QUARK DISTRIBUTION FUNCTIONS IN THE CHIRAL QUARK–SOLITON MODEL: CANCELLATION OF QUANTUM ANOMALIES

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In the framework of the chiral quark–soliton model of the nucleon we investigate the properties of the polarized quark distribution. In particular we analyse the so called anomalous difference between the representations for the quark distribution functions in terms of occupied and of non-occupied quark states. By an explicit analytical calculation it is shown that this anomaly is absent in the polarized isoscalar distribution $\Delta u+\Delta d$, which is ultraviolet finite. In the case of the polarized isovector quark distribution $\Delta u-\Delta d$ the anomaly can be cancelled by a Pauli–Villars subtraction which is also needed for the regularization of the ultraviolet divergence.

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1. Introduction

Since the discovery of the axial anomaly [1] this phenomenon has been attracting the interest of physicists leading to such an amount of generalizations, variations and applications that nowadays it is rather hard to give a definition of "quantum anomalies" which could cover all cases. With this reservation, it is still reasonable to think that typically quantum anomalies are associated with a situation where

1. a naively vanishing quantity is actually different from zero due to the nontrivial role of the ultraviolet effects,

2. once these ultraviolet effects are taken into account, the nonzero result for this quantity can be computed analytically whatever complicated functional and operator constructions stand behind it.

In this paper we want to attract attention to a phenomenon that appears if one considers the nucleon in the limit of large number of colors N_c. It is well known [2] that in this limit the nucleon is described by a sort of Hartree (mean field) approximation where the nucleon parameters can be represented as sums over "occupied single-quark states" in the mean field corresponding to the solution of Hartree equations. Using the C parity argument we can alternatively rewrite these quantities as sums over "nonoccupied single-quark states". As a result at large N_c we have two equivalent representations for various nucleon parameters

$$\langle N|O|N\rangle = \sum_{n,\,\mathrm{occ}} \langle n|\mathit{\Gamma}_O|n\rangle = -\sum_{n,\,\mathrm{non-occ}} \langle n|\mathit{\Gamma}_O|n\rangle . \tag{1}$$

Here $\langle N|O|N\rangle$ is a nucleon matrix element of some operator O in the full theory whereas $\mathit{\Gamma}_O$ is the "image" of the observable O in the single-quark Hilbert space in the Hartree approximation justified by large N_c. Next, $|n\rangle$ is the full set of single-quark states appearing in the mean field approximation. Strictly speaking in Eq. (1) we must subtract the corresponding vacuum sums and take into account the translational and rotational zero modes of the large N_c mean field solution.

The equivalence of the two representations (1) relies on the general argument of C invariance but formally it is based on the identity

$$\sum_{n,\,\mathrm{occ}} \langle n|\mathit{\Gamma}_O|n\rangle + \sum_{n,\,\mathrm{non-occ}} \langle n|\mathit{\Gamma}_O|n\rangle = \sum_{n} \langle n|\mathit{\Gamma}_O|n\rangle = 0\,, \tag{2}$$

i.e.

$$\mathrm{Tr}\,\mathit{\Gamma}_O = \sum_{n} \langle n|\mathit{\Gamma}_O|n\rangle = 0\,. \tag{3}$$

At this moment one can meet the same problem as in the case of the Fujikawa approach [3] to the axial anomaly where naively one has

$$\mathrm{Tr}\ \gamma_5 = 0 \tag{4}$$

but actually the careful treatment of the ultraviolet regularization leads to a nonzero result for the axial anomaly.

The exact form of the large N_c Hartree equations for QCD is not known and one has to deal with models imitating the large N_c QCD. The subsequent consideration will proceed in the framework of the chiral quark–soliton model [4–6]. We shall be interested in two questions:

1. For which ultraviolet regularizations the naive identity $\mathrm{Tr}\,\Gamma_O = 0$ really holds?
2. If in some regularization the "anomaly" $\mathrm{Tr}\,\Gamma_O \neq 0$ occurs, is it possible to compute this anomaly analytically?

The first question has a direct physical meaning since for practical calculations of nucleon observables one should use a regularization preserving the equivalence of two representations (1). In certain unphysical regularizations one can have a nonvanishing anomaly $\mathrm{Tr}\,\Gamma_O \neq 0$. Its analytical calculation is of certain interest because the numerical calculation of physical observables (1) in the quark soliton model is usually rather involved technically and any analytical results that can be compared with the numerical output are extremely useful for the check of the numerical procedure.

These general issues are of importance for the calculation of parton distributions in the quark soliton model. Recently the problem of the equivalence of the two different representations (1) has caused certain troubles [7]. In this paper we clarify the situation by a straightforward calculation of the "anomaly" associated with polarized quark distributions and demonstrate that this anomaly is cancelled by the Pauli–Villars subtraction.

2. Parton distributions in the quark soliton model

Recently a rather successful program of computing the quark distribution functions in the framework of the effective quark–soliton model was developed [7–12]. The quark soliton model [4–6] includes the chiral pion field $U = \mathrm{e}^{i\pi^a\tau^a/F_\pi}$ and the quark field ψ whose interaction is described by the Lagrangian

$$L = \bar{\psi}(i\gamma^\mu\partial_\mu - MU^{\gamma_5})\psi\,. \tag{5}$$

In the mean field approximation (justified in the limit of the large number of quark colors N_c [2]) the nucleon arises as a soliton of the chiral field U

$$U(x) = \exp[i(n^a\tau^a)P(r)], \quad n^a = \frac{x^a}{r}, \quad r = |x|\,. \tag{6}$$

This effective theory allows a quantum field-theoretical approach to the calculation of the quark and antiquark distributions in the nucleon. In contrast to naive quark composite models and to the bag model here we have a consistent approach reproducing the main features of the QCD parton model like positivity of the quark and antiquark distributions, various sum rules *etc.*

In terms of the quark degrees of freedom this picture of the nucleon corresponds to occupying with $N_{\rm c} = 3$ quarks the negative continuum levels as well as the valence level of the one-particle Dirac Hamiltonian H

$$H = -i\gamma^0\gamma^k\partial_k + M\gamma^0 U^{\gamma_5}\,, \tag{7}$$

in the background soliton field U. For the pion field (6) one can find the spectrum of the Hamiltonian (7)

$$H|n\rangle = E_n|n\rangle\,. \tag{8}$$

According to Eq. (1) various nucleon observables can be naturally represented as sums over eigenstates $|n\rangle$ of the Dirac Hamiltonian H. For example, the nucleon mass M_N is given by

$$M_N = N_{\rm c}\sum_{n,{\rm occ}}(E_n - E_n^{(0)}) = -N_{\rm c}\sum_{n,{\rm non-occ}}(E_n - E_n^{(0)})\,. \tag{9}$$

Here the energy of the vacuum is subtracted which is given by the sum of the eigenvalues $E_n^{(0)}$ of the free Hamiltonian

$$H_0|n^{(0)}\rangle = E_n^{(0)}|n^{(0)}\rangle, \quad H_0 = -i\gamma^0\gamma^k\partial_k + M\gamma^0. \tag{10}$$

The physical reason for the existence of the two equivalent expressions in (9) is that the polarized Dirac sea picture can be formulated either in terms of quark or in terms of antiquark states (occupied antiquark states correspond to non-occupied quark states).

