, as it has been assumed. When checking identity (20), boundary terms like, *e.g.*, $(\partial_z\psi_l)\varphi_k|_{-\infty}^{+\infty}$, vanish because

$$\varphi_k \to \text{const.} \quad \text{for} \quad z \to \pm\infty\,, \tag{27}$$

in accordance with the boundary condition (10).

Equations (16) can be solved in a standard way [8]. Adopting formulas given in [8] to the case at hand we find that

$$\varphi_k = \psi_2(z)\int\limits_{-\infty}^{z} d\zeta\, \mathrm{e}^{v_0\zeta}\psi_r(\zeta)f_k(\zeta) - \psi_r(z)\int\limits_{0}^{z} d\zeta\, \mathrm{e}^{v_0\zeta}\psi_2(\zeta)f_k(\zeta)\,, \tag{28}$$

where

$$\psi_2(z) = \psi_r(z)\int\limits_{0}^{z} d\zeta\, \mathrm{e}^{-v_0\zeta}\psi_r^{-2}(\zeta)\,. \tag{29}$$

The functions ψ_r, ψ_2 form a pair of linearly independent solutions of the homogeneous equation $\hat{L}\varphi = 0$. Notice that the integral $\int_{-\infty}^{z}$ present in the first term on the r.h.s. of formula (28) vanishes for $z \to +\infty$ due to the integrability conditions (21). It turns out that in the limits $z \to \pm\infty$ both terms on the r.h.s. of formula (28) approach constants, and φ_k obey the conditions (27).

General solution of Eqs. (16) is obtained by adding to the r.h.s. of formula (28) the general solution of the corresponding homogeneous equation, that is the function $c_k\psi_r(z) + d_k\psi_2(z)$. However, because $\psi_2(z)$ exponentially grows for large $|z|$ we have to put $d_k = 0$. In order to determine the constants c_k we have to impose a condition on the interface solution $\varphi(z)$ in addition to the boundary conditions (10). Actually, Eq. (9) and the boundary conditions (10) do not determine $\varphi(z)$ uniquely — due to invariance with respect to translations in z we can take $\varphi(z - z_0)$ with arbitrary z_0. We will require that

$$\varphi(0) = \varphi_0(0)\,, \tag{30}$$

where $\varphi_0(z)$ is a concrete, explicitly given function describing the unperturbed interface with fixed location on the z axis. The condition (30) breaks the translational invariance. It implies that

$$\varphi_k(0) = 0\,. \tag{31}$$

The solutions $\varphi_k(z)$ given by formula (28) obey this condition, therefore, also the constants c_k are vanishing. Thus, the perturbative scheme supplemented with the condition (30) yields unique interface solution $\varphi(z)$. In this solution we can, of course, shift the variable z, that is to substitute $z \to z - z_0$ simultaneously in all contributions φ_0, φ_k, in order to obtain the other solutions implied by the translational invariance.

3. Effects of the perturbations of the form $V_1 = \frac{1}{2}\varepsilon_1\varphi^2\nabla\varphi\nabla\varphi + \frac{1}{5}\varepsilon_2\varphi^5 + \frac{1}{6}\varepsilon_3\varphi^6$

Let us start from a description of the unperturbed interface $\varphi_0(z)$. The evolution equation has the form (4) with V replaced by V_0 given by formula (2). The interface solution is well-known, see, *e.g.*, [4]. Let us quote the relevant formulas from [4] and [6]. We use the following abbreviations

$$s = \frac{z}{4l_0}, \qquad a_+ = -\frac{1}{2\sqrt{2}l_0},$$

where

$$l_0{}^{-1} = \frac{1}{2\sqrt{2}}\left(3B + \sqrt{9B^2 - 32A}\right). \tag{32}$$

Then

$$\varphi_0(z) = \frac{a_+}{2}\left(1 + \tanh s\right), \tag{33}$$

and

$$\begin{aligned}
\psi_r(z) &= \frac{a_+}{8l_0}\frac{1}{\cosh^2 s}, \\
\psi_2(z) &= \frac{2l_0{}^2}{a_+}\frac{1}{\cosh^2 s}\int_0^s d\sigma \exp[-2(2\alpha+1)\sigma]\left[1+\exp(2\sigma)\right]^4.
\end{aligned}$$

