ON THE LAGRANGIAN DERIVATION OF THE INTERACTIONS BETWEEN A CHERN–SIMONS TERM AND A COMPLEX SCALAR FIELD

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Consistent interactions that can be added between a Chern–Simons term and a massless complex scalar field are investigated by means of cohomological arguments in the framework of the antibracket-antifield BRST formalism.

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An important step in the evolution of the antibracket-antifield method was the cohomological understanding of the Lagrangian BRST symmetry [1, 2] which provided a useful tool for analysing many interesting topics, such as the construction of consistent interactions in gauge theories [3–6] by means of the deformation theory applied to the solution of the master equation combined with cohomological techniques. This treatment offered an appropriate background for inferring many models of deep interest in theoretical physics, like Yang–Mills theories [7], the Chapline–Manton model [8], p-forms and chiral p-forms [9–13], as well as nonlinear gauge theories [14]. Along the same line, Einstein's gravity theory [15] and four- and eleven-dimensional supergravity [16] have been approached from the point of view of their deformations. Lately, the problem of obtaining consistent deformations has been extended also at the Hamiltonian level [17–20].

The aim of this paper is to study the consistent Lagrangian interactions that can be added between a Chern–Simons term and a massless complex scalar field within the antibracket-antifield deformation setting. Our analysis goes as follows. We start from a free theory that describes an Abelian three-dimensional Chern–Simons term and a charged scalar field, and construct the associated free Lagrangian BRST differential s, which simply decomposes as the sum between the Koszul–Tate differential δ and the exterior longitudinal derivative along the gauge orbits γ, $s=\delta+\gamma$. From these elements, we construct the consistent deformations of the solution to the master equation. In order to generate the non-integrated first-order deformation, which belongs to $H^0\left(s|d\right)$ ($H^0\left(s|d\right)$ denotes the zeroth order cohomological space of s modulo the exterior space-time derivative d), we perform its expansion according to an auxiliary degree, called antighost number, and assume that we can take the last representative of this expansion to be annihilated by γ. In consequence, we have to know the cohomology of γ, $H\left(\gamma\right)$. In the meantime, the computation of the before last term of this expansion requires the knowledge of $H\left(\delta|d\right)$. After the computation of these cohomologies, we appropriately solve the deformation equations, finally obtaining the deformed Lagrangian action and its gauge transformations. The antighost number zero piece in the first-order deformation takes the form $j^\mu A_\mu$, where j^μ stands for a conserved current of the massless complex scalar field corresponding to a global one-parameter invariance, while the antighost number one component shows that in the context of the deformed theory the matter fields will carry some gauge invariances, representing nothing but the gauge version of the above global one-parameter symmetry. As the conserved current j^μ is not invariant under the above mentioned gauge transformations of the matter fields, there appear nontrivial second-order deformations. The resulting model describes precisely the three-dimensional minimal coupling between a Chern–Simons term and a massless complex scalar field.

We begin with a three-dimensional system describing an Abelian Chern–Simons term and a massless complex scalar field

$$S_0^L\left[A_\mu,\varphi,\bar{\varphi}\right]=\int d^3x\left(\tfrac{1}{2}\varepsilon^{\mu\nu\rho}A_\mu F_{\nu\rho}+\left(\partial_\mu\varphi\right)\left(\partial^\mu\bar{\varphi}\right)\right), \tag{1}$$

where $F_{\nu\rho}=\partial_\nu A_\rho-\partial_\rho A_\nu$ and the bar operation signifies complex conjugation. This free model is invariant under the gauge transformations

$$\delta_\varepsilon A_\mu=\partial_\mu\varepsilon\,,\quad \delta_\varepsilon\varphi=0\,,\quad \delta_\varepsilon\bar{\varphi}=0\,. \tag{2}$$