Formally the equivalence of two representations (9) for M_N follows from the identity

$$\left(\sum_{n,{\rm occ}} E_n + \sum_{n,{\rm non-occ}} E_n\right) - (E_n \to E_n^{(0)}) = \sum_n (E_n - E_n^{(0)}) = {\rm Tr}\,(H - H_0) = 0\,. \tag{11}$$

At the last step we took into account that the traces of H and of H_0 over the spin indices vanish. Strictly speaking, this naive argument is not safe since the sums (9) over the occupied and non-occupied states are ultraviolet divergent and must be regularized. In principle, the ultraviolet regularization could lead to an anomalous difference between the summation over occupied and non-occupied states but in the case of the nucleon mass (9) one can check that in the regularizations suppressing the contributions of high eigenvalues the anomaly is absent:

$$\lim_{\Lambda\to\infty} {\rm Tr}\left[Hf\left(\frac{H}{\Lambda}\right) - H_0 f\left(\frac{H_0}{\Lambda}\right)\right] = 0\,, \tag{12}$$

where f is an arbitrary even function vanishing at infinity fast enough ($f(\pm\infty) = 0$) and such that $f(0) = 1$.

We can reformulate this verbally as the "absence of the anomaly" in the nucleon mass M_N (in the above regularization). The usage of the word "anomaly" is invoked by the similarity with the axial anomaly which can be interpreted as a nonvanishing trace of γ_5 computed in the basis of eigenstates of the Dirac operator in a background gauge field with a regularization suppressing the contribution of large eigenvalues [3].

The main object of interest in this paper is the study of the quark distribution functions. In the mean field approach (justified in the large $N_{\rm c}$ limit) the quark distributions can be represented as single or double sums over occupied or non-occupied one-particle eigenstates (8) of the Dirac Hamiltonian (7). We shall see that for the same parton distribution one can write two naively equivalent representations but whether this equivalence persists or not when one takes into account the ultraviolet regularization is a rather subtle question and the situation is different for different distributions. Moreover, even in the limit of the large cutoff, the cancellation of this anomalous difference between the naively equivalent representations is sensitive to the regularization used.

Let us start from the unpolarized isosinglet quark distribution $u(x)+d(x)$ which is given by the following expressions [8] in the leading order of the $1/N_{\rm c}$ expansion

$$\begin{aligned} u(x)+d(x) &= N_{\rm c}\sum_{n,{\rm occ}}\int\frac{d^3p}{(2\pi)^3}\delta\left(\frac{p^3+E_n}{M_N}-x\right)\langle n|\boldsymbol{p}\rangle(1+\gamma^0\gamma^3)\langle\boldsymbol{p}|n\rangle \\ &= -N_{\rm c}\sum_{n,{\rm non-occ}}\int\frac{d^3p}{(2\pi)^3}\delta\left(\frac{p^3+E_n}{M_N}-x\right)\langle n|\boldsymbol{p}\rangle(1+\gamma^0\gamma^3)\langle\boldsymbol{p}|n\rangle\,. \end{aligned} \tag{13}$$

Also here the subtraction of similar sums with the eigenstates and eigenvalues of the Hamiltonian (7) replaced by those of the free Hamiltonian (10) is implied. The result (13) has a transparent physical meaning of the probability to find a quark with momentum fraction x in the nucleon in the infinite momentum frame. In Ref. [8] it was shown that in the Pauli–Villars regularization the sums over occupied and non-occupied states in (13) really give the same result.

We stress that the fact of the equivalence of the two representations for parton distributions is crucial for the positivity of unpolarized distributions and for the validity of various sum rules inherited by the model from QCD [8]. Therefore the check of this equivalence is an essential part of the calculation of parton distributions in the chiral soliton model.

Now let us turn to the polarized quark distributions. In the leading order of the $1/N_{\rm c}$ expansion only the isovector polarized distribution survives

$$\Delta u(x) - \Delta d(x) \;=\; -\frac{1}{3}N_{\rm c}\sum_{n,\rm occ}\int\frac{d^3p}{(2\pi)^3}\delta\left(\frac{p^3+E_n}{M_N}-x\right) \times\langle n|\boldsymbol{p}\rangle(1+\gamma^0\gamma^3)\tau^3\gamma^5\langle\boldsymbol{p}|n\rangle\,. \tag{14}$$

Compared to the expression (13) for $u(x)+d(x)$ here we have an extra factor $\tau^3\gamma^5$ which reflects the fact that now we deal with the isovector polarized distribution. The factor of $1/3$ comes from the matrix element over the rotational wave functions of the soliton [13].

One can ask whether the summation over the occupied quark states in (14) can be replaced by the summation over non-occupied states

$$\Delta u(x) - \Delta d(x) \;=\; \frac{1}{3}N_{\rm c}\sum_{n,\rm non-occ}\int\frac{d^3p}{(2\pi)^3}\delta\left(\frac{p^3+E_n}{M_N}-x\right) \times\langle n|\boldsymbol{p}\rangle(1+\gamma^0\gamma^3)\tau^3\gamma^5\langle\boldsymbol{p}|n\rangle\,. \tag{15}$$

In this paper we shall show that in the case of the Pauli–Villars regularization (the sum over states n in (14) is logarithmically divergent) the two representations (14) and (15) are really equivalent.

We stress that the equivalence of the summation over the occupied and non-occupied states is very sensitive to the choice of the regularization. For example, if instead of the Pauli–Villars regularization we simply cut the summation over quark states in (14) including only states with $|E_n|<\omega_0$ then a nonzero difference between the two representations (14) and (15) will remain even in the limit of the infinite cutoff $\omega_0\to\infty$. The mechanism how this anomalous difference appears is similar in many respects to the famous axial anomaly. In particular, such similarity manifests itself in the fact that the anomalous difference between the two representations (14) and (15) can be computed analytically in the limit $\omega_0\to\infty$. The calculation of the anomalous difference is presented in this paper.

Although the regularization including only states with $|E_n|<\omega_0$ is not acceptable as a physical one and the Pauli–Villars regularization is more preferable in this respect, we want to emphasize that in the practical calculations based on the numerical diagonalization of the Dirac operator in the background soliton field, the $|E_n|<\omega_0$ regularization appears naturally. Indeed, in the numerical calculation one can work only with a finite amount of quark states so that one actually uses both Pauli–Villars subtraction (with the regulator mass $M_{\rm PV}$) and the $|E_n|<\omega_0$ regularization. The pure Pauli–Villars subtraction is simulated by working with $\omega_0\gg M_{\rm PV}$. The numerical calculation is rather involved and the analytical result for the anomaly in the $|E_n| < \omega_0$ regularization is very helpful for the control of numerics even if the anomaly cancels after the Pauli–Villars subtraction.

Now let us turn to the polarized isoscalar quark distribution $\Delta u(x) + \Delta d(x)$ which gets the first nonzero contribution only in the subleading order of the $1/N_c$ expansion

$$\begin{aligned} \Delta u(x) + \Delta d(x) \; = \; & \frac{N_c M_N}{2I} \sum_{m,\mathrm{all}} \sum_{n,\mathrm{occ}} \frac{1}{E_n - E_m} \\ & \times \langle n|\tau^3|m\rangle\langle m|(1+\gamma^0\gamma^3)\gamma^5\delta(E_n + P^3 - xM_N)|n\rangle \\ & + \frac{N_c}{4I}\frac{\partial}{\partial x}\sum_{n,\mathrm{occ}} \langle n|(1+\gamma^0\gamma^3)\tau^3\gamma^5\delta(E_n + P^3 - xM_N)|n\rangle \,. \end{aligned} \tag{16}$$

Here P^3 is the quark momentum projection on the third axis

$$P^3 = -i\frac{\partial}{\partial x^3}\,, \tag{17}$$

and I is the moment of inertia of the soliton.

Another representation for $\Delta u + \Delta d$ can be written in terms of the summation over non-occupied states n

$$\begin{aligned} \Delta u(x) + \Delta d(x) \; = & -\frac{N_c M_N}{2I} \sum_{m,\mathrm{all}} \sum_{n,\mathrm{non-occ}} \frac{1}{E_n - E_m} \\ & \times \langle n|\tau^3|m\rangle\langle m|(1+\gamma^0\gamma^3)\gamma^5\delta(E_n + P^3 - xM_N)|n\rangle \\ & - \frac{N_c}{4I}\frac{\partial}{\partial x}\sum_{n,\mathrm{non-occ}} \langle n|(1+\gamma^0\gamma^3)\tau^3\gamma^5\delta(E_n + P^3 - xM_N)|n\rangle \,. \end{aligned} \tag{18}$$

The numerical calculation of $\Delta u + \Delta d$ with the Pauli–Villars subtraction was presented in paper [7]. Unfortunately there the question about the equivalence of the two representation (16) and (18) was not investigated properly. Also the Pauli–Villars subtraction was used in paper [7] without proper justification.

