ON TWO RESUMMATION SCHEMES OF A HOT SCALAR FIELD MODEL

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Considering a charged scalar massless quantum field model with global gauge symmetry U(1), at finite temperature T, we analyze the light cone singular structure of the photonic two-point function, as derived in two different resummation schemes of leading thermal effects.

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1. Introduction

Because of an inherent non perturbative character, quantum field theories at high temperature require that original perturbative expansions be reorganized into effective perturbation theories. The so called *Resummation Program* (RP) [1] achieves this task, while encountering serious obstructions in the infrared (IR) sectors of hot quantum fields [2]. Infrared diverging results have been found begging the question of the reorganization of renormalized perturbative series. In some recent publications it has been stressed that the necessary resummation of leading thermal fluctuations could be consistently carried out, a perturbative way, with, in the end, very different ensuing IR sectors [3]. In this article, pursuing along this line of thinking, the collinear singularity problem of hot QCD is addressed [4], though, as a first step, through a much simpler charged scalar field version.

At $D = 6 - 2\varepsilon$ space-time dimensions, the scalar field self cubic interaction $g_0\varphi^3$ is renormalizable and asymptotically free: Having subtracted the ultraviolet poles at the renormalization mass scale λ, and introducing the new dimensionless coupling constant $g = g_0\lambda^{\varepsilon}$, the zero temperature Feynman rules can be given. At finite non zero temperature T, though, the model does not display the usual richness of gauge theories. In particular, no Hard Thermal Loops (HTL) are known to show up in the proper vertex functions [5], and thermal effective perturbation theory is accordingly determined by effective propagators only. In the R/A real time formalism [6] the latter read

$$^{\star}\Delta_{\alpha}(P) = \frac{i}{P^2 - \Sigma_{\alpha}(p_0, p) + i\epsilon_{\alpha} p_0}\,, \quad \alpha = R, A\,. \tag{1.1}$$

The function $\Sigma_{\alpha}(P)$ is the HTL self energy to be dealt with later on. In the course of practical calculations, this formalism gets completed by further conventions like, for example, $\epsilon_R = +1$, $\epsilon_A = -1$. Also, $\epsilon_{\overline{\alpha}} = -\epsilon_{\alpha}$. Some changes allow to raise the previous scalar field model up to the status of a gauge theory, endowed with a global U(1) symmetry. These changes are easily read from the Lagrangian density

$$\mathcal{L}(x) = \partial_{\mu}\varphi^* \partial^{\mu}\varphi + \frac{1}{2}\partial_{\mu}\phi^* \partial^{\mu}\phi + g_0 \phi\varphi\varphi^* + ie\varphi^* \overset{\leftrightarrow}{\partial_{\mu}} \varphi A^{\mu} \tag{1.2}$$

that is, the original Hermitian scalar field gets differentiated into a charged "scalar quark" field $\varphi(x)$, and a neutral "scalar gluonic" one $\phi(x)$, whereas the free photonic part of $\mathcal{L}(x)$ is omitted for short. Eventually, the charged scalar quark field interacts with the photon field A_{μ}, with coupling strength e. The bare "quark-gluon" vertex, $V^{(0)}$, and bare electromagnetic "quark" vertex, $\Gamma^{(0)}_{\mu}$ are given by

$$V^{(0)}(P_{\alpha}, P'_{\beta}, P"_{\delta}) = -ig\,, \quad \Gamma^{(0)}_{\mu}(P_{\alpha}, Q_{\beta}, P'_{\delta}) = -ie(p_{\mu} + p'_{\mu})\,, \tag{1.3}$$

where μ is the photon field Lorentz index, and P, P' the 6-momenta of external quark legs. For both zero and non zero temperature formalisms, bare R/A propagators can be read from (1.1) by omitting the self energy functions $\Sigma_{\alpha}(P)$. One may remark that by "minimally coupling", the pure scalar model could have been promoted to the status of a local, rather than global, U(1) gauge theory. The additional complexity, for both bare and effective perturbation theories, had brought the model in closer analogy with QCD. These extra structures, though, turn out to be unnecessary to our concern.

The article is organized as follows. The two elements that are necessary to define the required parts of both resummation schemes are the HTL scalar quark self energy and electromagnetic vertex: They are derived in Section 2. In Section 3, taking advantage of a powerful HTL-self energy representation, these elements are used to derive the (very) soft real photon proper two point function, within the customary *Resummation Program* (RP), and within a so-called *Perturbative Resummation* scheme (PR) of leading thermal fluctuations. Results are compared, while in order to alleviate the main text, a technical proof of that section is deferred to the Appendix. A short discussion of our results is presented in Section 4.

2. Elements of HTL-resummation schemes

In this section, we deal with the only quark self energy and electromagnetic vertex, evaluated at leading HTL order. The scalar quark field self energy is given by

$$\Sigma_\alpha(P) = -2ig^2 \int \frac{d^6K}{(2\pi)^5} \epsilon(k_0)\delta(K^2)[1+2n(k_0)]\Delta_\alpha(K+P)\,, \tag{2.1}$$

where $\epsilon(x)$ is the distribution "sign of x", the factor of 2, a factor of symmetry, and $n(k_0)$, the Bose Einstein statistical distribution defined without absolute value prescription [6], which makes the combination $\epsilon(k_0)[1+2n(k_0)]$ an even function of k_0. At $P^2=0$ thermal part of (2.1) is identically zero and in the kinematical regime $P^2 \ll p^2 \ll T^2$ one has [7]