A consistent deformation of the free action (1) and of its gauge invariances (2) defines a deformation of the corresponding solution to the master equation that preserves both the master equation and the field/antifield spectra. So, if $S_0^L\left[A_\mu,\varphi,\bar{\varphi}\right]+g\int d^3x\alpha_0+O\left(g^2\right)$ stands for a consistent deformation of the free action, with deformed gauge transformations $\tilde{\delta}_\varepsilon A_\mu=\partial_\mu\varepsilon+g\beta_\mu+O\left(g^2\right)$, $\tilde{\delta}_\varepsilon\varphi=g\rho+O\left(g^2\right)$, $\tilde{\delta}_\varepsilon\bar{\varphi}=g\lambda+O\left(g^2\right)$, then the deformed solution to the master equation

$$\tilde{S}=S+g\int d^3x\alpha+O\left(g^2\right), \tag{3}$$

satisfies $\left(\tilde{S},\tilde{S}\right)=0$, where the symbol $(,)$ signifies the antibracket and

$$S=S_0^L\left[A_\mu,\varphi,\bar{\varphi}\right]+\int d^3xA^{*\mu}\partial_\mu\eta, \tag{4}$$

represents the solution to the master equation for the free theory, while $\alpha=\alpha_0+A^{*\mu}\tilde{\beta}_\mu+\varphi^*\tilde{\rho}+\bar{\varphi}^*\tilde{\lambda}+$'more' ($g$ is the so-called deformation parameter or coupling constant). The terms $\tilde{\beta}_\mu$, $\tilde{\rho}$ and $\tilde{\lambda}$ are obtained from the functions β_μ, ρ and λ where we replace the gauge parameter ε with the fermionic ghost η. The fields carrying a star denote the antifields of the corresponding fields or ghosts. The Grassmann parity of an antifield is opposite to that of the corresponding field/ghost.

The pure ghost number (pgh) and the antighost number (antigh) of the fields, ghosts and antifields are valued like

$$\mathrm{pgh}\left(\Phi^{\alpha_0}\right)=\mathrm{pgh}\left(\Phi^*_{\alpha_0}\right)=0\,,\quad \mathrm{pgh}\left(\eta\right)=1\,,\quad \mathrm{pgh}\left(\eta^*\right)=0 \tag{5}$$

$$\mathrm{antigh}\left(\Phi^{\alpha_0}\right)=0\,,\ \mathrm{antigh}\left(\Phi^*_{\alpha_0}\right)=1\,,\ \mathrm{antigh}\left(\eta\right)=0\,,\ \mathrm{antigh}\left(\eta^*\right)=2\,, \tag{6}$$

where we employed the notations

$$\Phi^{\alpha_0}=\left(A_\mu,\varphi,\bar{\varphi}\right),\quad \Phi^*_{\alpha_0}=\left(A^{*\mu},\varphi^*,\bar{\varphi}^*\right). \tag{7}$$

The BRST symmetry of the free theory $s\bullet=(\bullet,S)$ simply decomposes as the sum between the Koszul–Tate differential δ and the exterior derivative along the gauge orbits γ, $s=\delta+\gamma$, where the degree of δ is the antighost number ($\mathrm{antigh}\,(\delta)=-1$, $\mathrm{antigh}\,(\gamma)=0$), and that of γ is the pure ghost number ($\mathrm{pgh}\,(\gamma)=1$, $\mathrm{pgh}\,(\delta)=0$). The grading of the BRST differential is named ghost number (gh) and is defined in the usual manner like the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}\,(s)=1$. The actions of δ and γ on the generators from the BRST complex can be written as

$$\delta\Phi^{\alpha_0}=0,\quad \delta\eta=0,\quad \delta A^{*\mu}=-\varepsilon^{\mu\nu\rho}F_{\nu\rho}\,, \tag{8}$$

$$\delta\varphi^*=\partial_\mu\partial^\mu\bar{\varphi},\quad \delta\bar{\varphi}^*=\partial_\mu\partial^\mu\varphi\,,\quad \delta\eta^*=-\partial_\mu A^{*\mu}\,, \tag{9}$$

$$\gamma A_\mu=\partial_\mu\eta\,,\quad \gamma\varphi=\gamma\bar{\varphi}=0\,, \tag{10}$$

$$\gamma\eta=\gamma\Phi^*_{\alpha_0}=\gamma\eta^*=0\,. \tag{11}$$

The master equation $\left(\tilde{S},\tilde{S}\right)=0$ holds to order g if and only if