In this paper we show that if one cuts the sum over occupied (non-occupied) states n allowing only $|E_n| < \omega_0$ in the Eqs. (16), (18) then in the infinite cutoff limit $\omega_0 \to \infty$

1. both representations (16) and (18) have a finite limit (*i.e.* $\Delta u(x)+\Delta d(x)$ has no ultraviolet divergences),

2. the two representations (16), (18) give the same result.

Comparing the last terms in the rhs of representations (16) and (18) for $\Delta u+\Delta d$ with expressions (14) and (15) for $\Delta u-\Delta d$ we see that the total expression for $\Delta u+\Delta d$ contains a contribution proportional to $\frac{\partial}{\partial x}\left[\Delta u(x)-\Delta d(x)\right]$.

Therefore we start our analysis by investigating the anomaly of $\Delta u-\Delta d$ which we do in Section 3. In Section 4 we show by explicit calculation that for the quark distribution $\Delta u+\Delta d$ there is no anomalous difference between the summations over occupied and non-occupied states. In Section 5 we discuss the numerical results and compare them to the GRSV parametrization of experimental data.

3. Anomaly of $\Delta u(x)-\Delta d(x)$

As it was explained in the introduction one of our aims is to investigate whether the two representations (14) and (15) for the polarized isovector quark distribution $\Delta u(x)-\Delta d(x)$ are equivalent. The answer to this question is sensitive to the ultraviolet regularization. Let us start from the regularization that allows only the quark states n with $|E_n|<\omega_0$. In this regularization Eq. (14) can be rewritten as follows.

$$\begin{aligned}&\left[\Delta u(x)-\Delta d(x)\right]^{\omega_0}_{\rm occ}\\&=-\frac{1}{3}N_{\rm c}M_N\int\limits_{-\omega_0}^{E_{\rm lev}+0}d\omega\,{\rm Tr}\left[\delta(H-\omega)\delta(\omega+P^3-xM_N)\tau^3(1+\gamma^0\gamma^3)\gamma_5\right].\end{aligned}\tag{19}$$

Here H is the Dirac Hamiltonian (7) and P^3 is momentum operator (17). Similarly, representation (15) becomes

$$\begin{aligned}&\left[\Delta u(x)-\Delta d(x)\right]^{\omega_0}_{\rm non-occ}\\&=\frac{1}{3}N_{\rm c}M_N\int\limits_{E_{\rm lev}+0}^{\omega_0}d\omega\,{\rm Tr}\left[\delta(H-\omega)\delta(\omega+P^3-xM_N)\tau^3(1+\gamma^0\gamma^3)\gamma_5\right].\end{aligned}\tag{20}$$

The main results of this section can be formulated as follows:

1. Both $\left[\Delta u(x)-\Delta d(x)\right]^{\omega_0}_{\rm occ}$ and $\left[\Delta u(x)-\Delta d(x)\right]^{\omega_0}_{\rm non-occ}$ are logarithmically divergent in the limit of large cutoff $\omega_0\to\infty$

$$[\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm occ} \sim [\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm non-occ} = \frac{N_{\rm c} M_N M^2}{12\pi^2} \ln \frac{\omega_0}{M} \times \int \frac{d^3 k}{(2\pi)^3} {\rm Sp}_{fl} \left[(\tilde{U}[\boldsymbol{k}])^+ \tau^3 \tilde{U}(\boldsymbol{k}) \right] \theta \left(k^3 - |x| M_N \right) + \dots . \quad (21)$$

2. In the difference $[\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm occ} - [\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm non-occ}$ the ultraviolet divergences cancel and the $\omega_0 \to \infty$ limit of this difference reduces to the following finite expression

$$\lim_{\omega_0 \to \infty} \left\{ [\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm occ} - [\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm non-occ} \right\} = -\frac{1}{12\pi^2} N_{\rm c} M_N M^2 \int \frac{d^3 k}{(2\pi)^3} \ln \frac{|x M_N + k^3|}{|x M_N|} {\rm Sp}_{fl} \left[\tau^3 (\tilde{U}[\boldsymbol{k}])^+ \tilde{U}(\boldsymbol{k}) \right] , \quad (22)$$

where $\tilde{U}(\boldsymbol{k})$ is the Fourier transform of the chiral mean field $U(\boldsymbol{r})$ entering the Dirac Hamiltonian (7)

$$\tilde{U}(\boldsymbol{k}) = \int d^3 r \, {\rm e}^{-i(\boldsymbol{k}\boldsymbol{r})} \left[U(\boldsymbol{r}) - 1 \right] . \quad (23)$$

Note that $[\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm occ}$ and $[\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm non-occ}$ separately are given by complicated functional traces (19) and (20) which can be computed only numerically. The fact that the anomalous difference between the representations in terms of the occupied and non-occupied states reduces to a simple momentum integral (22) is highly nontrivial and is similar to the well known fact that the famous axial anomaly gets its contribution only from the simplest diagram.

The fact that the divergence (21) is proportional to M^2 means that this divergence can be removed by the Pauli–Villars subtraction so that the following combinations are finite[1]

$$[\Delta u(x) - \Delta d(x)]^{\rm PV}_{\rm occ} = \lim_{\omega_0 \to \infty} \left\{ [\Delta u(x) - \Delta d(x)]^{\omega_0, M}_{\rm occ} - \frac{M^2}{M^2_{\rm PV}} [\Delta u(x) - \Delta d(x)]^{\omega_0, M_{\rm PV}}_{\rm occ} \right\} , \quad (24)$$

[1] Following Refs. [17,18], in our numerical calculations we define $[\Delta u - \Delta d]^{\omega_0, M_{\rm PV}}_{\rm occ}$ by Eq. (19) with $M \to M_{\rm PV}$ but do not include the discrete valence level of the regulator Dirac Hamiltonian (*i.e.* we replace $E_{\rm lev} + 0 \to E^{M_{\rm PV}}_{\rm lev} - 0$ in Eq. (19)). In contrast, our regulator analogue of Eq. (20) for $[\Delta u - \Delta d]^{\omega_0, M_{\rm PV}}_{\rm non-occ}$ includes this valence level. Obviously the difference $[\Delta u - \Delta d]^{\omega_0, M_{\rm PV}}_{\rm occ} - [\Delta u - \Delta d]^{\omega_0, M_{\rm PV}}_{\rm non-occ}$ is insensitive to this treatment of the level. Therefore this subtlety of the regularization does not affect the anomaly study in this paper.

$$[\Delta u(x) - \Delta d(x)]^{\rm PV}_{\rm non-occ}$$
$$= \lim_{\omega_0\to\infty} \left\{ [\Delta u(x) - \Delta d(x)]^{\omega_0,M}_{\rm non-occ} - \frac{M^2}{M^2_{\rm PV}} [\Delta u(x) - \Delta d(x)]^{\omega_0,M_{\rm PV}}_{\rm non-occ} \right\}. \tag{25}$$

Next, since the anomaly (22) is proportional to M^2 we see that in the Pauli–Villars regularization the summation over occupied and non-occupied states gives the same results:

$$[\Delta u(x) - \Delta d(x)]^{\rm PV}_{\rm occ} = [\Delta u(x) - \Delta d(x)]^{\rm PV}_{\rm non-occ} \,. \tag{26}$$