$$\Sigma_\alpha^{\mathrm{HTL}}(P) = m^2 \frac{P^2}{p^2}\left(1 - \frac{p_0}{2p}\left(\ln\left|\frac{p_0+p}{p_0-p}\right| - i\pi\epsilon(\epsilon_\alpha)\Theta(-P^2)\right)\right)\,, \quad m^2 = \frac{g^2T^2}{48\pi} \tag{2.2}$$

it therefore entails a HTL piece of leading order g^2T^2 which makes it necessary to use (1.1) as the scalar quark effective propagator, whenever p_μ is soft, that is, of order gT. Now, the modifications adopted above for scalar fields bring no change for the three point function which remains free of HTL counterpart. For the quark electromagnetic vertex the situation is different. At zeroth order in g the latter satisfies an obvious tree level Ward identity

$$q^\mu \Gamma_\mu^{(0)}(P_\alpha, Q_\beta, P'_\delta) = e\left(\Delta^{(0)}(P')\right)^{-1} - e\left(\Delta^{(0)}(P)\right)^{-1} = -ie\; 2Q\cdot P\,, \tag{2.3}$$

where, on the right hand side, R/A indices are irrelevant. Identity (2.3) keeps being satisfied at renormalized pure one-loop order, thermal contributions included, and one must have

$$q^\mu \Gamma_\mu^{(1)}(P_\alpha, Q_\beta, -P'_\delta) = ie\left(\Sigma_\delta(-P') - \Sigma_\alpha(P)\right) = ie\left(\Sigma_{\overline{\delta}}(P') - \Sigma_\alpha(P)\right)\,, \tag{2.4}$$

where properties specific to massless fields have been used [6]. Focusing on thermal contributions (2.2) only, one gets, considering orders of magnitudes of (2.4), estimated over soft momenta P, P'

$$\mathcal{O}(q_\mu)\times\mathcal{O}(\Gamma_\mu^{(1)}) = \mathcal{O}(P'^2 - P^2)\times\mathcal{O}(1) = \mathcal{O}(q_\mu)\times\mathcal{O}(p_\mu)\times\mathcal{O}(1)\,. \tag{2.5}$$

This indicates the possibility for $\Gamma_\mu^{(1)}$ of being of the same order of magnitude as p_μ, that is, of $\Gamma_\mu^{(0)}$ itself. Of course, *Power Counting* is not sufficient to conclude that the electromagnetic quark vertex entails such a HTL piece, and one has to recourse to actual calculations: Within the R/A formalism standard notations one may write the one-loop vertex correction as the sum of three terms,

$$\Gamma^{(1)}_\mu(P_\alpha, Q_\beta, -P'_\delta) = (A_\mu + B_\mu + C_\mu)\,(P_\alpha, Q_\beta, -P'_\delta) \tag{2.6}$$

with

$$\begin{aligned} A_\mu &= 2ieg^2 \int \frac{d^6K}{(2\pi)^5}\,[2k_\mu]\epsilon(k_0)\left(\frac{1}{2}+n(k_0)\right)\delta(K^2)\,\{\Delta_\alpha(P+K)\Delta_{\overline{\delta}}(K+P') \\ &\quad +\Delta_\beta(Q+K)\Delta_{\overline{\alpha}}(K-P) + \Delta_{\overline{\beta}}(K-Q)\Delta_\delta(K-P')\}\,, \qquad (2.7) \\ B_\mu &= 2ieg^2[p_\mu + p'_\mu] \int \frac{d^6K}{(2\pi)^5}\,\epsilon(k_0)\left(\frac{1}{2}+n(k_0)\right)\delta(K^2) \\ &\quad \times\{\Delta_\alpha(P+K)\Delta_{\overline{\delta}}(K+P')\}\,, \qquad (2.8) \\ C_\mu &= 2ieg^2[q_\mu] \int \frac{d^6K}{(2\pi)^5}\epsilon(k_0)\left(\frac{1}{2}+n(k_0)\right)\delta(K^2) \\ &\quad \times\{\Delta_\beta(Q+K)\Delta_{\overline{\alpha}}(K-P) - \Delta_{\overline{\beta}}(K-Q)\Delta_\delta(K-P')\}\,. \qquad (2.9) \end{aligned}$$

Whereas it is easy to check that Ward identity (2.4) is satisfied by the last four equations, rather specific features come about when the HTL approximation is taken. Indeed, it is tempting to believe that the form of most HTL vertices is universal and dictated by tree like Ward identities. In QCD the effective quark–photon vertex reads, with $\widehat{K} = (1, \widehat{k})$ and $\widehat{K}^2 = 0$,

$$\Gamma^{(1)}_\mu(P_\alpha, Q_\beta, P'_\delta) = iem^2_{\mathrm{QCD}} \int \frac{d\widehat{K}}{4\pi} \frac{\widehat{k}_\mu\, \not{\widehat{K}}}{(\widehat{K}\cdot P + i\epsilon_\alpha)(\widehat{K}\cdot P + i\epsilon_\delta)}\,. \tag{2.10}$$