The integral over σ can be easily calculated, but the resulting formula for ψ_2 is quite long. In the absence of perturbations, $z = x^3 - v_0 t$, where

$$v_0 = \frac{3}{4\sqrt{2}}\left(B - \sqrt{9B^2 - 32A}\right). \tag{34}$$

The interface φ_0 exists and it is stable when $0 < A < \frac{9B^2}{32}$. Usually, the coefficient B is regarded as independent of temperature, while $A = a(T - T_*)$, where $a > 0$. The inequality given above fixes the temperature range (T_*, T_c) in which the interface exists. Notice that at the temperature T_0 such that

$A = \frac{B^2}{4}$ the velocity v_0 vanishes. It is clear that $T_* < T_0 < T_c$. The left zero-mode has the form

$$\psi_l(z) = \frac{a_+}{8l_0} \exp\left[2(2\alpha - 1)s\right] \frac{1}{\cosh^2 s}\,, \tag{35}$$

where

$$\alpha = v_0 l_0 + \tfrac{1}{2}\,.$$

The value of the parameter α monotonically increases from 0 to 1 when A varies from 0 to $\frac{9B^2}{32}$.

The first order correction v_1 to the velocity is calculated from formula (22). In the present case

$$V_1'(\varphi_0) = -\varepsilon_1 \varphi_0 {\psi_r}^2 - \varepsilon_1 {\varphi_0}^2 \partial_z \psi_r + \varepsilon_2 {\varphi_0}^4 + \varepsilon_3 {\varphi_0}^5\,. \tag{36}$$

The integrations over z (or equivalently over s) can be related to the Euler B function [9] ,

$$B(x,y) = \int_0^\infty dt \frac{t^{x-1}}{(1+t)^{x+y}}\,,$$

by the substitution $y = \exp s$. The resulting expressions can be simplified with the help of the well-known formulas

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}\,, \qquad \Gamma(x+1) = x\Gamma(x)\,.$$

After straightforward calculations we obtain

$$v_1 = \frac{(\alpha+1)(2\alpha+1)}{160{l_0}^3} \left[\left(\alpha - \frac{1}{2}\right)\varepsilon_1 - \frac{l_0}{\sqrt{2}} \frac{2\alpha+3}{1-\alpha}\varepsilon_2 + \frac{(2\alpha+3)(\alpha+2)}{12(1-\alpha)}\varepsilon_3\right]. \tag{37}$$

Formula (37) shows that the $\varepsilon_2, \varepsilon_3$ corrections become more and more pronounced as α increases towards 1, that is as the temperature increases towards T_c. The singularity at $\alpha = 1$ is due to the fact that for $\alpha = 1$ the left zero-mode ψ_l and $V_1'(\varphi_0)$ do not vanish when $s \to +\infty$, and therefore the integral in formula (22) is divergent.

The ε_1 term in formula (37) vanishes for $\alpha = \frac{1}{2}$. The reason is that the corresponding term in the perturbation potential V_1 does not contribute to the values of the full potential V at the two minima $\varphi_\pm$. Therefore, it can influence the velocity only by changing the denominator in formula (8). For $\alpha = \frac{1}{2}$ the numerator in (8) vanishes if $\varepsilon_2 = \varepsilon_3 = 0$ and the ε_1 term can not contribute to the velocity.

The facts that ε_2 term in (37) is negative, and that the ε_3 term is positive, can be explained by the observation that the ε_2 term in the potential diminishes the potential energy difference across the interface, while the ε_3 term in the potential works in the opposite direction.