Now let us turn to the derivation of the result (22) for the anomalous difference between the summation over occupied and non-occupied states. Subtracting (20) from (19) we obtain

$$[\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm occ} - [\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm non-occ}$$
$$= -\frac{1}{3} N_{\rm c} M_N \int\limits_{-\omega_0}^{\omega_0} d\omega {\rm Tr} \left[\delta(H-\omega)\delta(\omega + P^3 - xM_N)\tau^3(1+\gamma^0\gamma^3)\gamma_5\right]. \tag{27}$$

We use the following representation for the operator delta function $\delta(H-\omega)$

$$\delta(H-\omega) = \frac{{\rm sign}\omega}{2\pi i}\left[\frac{1}{H^2-\omega^2-i0} - \frac{1}{H^2-\omega^2+i0}\right](H+\omega)\,. \tag{28}$$

The squared Dirac Hamiltonian (7) is

$$H^2 = -\partial^2 + M^2 + iM(\gamma^k\partial_k U^{\gamma_5})\,. \tag{29}$$

Now (27) takes the form

$$[\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm occ} - [\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm non-occ} = -\frac{2}{3} N_{\rm c} M_N {\rm Im} \int\limits_{-\omega_0}^{\omega_0} \frac{d\omega}{2\pi} {\rm sign}\omega$$
$$\times {\rm Tr}\left\{ \frac{1}{-\partial^2 + M^2 - \omega^2 - i0 + iM(\gamma^k\partial_k U^{\gamma_5})} (\omega - i\gamma^0\gamma^k\partial_k + \gamma^0 M U^{\gamma_5}) \right.$$
$$\left. \times\delta(\omega + P^3 - xM_N)\tau^3(1+\gamma^0\gamma^3)\gamma_5 \right\}. \tag{30}$$

Next we expand the "propagator" in the rhs in powers of $iM(\gamma^k\partial_k U^{\gamma_5})$

$$\frac{1}{-\partial^2+M^2-\omega^2-i0+iM(\gamma^k\partial_k U^{\gamma_5})}=\frac{1}{-\partial^2+M^2-\omega^2-i0}$$
$$-\frac{1}{-\partial^2+M^2-\omega^2-i0}iM(\gamma^k\partial_k U^{\gamma_5})\frac{1}{-\partial^2+M^2-\omega^2-i0}+\ldots. \quad (31)$$

The first nonvanishing contribution to (30) comes from the term linear in $iM(\gamma^k\partial_k U^{\gamma_5})$

$$[\Delta u(x)-\Delta d(x)]^{\omega_0}_{\rm occ}-[\Delta u(x)-\Delta d(x)]^{\omega_0}_{\rm non-occ}=-\frac{2}{3}N_{\rm c}M_N{\rm Im}\int\limits_{-\omega_0}^{\omega_0}\frac{d\omega}{2\pi}$$
$$\times{\rm Tr}\left\{\frac{1}{-\partial^2+M^2-\omega^2-i0}[-iM(\partial_k U^{\gamma_5})]\frac{1}{-\partial^2+M^2-\omega^2-i0}\right.$$
$$\left.\times(\omega-i\gamma^0\gamma^k\partial_k+\gamma^0MU^{\gamma_5})(1+\gamma^0\gamma^3)\gamma_5\gamma^k\delta(\omega+P^3-xM_N)\tau^3\right\}. \quad (32)$$

Computing the trace over the spin indices and turning to the momentum representation according to (23) we arrive at

$$[\Delta u(x)-\Delta d(x)]^{\omega_0}_{\rm occ}-[\Delta u(x)-\Delta d(x)]^{\omega_0}_{\rm non-occ}$$
$$=-\frac{8}{3}N_{\rm c}M_NM^2{\rm Im}\int\frac{d^3k}{(2\pi)^3}k^3{\rm Sp}_{fl}\left\{(\tilde U[\boldsymbol{k}])^+\tilde U(\boldsymbol{k})\tau^3\right\}$$
$$\times\int\limits_{-\omega_0}^{\omega_0}\frac{d\omega}{2\pi}\int\frac{d^3p}{(2\pi)^3}\frac{{\rm sign}\omega}{|\boldsymbol{k}+\boldsymbol{p}|^2+M^2-\omega^2-i0}\frac{\delta(\omega+p^3-xM_N)}{|\boldsymbol{p}|^2+M^2-\omega^2-i0}. \quad (33)$$

We first integrate over ω and p^3

$$[\Delta u(x)-\Delta d(x)]^{\omega_0}_{\rm occ}-[\Delta u(x)-\Delta d(x)]^{\omega_0}_{\rm non-occ}$$
$$=\frac{4}{3}N_{\rm c}M_NM^2\int\frac{d^3k}{(2\pi)^3}\int\frac{d^2\boldsymbol{p}^\perp}{2(2\pi)^3}k^3{\rm Sp}_{fl}\left[(\tilde U[\boldsymbol{k}])^+\tilde U(\boldsymbol{k})\tau^3\right]$$
$$\times\left\{(xM_N+k^3)|\boldsymbol{p}^\perp|^2-xM_N|\boldsymbol{p}^\perp+\boldsymbol{k}^\perp|^2+k^3[M^2-xM_N(xM_N+k^3)]\right\}^{-1}$$
$$\times\left[\theta\left(\omega_0-\frac{|\boldsymbol{p}^\perp|^2+M^2+(xM_N)^2}{2|x|M_N}\right)\theta\left(-\omega_0+\frac{|\boldsymbol{p}^\perp+\boldsymbol{k}^\perp|^2+M^2+(xM_N+k^3)^2}{2|xM_N+k^3|}\right)\right.$$
$$\left.-\,\theta\left(-\omega_0+\frac{|\boldsymbol{p}^\perp|^2+M^2+(xM_N)^2}{2|x|M_N}\right)\theta\left(\omega_0-\frac{|\boldsymbol{p}^\perp+\boldsymbol{k}^\perp|^2+M^2+(xM_N+k^3)^2}{2|xM_N+k^3|}\right)\right]. \quad (34)$$

In the limit of large cutoff ω_0 we have for any fixed A, B

$$\lim_{\omega_0\to\infty}\theta(\omega_0-A)\theta(-\omega_0+B)=0\,. \tag{35}$$

Nevertheless the integral in the rhs of (34) does not vanish in the limit $\omega_0\to\infty$ since this limit gets contributions from the region

$$\omega_0\sim\frac{|\boldsymbol{p}^\perp|^2}{|x|M_N}\gg M\sim|x|M_N\sim|k|\,, \tag{36}$$

where $|\boldsymbol{p}^\perp|$ grows with ω_0. In this region (34) simplifies to

$$\begin{aligned}
&\lim_{\omega_0\to\infty}\left\{[\Delta u(x)-\Delta d(x)]^{\omega_0}_{\mathrm{occ}}-[\Delta u(x)-\Delta d(x)]^{\omega_0}_{\mathrm{non-occ}}\right\}\\
&=\frac{4}{3}N_\mathrm{c}M_NM^2\int\frac{d^3k}{(2\pi)^3}\int\frac{d^2\boldsymbol{p}^\perp}{2(2\pi)^3}\mathrm{Sp}_{fl}\left[(\tilde U[\boldsymbol{k}])^+\tilde U(\boldsymbol{k})\tau^3\right]\\
&\times\frac{1}{|\boldsymbol{p}^\perp|^2}\left[\theta\left(\omega_0-\frac{|\boldsymbol{p}^\perp|^2+M^2+(xM_N)^2}{2|x|M_N}\right)\theta\left(-\omega_0+\frac{|\boldsymbol{p}^\perp|^2}{2|xM_N+k^3|}\right)\right.\\
&\left.-\,\theta\left(-\omega_0+\frac{|\boldsymbol{p}^\perp|^2}{2|x|M_N}\right)\theta\left(\omega_0-\frac{|\boldsymbol{p}^\perp|^2}{2|xM_N+k^3|}\right)\right]\,.
\end{aligned} \tag{37}$$

Now the integral over $\boldsymbol{p}^\perp$ becomes trivial and we arrive at the final result (22).