$$s\alpha=\partial_\mu k^\mu, \tag{12}$$

for some local k^{μ}. This means that the nontrivial first-order consistent interactions belong to $H^{0}(s|d)$. In the case where α is a coboundary modulo d $(\alpha = sb + \partial_{\mu}c^{\mu})$, then the deformation is trivial (it can be eliminated by a redefinition of the fields). In order to investigate the solution of (12), we develop α according to the antighost number

$$\alpha = \alpha_0 + \alpha_1 + \dots \alpha_J, \quad \mathrm{antigh}\,(\alpha_k) = k\,, \tag{13}$$

where the last term can be assumed to be annihilated by γ, $\gamma\alpha_J = 0$. Thus, we need to know the cohomology of γ, $H(\gamma)$, in order to determine the terms of highest antighost number in α. From (10),(11) it is easy to see that the cohomology of γ is generated by $F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$, φ, $\bar{\varphi}$, the antifields together with their derivatives, as well as by the undifferentiated ghost η. If we denote by $e^{M}(\eta)$ a basis in the space of the polynomials in the ghosts, it follows that the general solution to the equation $\gamma a = 0$ takes the form

$$a = a_M\left([F_{\mu\nu}], [\varphi], [\bar{\varphi}], \left[\Phi^{*}_{\alpha_0}\right], [\eta^{*}]\right) e^{M}(\eta)\ , \tag{14}$$

where the notation $f[q]$ signifies that f depends on q and its derivatives up to a finite order. As there is a single ghost field, which in addition is fermionic, it follows that the only nontrivial element of the basis $e^{M}(\eta)$ is η itself, hence we find that

$$a = a_1\left([F_{\mu\nu}], [\varphi], [\bar{\varphi}], \left[\Phi^{*}_{\alpha_0}\right]\right)\eta\,. \tag{15}$$

In this way, the expansion (13) stops after the first two terms, $\alpha = \alpha_0 + \alpha_1$, where

$$\alpha_1 = \tilde{\alpha}_1\left([F_{\mu\nu}], [\varphi], [\bar{\varphi}], \left[\Phi^{*}_{\alpha_0}\right]\right)\eta\,. \tag{16}$$

The equation (12) projected on antighost number zero becomes $\delta\alpha_1 + \gamma\alpha_0 = \partial_{\mu}n^{\mu}$. For the last equation to possess solution, it is necessary that $\tilde{\alpha}_1$ belongs to $H_1(\delta|d)$, hence

$$\delta\tilde{\alpha}_1 = \partial_{\mu}m^{\mu}\,. \tag{17}$$

Using (9) we obtain

$$\delta\left[i\left(\bar{\varphi}^{*}\bar{\varphi} - \varphi^{*}\varphi\right)\right] = \partial_{\mu}\left[i\left(\bar{\varphi}\partial^{\mu}\varphi - \varphi\partial^{\mu}\bar{\varphi}\right)\right]. \tag{18}$$

If we compare (18) with (17), we infer that $\tilde{\alpha}_1 = i\left(\bar{\varphi}^{*}\bar{\varphi} - \varphi^{*}\varphi\right)$, which further yields

$$\alpha_1 = i\left(\bar{\varphi}^{*}\bar{\varphi} - \varphi^{*}\varphi\right)\eta\,. \tag{19}$$

On the other hand, the equation (18) expresses nothing but the cohomological formulation of Noether's theorem, which provides the conservation of the current

$$j^{\mu} = i\left(\bar{\varphi}\partial^{\mu}\varphi - \varphi\partial^{\mu}\bar{\varphi}\right)\ , \tag{20}$$

corresponding to the global one-parameter invariance $\Delta\varphi = -i\varphi\xi$, $\Delta\bar{\varphi} = i\bar{\varphi}\xi$ of the complex scalar field action. With the help of (19), (10) and (11) we deduce that

$$\alpha_0 = -i\left(\bar{\varphi}\partial^\mu\varphi - \varphi\partial^\mu\bar{\varphi}\right)A_\mu\,, \tag{21}$$

which further leads to

$$\delta\alpha_1 + \gamma\alpha_0 = \partial_\mu\left[-i\left(\bar{\varphi}\partial^\mu\varphi - \varphi\partial^\mu\bar{\varphi}\right)\eta\right]. \tag{22}$$