Considering that in our present situation the role of Dirac matrices γ_μ is played by the dimensionful vectorial coupling $2k_\mu + p_\mu + p'_\mu$, one would get for $\Gamma^{(1)}_\mu(P_\alpha, Q_\beta, -P'_\delta)$ an expression like

$$\begin{aligned} &iem^2 \int d\widehat{K}\widehat{k}_\mu \frac{\widehat{k}^\lambda(2k_\lambda + p_\lambda + p'_\lambda)}{(\widehat{K}\cdot P + i\epsilon_\alpha)(\widehat{K}\cdot P' + i\epsilon_{\overline{\delta}})} \\ &= iem^2 \int d\widehat{K} \frac{\widehat{k}_\mu\ \widehat{K}\cdot(P+P')}{(\widehat{K}\cdot P + i\epsilon_\alpha)(\widehat{K}\cdot P' + i\epsilon_{\overline{\delta}})} \qquad (2.11) \end{aligned}$$

which over soft values of P, P' is effectively of the same order of magnitude as $\Gamma^{(0)}_\mu$, the bare electromagnetic vertex, but on the other hand, is hardly seen to obey (2.4). Indeed, in an actual calculation of (2.6), it is convenient to decompose the R/A propagators into the so called "Landau damping" and "Particle production" terms [8], writing

$$\begin{aligned}\Delta_\alpha(K+P) &= \frac{1}{2|\vec{k}+\vec{p}|}\\ &\quad\times\left(\frac{i}{k+p_0-|\vec{k}+\vec{p}|+i\epsilon_\alpha}-\frac{i}{k+p_0+|\vec{k}+\vec{p}|+i\epsilon_\alpha}\right)\\ &\overset{\text{HTL}}{=} \frac{1}{2k}\left(\frac{i}{\widehat{K}\cdot P+i\epsilon_\alpha}-\frac{i}{2k+i\epsilon_\alpha}\right), \qquad (2.12)\end{aligned}$$

where the second equality holds in the HTL sense only. A simple *Power Counting* argument then shows that, contrary to the cases of QED and QCD for which only the first type of terms matters, a mixing of both "Landau damping" and "Particle production" terms is here required to yield a proper HTL behavior, to wit,

$$\begin{aligned}\Delta_\alpha(P+K)\Delta_{\overline{\delta}}(K+P') &\mapsto \left(\frac{1}{2k}\right)^2\\ &\quad\times\left(\frac{i}{\widehat{K}\cdot P+i\epsilon_\alpha}\frac{-i}{2k+i\epsilon_{\overline{\delta}}}+\frac{-i}{2k+i\epsilon_\alpha}\frac{i}{\widehat{K}\cdot P'+i\epsilon_{\overline{\delta}}}\right)\\ &= \left(\frac{1}{2k}\right)^3\left(\frac{\widehat{K}\cdot(P+P')}{(\widehat{K}\cdot P+i\epsilon_\alpha)(\widehat{K}\cdot P'+i\epsilon_{\overline{\delta}})}\right). \qquad (2.13)\end{aligned}$$

Calculations are hereafter straightforward and yield a full cancellation of type (2.11), (2.13)-terms :

$$A_\mu^{\text{HTL}}(P_\alpha, Q_\beta, -P'_\delta) = 0\,. \qquad (2.14)$$

The only HTL pieces are those of B_μ and C_μ, respectively, given by

$$B_\mu^{\text{HTL}}(P_\alpha, Q_\beta, -P'_\delta) = -ie[p_\mu+p'_\mu]\, m^2\int\frac{d\widehat{K}}{8\pi^2}\frac{1}{(\widehat{K}\cdot P+i\epsilon_\alpha)(\widehat{K}\cdot P'+i\epsilon_{\overline{\delta}})}, \qquad (2.15)$$

$$C_\mu^{\text{HTL}}(P_\alpha, Q_\beta, -P'_\delta) = +ie[q_\mu]\, m^2\int\frac{d\widehat{K}}{8\pi^2}\frac{1}{(\widehat{K}\cdot P+i\epsilon_\alpha)(\widehat{K}\cdot P'+i\epsilon_{\overline{\delta}})} \qquad (2.16)$$

and the effective electromagnetic vertex to be treated on the same footing as (1.3) reads, therefore,

$$\Gamma_\mu^{(1)}(P_\alpha, Q_\beta, -P'_\delta) = -ie\,[p_\mu+p'_\mu-q_\mu]\, m^2\int\frac{d\widehat{K}}{8\pi^2}\frac{1}{(\widehat{K}\cdot P+i\epsilon_\alpha)(\widehat{K}\cdot P'+i\epsilon_{\overline{\delta}})}\,. \qquad (2.17)$$

This rather peculiar result is, of course, inherent to the model under consideration, with its dimensionful vectorial "quark-photon" coupling [9]. Likewise, one may observe that, contrary to QED and QCD, the phase space factor of $8\pi^2$ is not, and cannot be (in view of (2.2), (2.4) and (2.17)), the total solid angle of the model, which is only one third of it. As the diagrams, we will be interested in shortly, do not involve higher points HTL vertices, they will not be considered in the sequel. Now, in an emission rate calculation, where both HTL self energies and effective vertices are involved, it is crucial that Ward identity (2.4) be satisfied. One has, however,

$$q^\mu \Gamma^{(1)}_\mu(P_\alpha, Q_\beta, P'_\delta) = -ie\, m^2[P'^2 - P^2]\int \frac{d\widehat{K}}{8\pi^2}\frac{1}{(\widehat{K}\cdot P + i\epsilon_\alpha)(\widehat{K}\cdot P' + i\epsilon_\delta)}\,, \tag{2.18}$$

that is (2.4) and (2.18) do not coincide. This is not surprising since we are dealing with HTL approximated forms which, in particular, satisfy (2.14). A remnant of the original symmetry is recovered, though, in the limit of a calculation involving only very soft photons satisfying the condition $q_\mu/p_\mu \ll 1$. For example, by taking q_μ of order g^2T, one would mimic the coupling mediated by the QCD effective vertices, of the very soft scale fluctuations of the order of g^2T, to the soft ones, of the order of gT [10]. Over these very soft photon field configurations, one has, at least in a formal sense

$$\begin{aligned} \Sigma^{\mathrm{HTL}}_\alpha(P') &= \Sigma^{\mathrm{HTL}}_\alpha(P)\left(1 + \mathcal{O}\left(\frac{q}{p}\right)\right)\,, \\ m^2\int \frac{d\widehat{K}}{8\pi^2}\frac{1}{(\widehat{K}\cdot P + i\epsilon_\alpha)(\widehat{K}\cdot P' + i\epsilon_\alpha)} &= -\frac{\Sigma^{\mathrm{HTL}}_\alpha(P)}{P^2}\left(1 + \mathcal{O}\left(\frac{q}{p}\right)\right), \end{aligned} \tag{2.19}$$

where for the second equality, anticipating the next section, both P and P' internal lines bear the same R/A index, ϵ_α, thanks to the massless character of the involved scalar quark fields. Up to corrections of relative order q/p that form of (2.4), which is relevant to the next calculations, is eventually preserved.