An important feature of our calculation is that in the limit of large cutoff $\omega_0\to\infty$ the integral gets a contribution only from large $\boldsymbol{p}^\perp$. Actually the situation is analogous to the calculation of the axial anomaly which can also be formulated as saturated by the ultraviolet region.

The analogy with the axial anomaly goes further if we go to the higher terms of the expansion (31): these higher terms vanish in the limit of the large cutoff $\omega_0\to\infty$. The mechanism of this vanishing is as follows. Similarly to the leading term (34) one finds that in the limit $\omega_0\to\infty$ the integral over $\boldsymbol{p}^\perp$ comes from large $\boldsymbol{p}^\perp$. But a simple dimensional counting shows that in the higher order terms the integrand decays at large $\boldsymbol{p}^\perp$ too fast so that the integral vanishes in the limit $\omega_0\to\infty$. Since the higher order terms vanish our result for the anomaly (22) is actually exact.

Restricting the integration over ω in Eq. (30) to the interval $-\omega_0<\omega<0$ or to $0<\omega<\omega_0$ we can investigate separate distribution functions $[\Delta u(x)-\Delta d(x)]^{\omega_0}_{\mathrm{occ}}$ or $[\Delta u(x)-\Delta d(x)]^{\omega_0}_{\mathrm{non-occ}}$. In this case one gets nonzero contributions from all terms of the infinite series (31). However, it is not difficult to check that only the first nonvanishing term of this expansion is logarithmically divergent in the limit of large cutoff $\omega_0\to\infty$ and this logarithmic divergence is given by (21). This logarithmic divergence is proportional to M^2 and therefore in our previous calculation of

$\Delta u - \Delta d$ we could regularize it by the Pauli–Villars subtraction. Moreover, since the anomaly (22) is also proportional to M^2 it is cancelled by the same Pauli–Villars subtraction [9].

4. Cancellation of the anomaly of $\Delta u(x) + \Delta d(x)$

Now we turn to the investigation of $\Delta u(x)+\Delta d(x)$. The ω_0 cutoff version of (16) is

$$\begin{aligned}[\Delta u(x) \ &+\Delta d(x)]^{\omega_0}_{\rm occ} = \frac{N_{\rm c} M_N}{2I} \sum_m \sum_{-\omega_0 < E_n \leq E_{\rm lev}} \frac{1}{E_n - E_m} \\ &\times \langle n|\tau^3|m\rangle\langle m|(1+\gamma^0\gamma^3)\gamma^5\delta(E_n+P^3-xM_N)|n\rangle \\ &+\frac{N_{\rm c}}{4I}\frac{\partial}{\partial x}\sum_{-\omega_0 < E_n \leq E_{\rm lev}} \langle n|(1+\gamma^0\gamma^3)\tau^3\gamma^5\delta(E_n+P^3-xM_N)|n\rangle . \qquad (38)\end{aligned}$$

Although we use notations corresponding to the discrete spectrum, actually most of the spectrum is continuous. The singularities corresponding to $E_m = E_n$ are assumed to be regularized according to the principal value prescription.

Making use of (19) we find

$$[\Delta u(x) + \Delta d(x)]^{\omega_0}_{\rm occ} = [\Delta u(x)+\Delta d(x)]^{(1)\omega_0}_{\rm occ} - \frac{3}{4IM_N}\frac{\partial}{\partial x}[\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm occ} , \qquad (39)$$

where

$$\begin{aligned}[\Delta u(x) + \Delta d(x)]^{(1)\omega_0}_{\rm occ} &= \frac{N_{\rm c} M_N}{2I} \sum_m \sum_{-\omega_0 < E_n \leq E_{\rm lev}} \frac{1}{E_n - E_m} \\ &\times \langle n|\tau^3|m\rangle\langle m|(1+\gamma^0\gamma^3)\gamma^5\delta(E_n+P^3-xM_N)|n\rangle . \qquad (40)\end{aligned}$$

Similarly (18) leads to

$$\begin{aligned}[\Delta u(x) + \Delta d(x)]^{\omega_0}_{\rm non-occ} &= [\Delta u(x) + \Delta d(x)]^{(1)\omega_0}_{\rm non-occ} \\ &-\frac{3}{4IM_N}\frac{\partial}{\partial x}[\Delta u(x) - \Delta d(x)]^{\omega_0}_{\rm non-occ} , \qquad (41)\end{aligned}$$

where

$$\begin{aligned}[\Delta u(x) + \Delta d(x)]^{(1)\omega_0}_{\rm non-occ} &= -\frac{N_{\rm c} M_N}{2I} \sum_m \sum_{E_{\rm lev} < E_n < \omega_0} \frac{1}{E_n - E_m} \\ &\times \langle n|\tau^3|m\rangle\langle m|(1+\gamma^0\gamma^3)\gamma^5\delta(E_n+P^3-xM_N)|n\rangle . \qquad (42)\end{aligned}$$

We see that

$$
\begin{aligned}
&[\Delta u(x)+\Delta d(x)]^{\omega_0}_{\rm occ}-[\Delta u(x)+\Delta d(x)]^{\omega_0}_{\rm non-occ} \\
&=[\Delta u(x)+\Delta d(x)]^{(1)\omega_0}_{\rm occ}-[\Delta u(x)+\Delta d(x)]^{(1)\omega_0}_{\rm non-occ} \\
&-\frac{3}{4IM_N}\frac{\partial}{\partial x}\left\{[\Delta u(x)-\Delta d(x)]^{\omega_0}_{\rm occ}-[\Delta u(x)-\Delta d(x)]^{\omega_0}_{\rm non-occ}\right\}\,. \qquad (43)
\end{aligned}
$$

Here

$$
\begin{aligned}
&[\Delta u(x)+\Delta d(x)]^{(1)\omega_0}_{\rm occ}-[\Delta u(x)+\Delta d(x)]^{(1)\omega_0}_{\rm non-occ} \\
&=\frac{N_c M_N}{2I}\sum_m \sum_{-\omega_0<E_n<\omega_0}\left(\frac{1}{E_n-E_m}\right)_{\rm PV} \\
&\times\langle n|\tau^3|m\rangle\langle m|(1+\gamma^0\gamma^3)\gamma^5\delta(E_n+P^3-xM_N)|n\rangle\,. \qquad (44)
\end{aligned}
$$

We remind that here the principal value prescription for $(E_n-E_m)^{-1}$ is implied. This can be rewritten in the form

$$
\begin{aligned}
&[\Delta u(x)+\Delta d(x)]^{(1)\omega_0}_{\rm occ}-[\Delta u(x)+\Delta d(x)]^{(1)\omega_0}_{\rm non-occ}=-\frac{M_N N_{\rm c}}{4I}\int\limits_{-\omega_0}^{\omega_0} d\omega \\
&\times{\rm Sp}\left\{\left[\left(\frac{1}{H-\omega}\right)_{\rm PV}\tau^3\delta(H-\omega)+\delta(H-\omega)\tau^3\left(\frac{1}{H-\omega}\right)_{\rm PV}\right]\right. \\
&\times\left.\delta(\omega+P^3-xM_N)(1+\gamma^0\gamma^3)\gamma^5\right\}=-\frac{iM_N N_{\rm c}}{4I}\int\limits_{-\omega_0}^{\omega_0}\frac{d\omega}{2\pi} \\
&\times{\rm Sp}\left\{\left[\frac{1}{H-\omega+i0}\tau^3\frac{1}{H-\omega+i0}\right]\delta(\omega+P^3-xM_N)(1+\gamma^0\gamma^3)\gamma^5\right\} \\
&+\frac{iM_N N_{\rm c}}{4I}\int\limits_{-\omega_0}^{\omega_0}\frac{d\omega}{2\pi} \\
&\times{\rm Sp}\left\{\left[\frac{1}{H-\omega-i0}\tau^3\frac{1}{H-\omega-i0}\right]\delta(\omega+P^3-xM_N)(1+\gamma^0\gamma^3)\gamma^5\right\}\,. \qquad (45)
\end{aligned}
$$