In this way, we constructed the first-order deformation of the solution to the master equation like

$$S_1 = i\int d^3x\left(\left(\bar{\varphi}^*\bar{\varphi} - \varphi^*\varphi\right)\eta - \left(\bar{\varphi}\partial^\mu\varphi - \varphi\partial^\mu\bar{\varphi}\right)A_\mu\right). \tag{23}$$

The second-order deformation equation takes the form

$$s\sigma + \tfrac{1}{2}\chi = \partial_\mu p^\mu, \tag{24}$$

where $S_2 = \int d^3x\sigma$ and $(S_1, S_1) = \int d^3x\chi$. After some computation we find that (S_1, S_1) is non-vanishing

$$(S_1, S_1) = \int d^3x 4\eta\partial_\mu\left(\varphi\bar{\varphi}A^\mu\right) = \int d^3x\chi\,, \tag{25}$$

due to the fact that the current j^μ is not invariant under the gauge version of the rigid transformations of the complex scalar field (the antibracket (S_1, S_1) reduces to the antibracket between the first and second terms in the right-hand side of (23), that is proportional with the gauge variation of j^μ). Thus, we have to solve the second-order deformation equation (24), which requires that χ given in (25) is an s-coboundary modulo d. This is indeed the case because

$$\chi = s\left(-2\varphi\bar{\varphi}A^\mu A_\mu\right) + \partial_\mu\left(4\varphi\bar{\varphi}A^\mu\eta\right)\,, \tag{26}$$

such that

$$S_2 = \int d^3x\varphi\bar{\varphi}A^\mu A_\mu\,. \tag{27}$$

If we examine the third-order deformation equation, we observe that (S_1, S_2) is vanishing, hence we can safely take $S_3 = 0$. The higher-order equations are then satisfied with the choice $S_4 = S_5 = \cdots = 0$.

Putting together the above results, we infer that

$$\begin{aligned}\tilde{S} \;=\; & \int d^3x\left(\tfrac{1}{2}\varepsilon^{\mu\nu\rho}A_\mu F_{\nu\rho} + D_\mu\varphi\overline{D^\mu\varphi}\right.\\ & \left. + ig\left(\bar{\varphi}^*\bar{\varphi} - \varphi^*\varphi\right)\eta + A^*_\mu\partial^\mu\eta\right)\,,\end{aligned} \tag{28}$$

represents the full consistent solution to the master equation of our deformed problem, where the covariant derivative is defined through

$$D_\mu = \partial_\mu + igA_\mu \,. \tag{29}$$

The antifield-independent piece in (28)

$$\tilde{S}_0 = \int d^3x \left(\frac{1}{2} \varepsilon^{\mu\nu\rho} A_\mu F_{\nu\rho} + D_\mu \varphi \overline{D^\mu \varphi} \right) , \tag{30}$$

describes nothing but the Lagrangian interaction between a Chern–Simons term and a charged scalar field in three dimensions, while the terms linear in the antifields of the matter fields emphasize that the gauge transformation of the complex scalar field reads as

$$\tilde{\delta}_\varepsilon \varphi = -ig\varphi\varepsilon, \; \tilde{\delta}_\varepsilon \bar{\varphi} = ig\bar{\varphi}\varepsilon \,, \tag{31}$$

while that associated with the vector field is kept unchanged. Thus, the added interaction results in gauging the initial rigid symmetry of the matter fields at the level of the gauge transformations, although there appear coupling terms of order two in the deformation parameter.

To conclude with, in this paper we have investigated the consistent interactions that can be introduced between a three-dimensional Chern–Simons term and a massless complex scalar field. Our analysis is based on cohomological arguments. The first-order deformation belongs to $H^0(s|d)$ and takes the form $j^\mu A_\mu$, where j^μ is the conserved current of the matter theory corresponding to a global one-parameter symmetry. This current is correlated with $H_1(\delta|d)$ by means of Noether's theorem. As the conserved current j^μ is not invariant under the gauge version of the rigid transformations of the matter field, second-order interaction terms are present. As a consequence, we obtain the three-dimensional interaction vertices corresponding to the minimal coupling between the Chern–Simons term and a charged scalar field. Meanwhile, the scalar field becomes endowed with some gauge transformations, which are nothing but the gauge version of the initial rigid ones.

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