4. Summary and discussion

1. The purpose of this work is to provide a practical tool for analytic, approximate computations of characteristics of the perturbed planar interfaces within the framework of Ginzburg–Landau models. We have shown how one can systematically calculate corrections to the profile and the velocity of the interface. In particular, the scheme can be applied to perturbations of domain walls, which can be regarded as special, static interfaces appearing when the two minima of V_0 are degenerate. In the example discussed in Section 3 this is the case when $\alpha = 1/2$. The scheme is relatively simple. The final formulas (22), (23), (28), (29) contain one dimensional integrals which always can be tackled numerically, and in many cases calculated analytically.
2. We have not delved into the problem of convergence of the perturbative series. Such a mathematical investigation does not belong to the scope of our work. It is clear that if the perturbations shift the two minima of V_0 only slightly, the corresponding changes of the profile and of the velocity of the interface are also small, hence they can be calculated as small corrections. This does not necessarily mean that the series is convergent — it can belong to a wider class of asymptotic series, but it is acceptable from physical viewpoint.
3. Using the results of the present paper in combination with the curvature expansion mentioned in the Introduction, one can calculate evolution of curved interfaces in the perturbed Ginzburg–Landau models. Comparisons of such theoretical predictions with experimental observations of evolution of the interfaces could help to determine the best formula for the free energy of the system. Let us remind that the formulas for the free energy in the Ginzburg–Landau models rarely can be derived from underlying microscopic theories. In most cases they are postulated on basis of qualitative phenomenological considerations. General discussion of various formulas for the free energy in the context of theory of liquid crystals can be found in [10].

Appendix

Resummation of the perturbative contributions to the velocity

The perturbative corrections v_k to the velocity of the interface are given by formulas (22), (23). They have been obtained in somewhat indirect way, namely from the integrability conditions. On the other hand, the velocity is given by the exact formula (8). We would like to check that the sum (15) of the perturbative contributions v_k, below denoted by $\tilde{v}$,

$$\tilde{v} \stackrel{df}{=} v_0 + \varepsilon v_1 + \sum_{k=2}^{\infty} \varepsilon^k v_k \,, \tag{38}$$

does coincide with the velocity v given by formula (8). Actually, because formula (8) follows directly from Eq. (9), it is sufficient to check that $\tilde{v}$ coincides with the v present in that equation.

In the first step, we substitute for v_0, v_1 and v_k ($k \geq 2$) in (38) formulas (14), (22) and (23), respectively. Next, we notice that the definition of h_k given below formula (19) is equivalent to the following formula

$$\sum_{k=2}^{\infty} \varepsilon^k h_k = V_0'(\varphi) + \varepsilon V_1'(\varphi, \partial_z \varphi) - V_0'(\varphi_0) - \varepsilon V_1'(\varphi_0, \partial_z \varphi_0) - V_0''(\varphi_0)(\varphi - \varphi_0) \,,$$

where

$$\begin{aligned} V_0'(\varphi) + \varepsilon V_1'(\varphi, \partial_z \varphi) &= \partial_z{}^2 \varphi + v \partial_z \varphi \,, \\ V_0'(\varphi_0) &= \partial_z \psi_r + v_0 \psi_r \,, \end{aligned}$$

and

$$V_0''(\varphi_0)\varphi_0 = \partial_z \psi_r + v_0 \psi_r - \hat{L}\varphi_0 \,,$$

according to Eqs. (9), (13) and definition (17). We also use the formula

$$\sum_{k=2}^{\infty} \varepsilon^k \sum_{j=1}^{k-1} v_j \partial_z \varphi_{k-j} = \left(\sum_{i=1}^{\infty} \varepsilon^i v_i \right) \left(\sum_{j=1}^{\infty} \varepsilon^j \partial_z \varphi_j \right) = (\tilde{v} - v_0) \partial_z (\varphi - \varphi_0) \,.$$

After a simple calculation we obtain from formula (38) that

$$(v - \tilde{v}) \int_{-\infty}^{+\infty} dz \psi_l \partial_z \varphi = 0 \,. \tag{39}$$

Thus, indeed

$$v = \tilde{v} \,,$$

provided that

$$\int_{-\infty}^{+\infty} dz \psi_l \partial_z \varphi \neq 0 \,.$$

Because

$$\partial_z \varphi = \psi_r + \varepsilon \partial_z \varphi_1 + \cdots \,,$$

that last condition is satisfied if ε is small enough, that is, if the perturbation is not too strong.

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