Hence

$$
\begin{aligned}
&[\Delta u(x)+\Delta d(x)]^{(1)\omega_0}_{\rm occ}-[\Delta u(x)+\Delta d(x)]^{(1)\omega_0}_{\rm non-occ} \\
&=-{\rm Im}\frac{M_N N_{\rm c}}{2I}\int\limits_{-\omega_0}^{\omega_0}\frac{d\omega}{2\pi}{\rm Sp}\left\{\frac{1}{H^2-\omega^2-i0\,{\rm sign}\omega}(H+\omega)\tau^3(H+\omega)\right.
\end{aligned}
$$

$$\times \frac{1}{H^2 - \omega^2 - i0\mathrm{sign}\omega}\delta(\omega + P^3 - xM_N)(1+\gamma^0\gamma^3)\gamma^5\Big\}$$
$$= -\mathrm{Im}\frac{M_N N_c}{2I}\int\limits_{-\omega_0}^{\omega_0}\frac{d\omega}{2\pi}\mathrm{sign}\omega\mathrm{Sp}\Big\{\frac{1}{H^2-\omega^2-i0}(H+\omega)\tau^3(H+\omega)$$
$$\times \frac{1}{H^2-\omega^2-i0}\delta(\omega + P^3 - xM_N)(1+\gamma^0\gamma^3)\gamma^5\Big\}$$
$$= -\mathrm{Im}\frac{M_N N_c}{2I}\int\limits_{-\omega_0}^{\omega_0}\frac{d\omega}{2\pi}\mathrm{sign}\omega\mathrm{Sp}\Big\{\frac{1}{-\partial^2 + M^2 - \omega^2 - i0 + iM(\gamma^k\partial_k U^{\gamma_5})}$$
$$\times(\omega - i\gamma^0\gamma^k\partial_k + \gamma^0 M U^{\gamma_5})\tau^3(\omega - i\gamma^0\gamma^k\partial_k + \gamma^0 M U^{\gamma_5})$$
$$\times \frac{1}{-\partial^2 + M^2 - \omega^2 - i0 + iM(\gamma^k\partial_k U^{\gamma_5})}\delta(\omega - i\partial_3 - xM_N)(1+\gamma^0\gamma^3)\gamma^5\Big\}. \tag{46}$$

The rest of the calculation is similar to how we worked with expression (30) for the anomaly of $\Delta u(x) - \Delta d(x)$.

Nonzero contributions to the anomaly come from the expansion of the propagators up to terms linear and quadratic in $iM(\gamma^k\partial_k U^{\gamma_5})$:

$$[\Delta u(x) + \Delta d(x)]^{(1)\omega_0}_{\mathrm{occ}} - [\Delta u(x) + \Delta d(x)]^{(1)\omega_0}_{\mathrm{non-occ}}$$
$$= A_1(x) + A_2(x)\,. \tag{47}$$

Here $A_1(x)$ corresponds to terms linear in $iM(\gamma^k\partial_k U^{\gamma_5})$

$$A_1(x) = \mathrm{Im}\frac{M_N N_c}{2I}\int\limits_{-\omega_0}^{\omega_0}\frac{d\omega}{2\pi}\mathrm{sign}\omega\mathrm{Sp}\Big\{\Big[iM(\gamma^l\partial_l U^{\gamma_5})\frac{1}{-\partial^2 + M^2 - \omega^2 - i0}$$
$$\times(\omega - i\gamma^0\gamma^k\partial_k + \gamma^0 M U^{\gamma_5})\tau^3(\omega - i\gamma^0\gamma^k\partial_k + \gamma^0 M U^{\gamma_5})$$
$$+(\omega - i\gamma^0\gamma^k\partial_k + \gamma^0 M U^{\gamma_5})\tau^3(\omega - i\gamma^0\gamma^k\partial_k + \gamma^0 M U^{\gamma_5})$$
$$\times\frac{1}{-\partial^2 + M^2 - \omega^2 - i0}iM(\gamma^l\partial_l U^{\gamma_5})\Big]$$
$$\times\Big[\frac{1}{-\partial^2 + M^2 - \omega^2 - i0}\Big]^2\delta(\omega - i\partial_3 - xM_N)(1+\gamma^0\gamma^3)\gamma^5\Big\}, \tag{48}$$

and A_2 is quadratic in $iM(\gamma^k\partial_k U^{\gamma_5})$

$$
\begin{aligned}
A_2(x) = &-\mathrm{Im}\frac{M_N N_c}{2I}\int\limits_{-\omega_0}^{\omega_0}\frac{d\omega}{2\pi}\mathrm{sign}\omega\mathrm{Tr}\left\{\frac{1}{-\partial^2+M^2-\omega^2-i0}\right.\\
&\times\left[iM(\gamma^m\partial_m U^{\gamma_5})\frac{1}{-\partial^2+M^2-\omega^2-i0}iM(\gamma^n\partial_n U^{\gamma_5})\frac{1}{-\partial^2+M^2-\omega^2-i0}\right.\\
&\times(\omega-i\gamma^0\gamma^k\partial_k+\gamma^0 MU^{\gamma_5})\tau^3(\omega-i\gamma^0\gamma^l\partial_l+\gamma^0 MU^{\gamma_5})\\
&+iM(\gamma^m\partial_m U^{\gamma_5})\frac{1}{-\partial^2+M^2-\omega^2-i0}\\
&\times(\omega-i\gamma^0\gamma^k\partial_k+\gamma^0 MU^{\gamma_5})\tau^3(\omega-i\gamma^0\gamma^l\partial_l+\gamma^0 MU^{\gamma_5})\\
&\times\frac{1}{-\partial^2+M^2-\omega^2-i0}iM(\gamma^n\partial_n U^{\gamma_5})\\
&+(\omega-i\gamma^0\gamma^k\partial_k+\gamma^0 MU^{\gamma_5})\tau^3(\omega-i\gamma^0\gamma^l\partial_l+\gamma^0 MU^{\gamma_5})\\
&\times\frac{1}{-\partial^2+M^2-\omega^2-i0}iM(\gamma^m\partial_m U^{\gamma_5})\\
&\times\left.\frac{1}{-\partial^2+M^2-\omega^2-i0}iM(\gamma^n\partial_n U^{\gamma_5})\right]\\
&\times\left.\frac{1}{-\partial^2+M^2-\omega^2-i0}\delta(\omega-i\partial_3-xM_N)(1+\gamma^0\gamma^3)\gamma^5\right\}\,. \qquad (49)
\end{aligned}
$$

A straightforward calculation leads to the following results for $A_1(x)$ and $A_2(x)$

$$A_1(x) = -\frac{M^2 M_N N_c}{8\pi^2 I}\int\frac{d^3k}{(2\pi)^3}\frac{1}{k^3}\ln\left|1+\frac{k^3}{xM_N}\right|\mathrm{Sp}\left\{\tau^3\left[\tilde{U}(\boldsymbol{k})\right]^+\tilde{U}(\boldsymbol{k})\right\}, \qquad (50)$$

$$
\begin{aligned}
A_2(x) \;=\; &\frac{M_N N_c M^2}{8\pi^2 I}\int\frac{d^3k}{(2\pi)^3}\ln\left|\frac{k^3+xM_N}{xM_N}\right|\\
&\times\left(\frac{1}{k^3}+\frac{1}{2}\frac{\partial}{\partial k^3}\right)\mathrm{Sp}\left\{\tau^3\left[\tilde{U}(\boldsymbol{k})\right]^+\tilde{U}(\boldsymbol{k})\right\}\,. \qquad (51)
\end{aligned}
$$