3. A comparison of PR and RP calculations

We are now in a position to calculate the (very) soft real photon emission rate from the plasma, relying on effective propagators (1.1) and effective vertices

$$\Gamma^{\mathrm{eff}}_\mu(P_\alpha, Q_\beta, -P'_\delta) = \Gamma^{(0)}_\mu(P_\alpha, Q_\beta, -P'_\delta) + \Gamma^{(1)}_\mu(P_\alpha, Q_\beta, -P'_\delta)\,. \tag{3.1}$$

For the sake of later purpose it is instructive to recall the basic steps entering the soft photon emission rate calculation of thermal QCD. In the R/A

formalism being used, this result enjoys a simple and systematic derivation, which we follow here [8]. In view of (3.1) one gets three types of terms: A term with two bare vertices $\Gamma_\mu^{(0)}$, two terms with one bare vertex $\Gamma_\mu^{(0)}$ and the other $\Gamma_\mu^{(1)}$, and a term with two vertices $\Gamma_\mu^{(1)}$. In QCD the first three terms pose no problem: Terms of second type entail a collinear singularity that cancels out with a similar singularity coming from the last term. A residual collinear singularity remains, though induced by the latter, and we will, therefore, focus on that particular contribution including two vertices $\Gamma_\mu^{(1)}$. One gets within standard notations (Feynman gauge)

$$\begin{aligned}\Pi_R(Q) \;=\; & i\int \frac{d^4P}{(2\pi)^4}(1-2n_F(p_0))\, g^{\mu\nu}\,\mathrm{disc}\,\mathrm{Tr}\Big\{{}^\star S_R(P)\Gamma^{(1)}{}_\mu(P_R,Q_R,-P'_A) \\ & \times{}^\star S_R(P')\Gamma^{(1)}{}_\nu(P_R,Q_R,-P'_A)\Big\}\,. \end{aligned}\tag{3.2}$$

Substituting the relevant QCD expressions, one can write, with the convention $\epsilon_R = +\epsilon$,

$$\begin{aligned}\Pi_R(Q) \;=\; & -ie^2 m^4_{\mathrm{QCD}}\int \frac{d^4P}{(2\pi)^4}(1-2n_F(p_0))\int\frac{d\widehat{K}}{4\pi}\int\frac{d\widehat{K}'}{4\pi} \\ & \times\mathrm{disc}\,\frac{\widehat{K}\cdot\widehat{K}'\,\mathrm{Tr}\left({}^\star S_R(P)\widehat{\not K}\,{}^\star S_R(P')\widehat{\not K}'\right)}{(\widehat{K}\cdot P+i\epsilon)(\widehat{K}\cdot P'+i\epsilon)(\widehat{K}'\cdot P+i\epsilon)(\widehat{K}'\cdot P'+i\epsilon)}\,. \end{aligned}\tag{3.3}$$

Because of the factor $\widehat{K}\cdot\widehat{K}'$ appearing in the numerator there is no double pole but a simple collinear one at $\widehat{K} = \widehat{Q}$ whose residue just involves the Ward identity equivalent to (2.4), that is

$$m^2_{\mathrm{QCD}}\int\frac{d\widehat{K}'}{4\pi}\,\frac{[\widehat{Q}\cdot\widehat{K}']\,\widehat{\not K}'}{(\widehat{K}'\cdot P+i\epsilon)(\widehat{K}'\cdot P'+i\epsilon)} = \frac{1}{q}[\Sigma_R(P)-\Sigma_R(P')] \tag{3.4}$$

and yields for $\Pi_R(Q)$ the expression

$$\begin{aligned} & -i\frac{e^2}{q}\int\frac{d^4P}{(2\pi)^4}(1\;\;-2n_F(p_0))\,\mathrm{disc}\,m^2_{\mathrm{QCD}}\int\frac{d\widehat{K}}{4\pi}\frac{1}{(\widehat{K}\cdot P+i\epsilon)(\widehat{K}\cdot P'+i\epsilon)} \\ & \times\mathrm{Tr}\left({}^\star S_R(P)\widehat{\not Q}\,{}^\star S_R(P')[\Sigma_R(P)-\Sigma_R(P')]\right)\,. \end{aligned}\tag{3.5}$$

The discontinuity in p_0 can be taken by forming the difference of R and A-indiced P-dependent quantities. Then, an appropriate choice of the integration contour in the p_0-complex plane allows to write

$$
\begin{aligned}
\Pi_R(Q) \;=\; & -2\frac{e^2 m^2_{\rm QCD}}{q}\int\frac{d^4P}{(2\pi)^3}(1-2n_F(p_0))\int\frac{d\widehat{K}}{4\pi}\frac{\delta(\widehat{K}\cdot P)}{\widehat{K}\cdot Q+i\epsilon} \\
& \times{\rm Tr}\left({}^\star S_A(P)\widehat{\not Q}{}^\star S_R(P')\left[\Sigma_A(P)-\Sigma_R(P')\right]\right), \qquad (3.6)
\end{aligned}
$$

where a factor of 2 accounts for the two possibilities $\widehat{K}=\widehat{Q}$ and $\widehat{K}'=\widehat{Q}$, and where the relation $P'=P+Q$ has been used. The angular integration develops a collinear singularity at $\widehat{K}=\widehat{Q}$, and is responsible for that singular part of $\Pi_R(Q)$ which can be expressed as

$$
\begin{aligned}
& -2\frac{e^2 m^2_{\rm QCD}}{q}\left(\int\frac{d\widehat{K}}{4\pi}\frac{1}{Q\cdot\widehat{K}+i\epsilon}\right)\int\frac{d^4P}{(2\pi)^3}\;\delta(P\cdot\widehat{Q})\;(1-2n_F(p_0)) \\
& \times{\rm Tr}\left({}^\star S_A(P)\widehat{\not Q}{}^\star S_R(P')\;\left[\Sigma_A(P)-\Sigma_R(P')\right]\right). \qquad (3.7)
\end{aligned}
$$

The two terms involving one bare vertex $\Gamma^{(0)}_\mu$ and a one-loop HTL correction $\Gamma^{(1)}_\mu$, entail a similar singularity which, when combined with (3.7), leave uncanceled the $\Pi_R(Q)$ singular contribution

$$
\begin{aligned}
& -2i\frac{e^2 m^2_{\rm QCD}}{q^2}\left(\int\frac{d\widehat{K}}{4\pi}\frac{1}{\widehat{Q}\cdot\widehat{K}+i\epsilon}\right)\int\frac{d^4P}{(2\pi)^3}\;\delta(P\cdot\widehat{Q})\;(1-2n_F(p_0)) \\
& \times\left[{\rm Tr}\left({}^\star S_A(P)\widehat{\not Q}\right)-{\rm Tr}\left({}^\star S_R(P')\widehat{\not Q}\right)\right]. \qquad (3.8)
\end{aligned}
$$

Getting back to our scalar model, we analyze things starting from ordinary Perturbation Theory and keep in mind that the two RP and PR-HTL-resummation schemes just correspond to the two possible sequences along which one performs in loop integrals, the sum over N, the number of HTL self energy insertions, and the integral on p_0, the looping energy [3]. One has at leading order in q/p

$$
\begin{aligned}
\Pi_R(Q) \;\simeq\; & -ie^2\int\frac{d^5p}{(2\pi)^5}\sum_{N,N'=0}^{\infty}\int_{-\infty}^{+\infty}\frac{dp_0}{2\pi}(1+2n(p_0)) \\
& \times{\rm disc}\, m^2\int\frac{d\widehat{K}}{8\pi^2}\frac{1}{(\widehat{K}\cdot P+i\epsilon)(\widehat{K}\cdot P'+i\epsilon)} \\
& \times\Delta_R^{(N)}(P)\Delta_R^{(N')}(P')\left(2\Sigma_R(P)\right), \qquad (3.9)
\end{aligned}
$$

and where $\Delta_R^{(N)}$ is the partial effective propagator obtained by inserting N HTL-self energy functions $\Sigma_R(P)$ along the internal P-line

$$\Delta_R^{(N)}(P) = i\frac{\left(\Sigma_R^{\rm HTL}(P)\right)^N}{(P^2+i\epsilon p_0)^{N+1}}\,. \tag{3.10}$$

Permuting in (3.9) the sum (N,N') and integral (p_0) operations one recovers the sequence corresponding to the emission rate RP calculation. In order to take advantage of important simplifications and make the essential difference of RP and PR structures more transparent it is convenient to rely on the same order of approximation as used so far, (2.19), and write

$$\Delta_R^{(N)}(P)\Delta_R^{(N')}(P') = \frac{i}{P^2+i\epsilon p_0}\Delta_R^{(N+N')}(P)\left(1+\mathcal{O}\left(\frac{q}{p}\right)\right). \tag{3.11}$$

It is worth stressing that relying on (3.11) is in no way compulsory to make the point we are interested in, but amounts to simplifying calculations that, otherwise, would become extremely cumbersome, as can be read off the Appendix of [11]. One gets then, [3],

$$\begin{aligned}\Pi_R(Q) &\simeq -ie^2\sum_{N+N'=0}^{\infty}\int\frac{d^5p}{(2\pi)^5}\int_{-\infty}^{+\infty}\frac{dp_0}{2\pi}(1+2n(p_0))\,{\rm disc}\Bigg\{-\frac{\Sigma_R(P)}{P^2}\\ &\quad\times\frac{i}{P^2+i\epsilon p_0}\Delta_R^{(N+N')}(P)\,2\Sigma_R(P)\Bigg\}\,,\end{aligned} \tag{3.12}$$

where (2.19) has been used. Equation (3.12) can be written as

$$\begin{aligned}\Pi_R(Q) &\simeq -2ie^2\int\frac{d^5p}{(2\pi)^5}\sum_{N=0}^{\infty}\int_{-\infty}^{+\infty}\frac{dp_0}{2\pi}(1+2n(p_0))\\ &\quad\times{\rm disc}\left(-\frac{\Sigma_R(P)}{P^2}\,i\Delta_R^{(N+1)}(P)\right)\,,\end{aligned} \tag{3.13}$$

where (2.19), whose discontinuity we have just seen to be crucial in obtaining the factored out singular integral of (3.8) along the QCD-RP sequence, is singled out. Thus, in a PR scheme, $\Pi_R(Q)$ involves the series with general term

$$\int_{-\infty}^{+\infty}\frac{dp_0}{2\pi}(1+2n(p_0)){\rm disc}\left(-\frac{\Sigma_R(P)}{P^2}\;i\Delta_R^{(N+1)}(P)\;\right). \tag{3.14}$$