Now we insert these results into (47)

$$
\begin{aligned}
&[\Delta u(x)+\Delta d(x)]^{(1)\omega_0}_{\mathrm{occ}} - [\Delta u(x)+\Delta d(x)]^{(1)\omega_0}_{\mathrm{non-occ}}\\
&=\frac{M_N N_c M^2}{16\pi^2 I}\int\frac{d^3k}{(2\pi)^3}\ln\left|\frac{k^3+xM_N}{xM_N}\right|\frac{\partial}{\partial k^3}\mathrm{Sp}\left\{\tau^3\left[\tilde{U}(\boldsymbol{k})\right]^+\tilde{U}(\boldsymbol{k})\right\}\,. \qquad (52)
\end{aligned}
$$

Note that shifting the integration variable

$$k^3 \to k^3 - xM_N\,, \qquad (53)$$

we obtain

$$\int \frac{d^3k}{(2\pi)^3} \ln\left|\frac{k^3 + xM_N}{xM_N}\right| \frac{\partial}{\partial k^3} \mathrm{Sp}\left\{\tau^3 \left[\tilde{U}(\boldsymbol{k})\right]^+ \tilde{U}(\boldsymbol{k})\right\}$$
$$= -\frac{1}{M_N}\frac{\partial}{\partial x} \int \frac{d^3k}{(2\pi)^3} \ln\left|k^3 + xM_N\right| \mathrm{Sp}\left\{\tau^3 \left[\tilde{U}(\boldsymbol{k})\right]^+ \tilde{U}(\boldsymbol{k})\right\}. \quad (54)$$

Therefore

$$[\Delta u(x) + \Delta d(x)]^{(1)\omega_0}_{\mathrm{occ}} - [\Delta u(x) + \Delta d(x)]^{(1)\omega_0}_{\mathrm{non-occ}}$$
$$= -\frac{N_{\mathrm{c}} M^2}{16\pi^2 I}\frac{\partial}{\partial x} \int \frac{d^3k}{(2\pi)^3} \ln\left|\frac{k^3 + xM_N}{xM_N}\right| \mathrm{Sp}\left\{\tau^3 \left[\tilde{U}(\boldsymbol{k})\right]^+ \tilde{U}(\boldsymbol{k})\right\}. \quad (55)$$

Inserting this result and (22) into (43) we observe a complete cancellation:

$$\lim_{\omega_0 \to \infty} \left\{[\Delta u(x) + \Delta d(x)]^{\omega_0}_{\mathrm{occ}} - [\Delta u(x) + \Delta d(x)]^{\omega_0}_{\mathrm{non-occ}}\right\} = 0\,. \quad (56)$$

Thus the isoscalar polarized quark distribution $\Delta u(x)+\Delta d(x)$ is nonanomalous.

Using similar methods one can check that function $\Delta u(x) + \Delta d(x)$ is free of ultraviolet divergences: although the two separate terms in the rhs of (39) are UV divergent the total sum is finite.

5. Numerical results

The numerical results for the isovector polarized distribution function $\Delta u(x) - \Delta d(x)$ are given in [9]. For the computation of $\Delta u(x)+\Delta d(x)$ (16), (18) we use the numerical methods which were developed in [9] and later extended in [11] for the computation of the isovector unpolarized distribution.

The eigenvectors and eigenvalues of the Dirac Hamiltonian (7) are determined by diagonalizing in the free Hamiltonian basis (10). This basis is made discrete by placing the soliton in a three-dimensional spherical box of finite radius D and imposing the Kahana–Ripka boundary conditions [15]. Both $\Delta u(x) - \Delta d(x)$ and $\Delta u(x) + \Delta d(x)$ were computed using the standard value of the constituent quark mass $M = 350$ MeV as derived from the instanton vacuum [16].

In our calculation we use the self-consistent solitonic profile $P(r)$ (see *e.g.* Ref. [17, 18] for the details of the regularization procedure). However, performing the numerical calculations in the finite spherical box one should be careful about the large distance effects. To be safe, we artificially exponentially suppress the pion tail of the soliton profile at large distances so that the field vanishes outside the box (a similar problem in the calculation of g_A was studied in [19]).

In Fig. 1 we compare our numerical results for the anomaly of $\Delta u(x) - \Delta d(x)$ with the analytical result (22). We observe a rather good agreement.

Fig. 2 shows the numerical results for the Dirac sea contribution to $\Delta u(x)+\Delta d(x)$ based on the two representations (occupied and non-occupied).

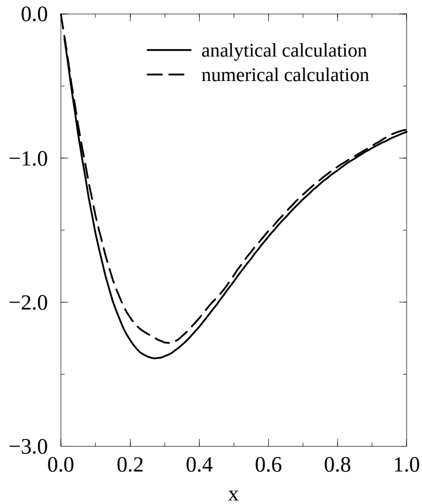

Fig. 1. Analytical (solid) and numerical (dashed) results for the anomalous difference $[\Delta u - \Delta d]_{\rm occ} - [\Delta u - \Delta d]_{\rm non-occ}$.

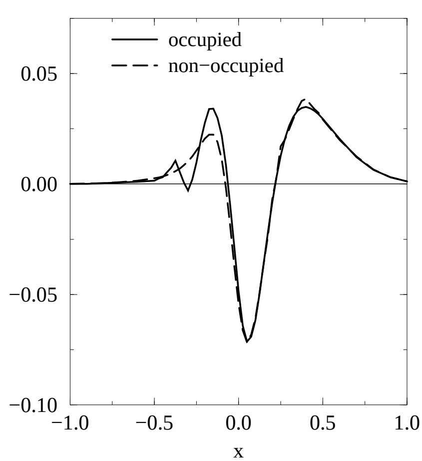

Fig. 2. Results for continuum contribution $[\Delta u + \Delta d]_{\rm sea}$ based on the occupied and non-occupied representations.

We see a reasonable agreement between the two results which confirms the absence of the anomaly in $\Delta u(x) + \Delta d(x)$. Some difference between the two curves at negative x is finite-box artefact. Increasing the size of the box one can see that this difference tends to disappear.

In Fig. 3 we compare the result of the calculation of $\Delta u(x) + \Delta d(x)$, $\Delta \bar{u}(x) + \Delta \bar{d}(x)$ with the GRSV-LO parametrization [22] at the low scale of the model $\mu = 600\,\mathrm{MeV}$. We see that the quark distribution $\Delta u(x) + \Delta d(x)$ is in a reasonable agreement with the GRSV parametrization whereas the antiquark distribution $\Delta \bar{u}(x) + \Delta \bar{d}(x)$ obtained in the model is considerably smaller than that of the GRSV parametrization. Note that the polarized antiquark distributions are not directly accessible in inclusive hard reactions. Due to the lack of data the GRSV parametrizations therefore are based on certain assumptions, *e.g.* in the GRSV analysis it was assumed that $\Delta \bar{u}(x) = \Delta \bar{d}(x)$. In contrast to this the QCD large N_c counting and the quark soliton model predict a large flavor asymmetry in the light polarized sea. Some physical applications of this have been studied in Refs. [21,23,24].