Because of mass singularities, though, (3.14) is not defined. A particular representation for the HTL self energy function (2.2) can be introduced [3]

$$\Sigma = m^2 \frac{P^2}{p^2} - \frac{m^2}{2} \frac{P^2}{p^2} \frac{p_0}{p} \lim_{\varepsilon=0} \frac{1}{\varepsilon} \left(1 - \left(\frac{p_0 - p}{p_0 + p}\right)^{\varepsilon}\right). \tag{3.15}$$

Thanks to the $(p_0 \mp p)^{\pm\varepsilon}$ factors this representation is able to provide mass/collinear singularities with the same regularization as a dimensional one would operate at $D = 6 + 2\varepsilon$ dimensions and is endowed with most interesting properties, [3,12]: The limit $\varepsilon = 0$ commutes with the sum over N, the integral on p_0, and, as will be illustrated shortly, with the prescription of discontinuity in the variable p_0. In a PR scheme calculation of $\Pi_R(Q)$ one, therefore, calculates the sum

$$\begin{aligned}
&\sum_{N=0}^{\infty} \lim_{\varepsilon=0} \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi}(1 + 2n(p_0))\, \mathrm{disc}\left(-\frac{\Sigma_R(P,\varepsilon)}{P^2}\, i\Delta_R^{(N+1)}(P,\varepsilon)\right) \\
&\quad = \sum_{N=0}^{\infty} \lim_{\varepsilon=0} \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi}(1 + 2n(p_0))\ \mathrm{disc}\left(-i\Delta_R^{(N+2)}(P,\varepsilon)\right),
\end{aligned} \tag{3.16}$$

where $\Delta_R^{(N)}(P,\varepsilon)$ is obtained by substituting in (3.10) the representation (3.15) for the HTL self energy (2.2); and likewise for the effective propagator of (1.1) giving then the expression ${}^\star\Delta_R(P,\varepsilon)$. Each term of (3.16) involves mass singular contributions all of them obeying finite series of arithmetical cancellation patterns

$$\left\{\frac{\varepsilon^k}{(2\varepsilon)^j}\right\} \times \sum_{m=0}^{j} C_j^m (-1)^m\ m^k = 0\,, \quad 1 \le k \le j-1 \tag{3.17}$$

so that $\Pi_R(Q)$ is eventually singularity free. Since the proof of (3.17) is a bit lengthy, we defer it to the Appendix.

The RP sequence ends up with a different *scenario* since inverting the sum and integral operations one has to calculate

$$\begin{aligned}
&\int_{-\infty}^{+\infty} \frac{dp_0}{2\pi}(1 + 2n(p_0))\, \mathrm{disc}\left(-\frac{\Sigma_R(P)}{P^2}\, i \sum_{N=0}^{\infty} \Delta_R^{(N+1)}(P)\right) \\
&= \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi}(1 + 2n(p_0))\, \mathrm{disc}\left(\lim_{\varepsilon=0}\ \left(-\frac{\Sigma_R(P,\varepsilon)}{P^2}\right)\, i \sum_{N=0}^{\infty}\ \lim_{\varepsilon=0} \Delta_R^{(N+1)}(P,\varepsilon)\right)
\end{aligned}$$

$$= \int\limits_{-\infty}^{+\infty} \frac{dp_0}{2\pi}(1+2n(p_0))$$

$$\times \operatorname{disc}\left(\lim_{\varepsilon=0}\left(-\frac{\Sigma_R(P,\varepsilon)}{P^2}\right)\, i \lim_{\varepsilon=0}\left[{}^\star\Delta_R(P,\varepsilon) - \Delta_R^{(0)}(P)\right]\right). \tag{3.18}$$

With the remaining integration over p, one gets for $\Pi_R(Q)$ the expression

$$-4ie^2 \int \frac{d^5p}{(2\pi)^5} \int\limits_{-\infty}^{+\infty} \frac{dp_0}{2\pi}(1+2n(p_0))$$

$$\times \operatorname{disc}\left(\lim_{\varepsilon=0}\left(-\frac{\Sigma_R(P,\varepsilon)}{P^2}\right)\, i \lim_{\varepsilon=0}\left[{}^\star\Delta_R(P,\varepsilon) - \Delta_R^{(0)}(P)\right]\right). \tag{3.19}$$

Then, we take the same calculational step as taken along the RP sequence of QCD passing from (3.5) to (3.6) and single out the contribution attached to the discontinuity of (2.19) which reads now

$$\operatorname{disc}\left(-\frac{\Sigma_R(P,\varepsilon)}{P^2}\right) = -i\Theta(-P^2)\frac{\sin(\pi\varepsilon)}{\varepsilon}\frac{m^2}{p^2}\frac{p_0}{p}\left|\frac{p_0-p}{p_0+p}\right|^{\varepsilon}. \tag{3.20}$$

In (3.20) the previously alluded commutativity of the $\varepsilon = 0$ limit with the discontinuity prescription is obvious since the limit at $\varepsilon = 0$ just reproduces the discontinuity of the (retarded) self energy function (2.2) divided by P^2. Letting aside for a while the contribution to (3.18) which is attached to ${}^\star\Delta_R(P,\varepsilon)$, we focus on that part of $\Pi_R(Q)$ which is due to the *Principal Part* component of the bare propagator $\Delta_R^{(0)}(P)$. It is

$$-4ie^2\,\frac{\sin(\pi\varepsilon)}{\varepsilon}\int\frac{d^5p}{(2\pi)^5}\frac{m^2}{p^2}\int\limits_0^{\infty}\frac{dp_0}{\pi}(1+2n(p_0))\,\frac{\boldsymbol{P}}{P^2}\operatorname{disc}\left(-\frac{\Sigma_R(P,\varepsilon)}{P^2}\right)$$

$$= +4e^2\,\frac{\sin(\pi\varepsilon)}{\pi\varepsilon}\int\frac{d^5p}{(2\pi)^5}\frac{m^2}{p^2}\int\limits_0^{p} dp_0\,(1+2n(p_0))\,\frac{p_0}{p}\,\frac{\left(\frac{p-p_0}{p+p_0}\right)^{\varepsilon}}{(p-p_0)(p+p_0)}, \tag{3.21}$$

where in the second line the *Principal Part* prescription can be given up in view of the ε-regularization supplied by (3.15): Clearly, a mass singularity develops at the light cone boundary $p_0 = p$, which eventually plagues the RP calculation of $\Pi_R(Q)$. That is, following the customary calculational steps of the *Resummation Program*, a divergent result is obtained due to some "residual" mass singularity. Now, the above comparison of the two

PR and RP calculations shed some interesting new light on both the origin and spurious character of this singularity. Proceeding along the RP sequence, the sum over N is performed first resulting in the effective propagator $[{}^\star\Delta_R(P,\varepsilon) - \Delta_R^{(0)}(P)]$ of (3.18). By the same token though, the arithmetical cancellation patterns (3.17) of mass singularities which along the PR sequence take place at all partial effective propagator $\Delta_R^{(N+2)}(P,\varepsilon)$ are definitely broken. As could be intuitively expected then [13] a "residual" mass singularity pops out of a given contribution to $\Pi_R(Q)$ and this appears, therefore, as a purely structural *artefact* resulting from the reorganization into the form (1.1) of the effective propagator original perturbative series (3.10).

In a somewhat surprising but remarkable analogy with QCD it is interesting to note that another collinear singularity is induced also by the two terms corresponding to one insertion of one-loop vertex (2.19), the other bare (1.3). Within the same approximations as used so far, and along the RP sequence, these two identical contributions to $\Pi_R(Q)$ effectively add up to

$$
\begin{aligned}
&-2ie^2 \int \frac{d^5p}{(2\pi)^5} \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi}(1+2n(p_0)) \\
&\times \sum_{N+N'=0}^{\infty} \mathrm{disc}\left(\frac{i}{P^2+i\epsilon p_0}\Delta_R^{(N+N')}(P,\varepsilon)2\Sigma_R(P,\varepsilon)\right) \\
= \; &-4ie^2 \int \frac{d^5p}{(2\pi)^5} \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi}(1+2n(p_0)) \sum_{N=0}^{\infty} \mathrm{disc}\left(\frac{\Sigma_R(P,\varepsilon)}{P^2} i\Delta_R^{(N)}(P,\varepsilon)\right).
\end{aligned}
\tag{3.22}
$$

Summing over N, and taking as before the discontinuity of the first term inside the parenthesis of (3.22), yields

$$
-4ie^2 \int \frac{d^5p}{(2\pi)^5} \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi}(1+2n(p_0))\, \mathrm{disc}\left(\frac{\Sigma_R(P,\varepsilon)}{P^2}\right) i[{}^\star\Delta_R(P,\varepsilon)]\,. \tag{3.23}
$$

Though, involving a residue different from the one in (3.21), a mass singularity can be shown to develop out of (3.23). However, it is readily seen that the whole expression cancels against the same term in (3.19). This compensation is a consequence of identity (2.19), relating, at the order of approximation we are calculating, the effective vertex to the self energy: As mentioned in the introduction of this section and after equation (3.7) the same mechanism is known to take place in thermal QCD as a consequence of the Ward identity which, in HTL approximation, relates self energy and electromagnetic vertex [8].

4. Discussion

On the basis of a simple charged scalar field model and restricting the photon field to very soft modes, $q/p \ll 1$, it is possible to propose a few arguments in favor of a net difference separating in the transposed context of the collinear singularity problem of thermal QCD the *Resummation Program* (RP) from the perturbative resummation scheme (PR) of thermal leading fluctuations.

Within a given approximation level of the order of zero in the small q/p--parameter expansion, the trace of the very soft photon polarization tensor appears as a mass (or collinear) singularity free quantity in the latter scheme, whereas it entails a mass singularity when the usual steps of the RP calculational scheme are taken. Out of this simplified example a possible mechanism responsible for the collinear singularity of QCD may be foreseen: Reorganizing for the propagator the original perturbative series into the effective form used in the RP results in a breaking of the mass singularity arithmetical cancellation patterns that are actual at any order of bare Perturbation Theory. More precisely, replacing $\Delta_R^{(N,N')}(P,P')$ by ${}^\star\Delta_R(P,P')$ would amount to isolating a mass singularity coming from the discontinuity of the (double) effective vertex insertion, from those induced by partial effective propagators $\Delta_R^{(N,N')}(P,P')$ themselves. The former is accordingly singled out with no other singular counterpart to cancel against.

Since we have been dealing with a simple, unphysical model, we do not think worth addressing the reliability issue of an expansion in term of the very soft photon parameter q/p. In full rigour, our results should accordingly be taken at the level of a formal, still encouraging indication that things could come out different in a PR treatment of the QCD situation and, in particular, not so singular. The physical case of QCD is, of course, more difficult to deal with but it is interesting to note that the formal indication provided by the present model gets supported by preliminary QCD results, obtained without approximations [14].