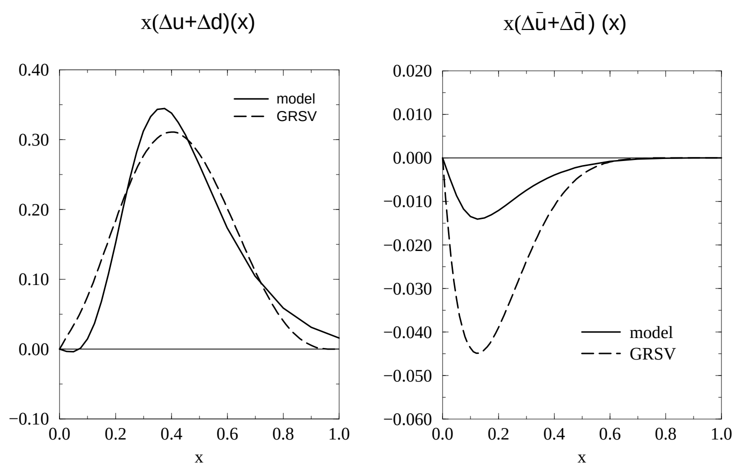

Fig. 3. The quark soliton model results for $x[\Delta u + \Delta d]$ and $x[\Delta \bar{u} + \Delta \bar{d}]$ versus LO-GRSV parametrization at the scale $\mu \approx 600$ MeV.

Fig. 4 shows our predictions for the polarized antiquark distributions $\Delta \bar{u}(x)$ and $\Delta \bar{d}(x)$ separately at the scale $\mu = 600\,\mathrm{MeV}$.

Since the quark distribution $\Delta u + \Delta d$ is finite, no ultraviolet regularization is needed for this quantity. There is even an argument against regularizing $\Delta u + \Delta d$ coming from the fact that the first moment of this distribution is related to the imaginary part of the quark determinant in the background soliton field which has to be left nonregularized if one wants to keep baryon

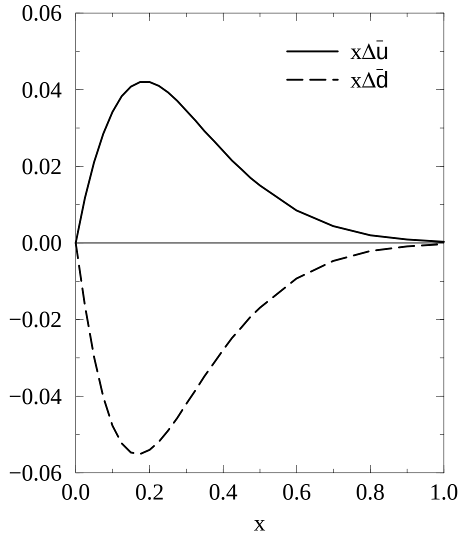

Fig. 4. The quark soliton model predictions for $x\Delta\bar{u}$ and $x\Delta\bar{d}$ at the scale $\mu \approx$ 600 MeV.

number conserved — this is an analog of the nonrenormalizability of the Wess–Zumino term in pure chiral models.

Several comments should be made about the calculations of $\Delta u + \Delta d$ within the same model by Wakamatsu *et al.* who published three different versions of the calculation in papers [7, 20, 21]. In paper [20] one of the terms was overlooked. This mistake was corrected by the authors of [11]. The revised version of calculation of Wakamatsu *et al.* was published in [7]. In this paper the question about the anomalous difference $[\Delta u(x) + \Delta d(x)]^{\omega_0}_{\rm occ} - [\Delta u(x) + \Delta d(x)]^{\omega_0}_{\rm non-occ}$ was investigated only numerically but the accuracy of the calculation did not allow the authors to draw any conclusions concerning whether this difference vanishes or not. Actually the numerical accuracy of the agreement between the two representations which we observe in our calculation (see Fig. 2), and which is necessary for a proper evaluation of the parton distributions, is of two orders of magnitude better than the same difference presented in [7]. The practical solution accepted in [7] was to use $[\Delta u(x) + \Delta d(x)]^{\omega_0}_{\rm occ}$ for $x > 0$ and $[\Delta u(x) + \Delta d(x)]^{\omega_0}_{\rm non-occ}$ for $x < 0$ (*i.e.* for the antiquark distribution). As it was explained above, $\Delta u(x) + \Delta d(x)$ should not be regularized contrary to what the authors of Ref. [7] do.

The first moment of the $\Delta u(x) + \Delta d(x)$ gives the singlet axial charge. Our result of $g_A^{(0)} = \int_{-1}^{1} dx (\Delta u + \Delta d)(x) = 0.35$ agrees with the calculation performed in other works [14,26]. Note that in the calculation of this charge no ultraviolet regularization was used.

6. Conclusions

We have proved that the representation of singlet polarized (anti)quark distributions in the chiral quark–soliton model as a sum over quark orbitals is ultraviolet finite and free of quantum anomalies. This is a serious check of the consistency of the quark–soliton model.

In fact, the cancellation of quantum anomalies in the model is related to the fact that certain basic properties of QCD as a local quantum field theory are realized in the model. The equivalence of the summation over occupied and non-occupied states is directly connected to anticommutativity of fermion fields at space-like intervals. Actually this locality property has a direct relation to the positivity of quark and antiquark densities in the quark soliton model [8, 9].

Another consequence of the cancellation of anomalies is that the model results for the parton distributions are compatible with the charge conjugation invariance: the quark distributions in nucleon coincide with the antiquark distributions in the antinucleon.

From the practical point of view the results presented in this paper allow us to conclude that for the calculation of the singlet polarized quark and antiquark distributions no Pauli–Villars subtraction is needed. Additionally the numerical check of the cancellation of the anomalies is a powerful tool to control the accuracy of the numerics.

We have computed the singlet polarized quark and antiquark distributions which arise in the subleading order of $1/N_{\rm c}$ expansion. We found the quark distribution $\Delta u(x) + \Delta d(x)$ to be in a reasonable agreement with GRSV [22] parametrization of parton distributions at low normalization point.

Concerning the comparison of the parton distributions computed in the current model with the fits to experimental data it is often asked whether the model deals with current or constituent quarks. Actually one should be rather careful with the term "constituent quark" since the object is strictly speaking absent in QCD and appears only in the context of various models and heuristic approximations. The chiral quark soliton model used in the present paper can be derived from QCD by assuming the QCD vacuum to be dominated by a gas of instantons. Thus in the present model the nucleon parton distributions are computed starting from QCD expressions. Certain approximations are used, in particular, the functional integral over the gluon fields is approximated by the statistical average over the instanton medium, large $N_{\rm c}$ limit is taken *etc.* However, whatever approximations are used, we always deal with the quark fields inherited from the QCD action and in this sense our quark distributions are usual *current quark distributions.* The structure of the model calculation also fixes the scale at which the parton distributions are computed — it is determined by the inverse average instanton size which is of order of 600 MeV. In this sense the comparison of our calculations with the phenomenological fits is quite justified (with a certain care about the accuracy of the model and the region of x where the model makes sense as well as about the assumptions made in the fits to experimental data).

A remarkable prediction of our model is that the polarized distributions of u and d antiquarks are essentially different, see Fig. 4. Usually, in parametrizations of polarized parton distributions, it was assumed that $\Delta\bar{u}(x) = \Delta\bar{d}(x)$, which is not confirmed by our model calculations (see Fig. 4). It would be extremely interesting to include into the fits of the data the flavor decomposition pattern for polarized antiquarks obtained in our model calculations. Future experiments at HERA and RHIC investigating Drell–Yan lepton pair production in polarized nucleon–nucleon collisions will clarify the situation. For a discussion see [23,24]. Let us note that in the singlet polarized channel under the evolution the quark distributions mix with polarized gluon distribution. Analysis of Refs. [8,27] in the framework of the instanton model of the QCD vacuum shows that the gluon distribution is parametrically smaller (suppressed by M^2/M_{PV}^2) than quark and antiquark distributions. In order to obtain a non-zero result one has to go beyond the zero-mode approximation of Ref. [27] and/or consider contributions of many instantons. Both ways would lead to extra powers of the packing fraction of instantons. This means that gluons at low normalization point inside the nucleons appear only at the level of M^2/M_{PV}^2.

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