Appendix A

The following proof is constructed out of the one given in [3], Section 4. The general term of the series is

$$\lim_{\varepsilon=0}\int\limits_{-\infty}^{+\infty}\frac{dp_0}{2\pi}(1+2n(p_0))\ \mathrm{disc}\left(-i\Delta_R^{(N+2)}(P,\varepsilon)\right) \tag{A.1}$$

with the discontinuity of (A.1) being given by

$$\begin{aligned}\mathrm{disc}\left(-i\Delta_R^{(N)}(P,\varepsilon)\right) &= -2\frac{(-1)^N}{N!}\pi\varepsilon(p_0)\delta^{(N)}(P^2)\mathcal{R}e\left(\Sigma_R(P,\varepsilon)\right)^N \\ &\quad +2\boldsymbol{P}\frac{\mathcal{I}m\left(\Sigma_R(P,\varepsilon)\right)^N}{(P^2)^{N+1}}\,.\end{aligned} \tag{A.2}$$

Changing the integration variable from p_0 to $x=|P^2|/p^2$, the contribution of the first part of $(A.2)$ is found to be

$$\begin{aligned}&\frac{1}{p}\left(\frac{m^2}{p^2}\right)^N\int\limits_0^1 dx\,\delta(x)\frac{1+2n(p\sqrt{1-x})}{\sqrt{1-x}}\frac{1}{N!}\left(\frac{d}{dx}\right)^N \\ &\times x^N\mathcal{R}e\left\{1-\frac{\sqrt{1-x}}{2\varepsilon}\left(1-\frac{\mathrm{e}^{i\pi\varepsilon}x^\varepsilon}{(1+\sqrt{1-x})^{2\varepsilon}}\right)\right\}^N \\ =\ &\frac{1+2n(p)}{p}\left(\frac{m^2}{p^2}\right)^N\left(1-\frac{1}{2\varepsilon}\right)^N,\end{aligned} \tag{A.3}$$

where the parity of the integrand under the transformation $p_0\to -p_0$ has been used. For the second term of (A.2), one finds

$$\frac{1}{p}\left(\frac{m^2}{p^2}\right)^N\frac{1}{\pi}\int\limits_0^1\frac{dx}{x}\frac{1+2n(p\sqrt{1-x})}{\sqrt{1-x}}\mathcal{I}m\left\{1-\frac{\sqrt{1-x}}{2\varepsilon}\left(1-\frac{\mathrm{e}^{i\pi\varepsilon}x^\varepsilon}{(1+\sqrt{1-x})^{2\varepsilon}}\right)\right\}^N. \tag{A.4}$$

Writing,

$$1+2n(p\sqrt{1-x})=1+2n(p)+[1+2n(p\sqrt{1-x})-1-2n(p)] \tag{A.5}$$

it is easy to see that the singular part of (A.4) is attached to the first term of (A.5), and reads

$$\frac{1+2n(p)}{p}\left(\frac{m^2}{p^2}\right)^N A^N(\varepsilon)A^{(N)}(\varepsilon)$$
$$= \frac{1}{\pi}\sum_{j=1}^{N} C_N^j\left(-\frac{1}{2\varepsilon}\right)^j \sum_{m=1}^{j} C_j^m(-1)^m \sin(\pi m\varepsilon) f_{jm}(\varepsilon)\,, \tag{A.6}$$

where the range $j \geq m \geq 1$ comes from the discontinuity (or imaginary part) prescription, and where the $f_{jm}(\varepsilon)$ are the functions:

$$f_{jm}(\varepsilon) = \frac{1}{m\varepsilon} + \left(\int_0^1 \frac{dx}{x}\frac{\sqrt{1-x}^{\,j-1}}{(1+\sqrt{1-x})^{2m\varepsilon}} - \frac{1}{m\varepsilon}\right). \tag{A.7}$$

In (A.7), the first term, $1/m\varepsilon$, contributes to $A^N(\varepsilon)$ an amount

$$1-\left(1-\frac{1}{2\varepsilon}\right)^N \tag{A.8}$$

which is readily seen to compensate for the singularities of (A.3). Now, the functions

$$\varepsilon \longmapsto g_{jm}(\varepsilon) = \left(\int_0^1 \frac{\mathrm{d}x}{x}\frac{\sqrt{1-x}^{\,j-1}}{(1+\sqrt{1-x})^{2m\varepsilon}} - \frac{1}{m\varepsilon}\right) \tag{A.9}$$

define analytic functions of ε in the half of the complex plane $\mathcal{P}_j = \{\varepsilon|\mathcal{R}e(\varepsilon)\rangle - \frac{1}{j}\}$, where they enjoy the Taylor expansion [3]

$$g_{jm}(\varepsilon) = \sum_{n=0}^{\infty}\frac{m^n\varepsilon^n}{n!}\,b_j^{(n)}. \tag{A.10}$$

The proof of statement $(A.10)$ can be found in the Appendix A of [3]. The coefficients $A^N(\varepsilon)$ read therefore

$$A^{(N)}(\varepsilon) = \frac{1}{\pi}\sum_{j=1}^{N} C_N^j(-1)^j\sum_{m=1}^{j} C_j^m(-1)^m\sum_{p=1}^{\infty}\left(\sum_{n+2k+1=p}(-1)^k\frac{\pi^{2k+1}}{(2k+1)!}\frac{b_j^{(n)}}{n!}\right)$$
$$\times\frac{m^p\varepsilon^p}{(2\varepsilon)^j} \tag{A.11}$$

and in the limit $\varepsilon = 0$, the $(N+1)(N+2)/2$ mass singularities cancel out according to the set of $N(N-1)/2$ arithmetical identities

$$\left\{ \frac{\varepsilon^p}{(2\varepsilon)^j} \right\} \times \sum_{m=0}^{j} C_j^m (-1)^m \, m^p = 0 \,, \quad 1 \le p \le j-1 \tag{A.12}$$

which is (3.17).

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