BOSE–EINSTEIN EFFECT FROM ASYMMETRIC SOURCES IN MONTE CARLO GENERATORS

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We discuss the implementations of the Bose–Einstein effect from asymmetric sources in Monte Carlo generators. A comparison of LEP data with results from the PYTHIA/JETSET code with the standard procedure imitating the effect and with the results from the weight method (with weights depending in various ways on components of momenta differences) is presented. We show that in this last method one can reproduce the experimental hierarchy of the source radii.

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1. Introductory remarks

Recently one observes a renewal of interest in analysing the space–time structure of sources in multiparticle production by means of Bose–Einstein (BE) interference [1]. Such analysis followed the example of astrophysical investigations of Hanbury-Brown and Twiss [2]. The main motivation of this renewal was the analysis of the $e^+e^- \to W^+W^-$ process which became available at the LEP2. It was suggested [3, 4] that the BE interference (and/or colour reconnection effects) between the strings from two W decays may shift the W mass value fitted from the two jet mass distributions by as much as a few hundred MeV, thus making this channel useless for precise tests of the standard model. However, other investigations suggested that such a big shift is unlikely [5–7]. Experimentally, the existence of interference effects between strings is still debatable [8].

Investigating such subtle effects became possible when instead of the standard approach [9] one started to model this effect in Monte Carlo generators. There are several methods of modelling: as the "afterburner" for which the original MC provides a source [10, 11], by shifting the momenta [12] or by adding weights to generated events [13, 14]. Another approach was set forward by Andersson and collaborators who used the symmetrisation inside fragmenting string [15] to model the effect for a single string [16]. Here we consider the most widely used methods of shifting momenta and weighting events.

Another reason to analyse the BE effect were the efforts to estimate size and shape of source of particle production in various processes (in particular for coming RHIC data). The analysis of BE effect in 3 dimensions is supposed to reflect the spatial source asymmetry. Such analysis was done for the LEP data at the Z^0 peak [17] which have very high statistics and good accuracy.

In this paper we compare the 3-dimensional data for BE effect from LEP with the results of the standard momentum shifting procedure and of the weight method. In the next section we present the data discussing in detail the definitions and the procedures used by the experimental groups. In the third section we compare them with the results obtained from the PYTHIA/JETSET MC generator using the original procedure modelling the effect by momentum shifting and with the results from the weight method with weights independent on spatial orientation of momenta. Fourth section contains the results for asymmetric weights. Our conclusions are presented in the last section.

2. Experimental data

Although the discussion of the shape of asymmetric sources in the framework of BE interference concerned most often the heavy ion collisions, the best experimental data with highest statistics exist for the e^+e^- annihilation at the Z^0 peak. In the following we concentrate our attention on the L3 data [18] which discuss the ratios using "uncorrelated background" and three different radii to parametrise the data. The DELPHI data [19] are parametrised with only two radii, and the OPAL data [20] use the like/unlike ratio which requires a cut off of the resonance affected regions even in double ratios.

As in the L3 paper [18] we use for each pair of identical pions three components of the invariant $Q^2 = -(p_1 - p_2)^2$: $Q^2_{\mathrm{L}}, Q^2_{\mathrm{out}}, Q^2_{\mathrm{side}}$ defined in the LCMS (Longitudinal Centre-of-Mass System), where the sum of three — vector momenta is perpendicular to the thrust axis. The Q_{out} component is measured along this sum, the Q_{L} along the thrust axis, and Q_{side} is the projection of Q on the axis perpendicular to these two directions [18,21].

We define a "double ratio" in the same way as in the L3 paper using a reference sample from mixed events:

$$R_2(p_1,p_2) = \frac{\frac{\rho_2}{\rho_2^{\mathrm{mix}}}}{\frac{\rho_2^{\mathrm{MC}}}{\rho_2^{\mathrm{mix,MC}}}} \,. \tag{1}$$

This "double ratio" is parametrised by

$$\begin{aligned} R_2(Q_{\mathrm{L}},Q_{\mathrm{out}},Q_{\mathrm{side}}) &= \gamma[1+\delta Q_{\mathrm{L}}+\varepsilon Q_{\mathrm{out}}+\zeta Q_{\mathrm{side}}] \\ &\times \left[1+\lambda\exp(-R_{\mathrm{L}}^2Q_{\mathrm{L}}^2-R_{\mathrm{out}}^2Q_{\mathrm{out}}^2-R_{\mathrm{side}}^2Q_{\mathrm{side}}^2-2\rho_{\mathrm{L,out}}R_{\mathrm{L}}R_{\mathrm{out}}Q_{\mathrm{L}}Q_{\mathrm{out}})\right] . \end{aligned} \tag{2}$$

The first bracket reflects possible traces of long-distance correlations; the last term in the second bracket seems to be negligible when fitting data and will be omitted in the following.

By fitting the parameters R_{L} and R_{side} we get some information on the geometric radii in the longitudinal and transverse directions (respective to the thrust axis). R_{out} reflects both the spatial extension and time duration of the emission process.

In the L3 data the fit region in all three variables extends to 1.04 GeV and is divided into 13 bins, which gives 2197 points fitted with 8 parameters. The fit parameters δ,ε and ζ are rather small; this means that the observed BE enhancement is rather well approximated with a Gaussian. The value of the parameter λ is fitted as 0.41 ± 0.01.

The fitted values of radii (in fm) are as follows:

$$R_{\mathrm{L}}=0.74\pm0.02^{+0.04}_{-0.03}\,,\quad R_{\mathrm{out}}=0.53\pm0.02^{+0.05}_{-0.06}\,,\quad R_{\mathrm{side}}=0.59\pm0.01^{+0.03}_{-0.13}\,.$$

We see clear evidence for source elongation: $R_{\mathrm{side}}/R_{\mathrm{L}}$ is smaller than one by more than four standard deviations.

It is instructive to inspect the projections of the double ratio on the three axes Q_{L}, Q_{out} and Q_{side}. This is done by restricting the values of two other variables to less than 0.24 GeV, plotting the histograms in the third variable in bins of width 0.08 GeV and constructing the double ratio in this variable. The results are shown in Fig. 1 as presented by the L3 collaboration [18]. The values of double ratios fall down smoothly from the maxima of about 1.25 at Q_i close to zero to the plateau at 1. It is rather difficult to see the differences between three plots, but superposing them one may note that the fall is fastest for Q_{L} as expected from the fact that the fitted value of parameter R_{L} is bigger than the values of R_{out} and R_{side} quoted above.

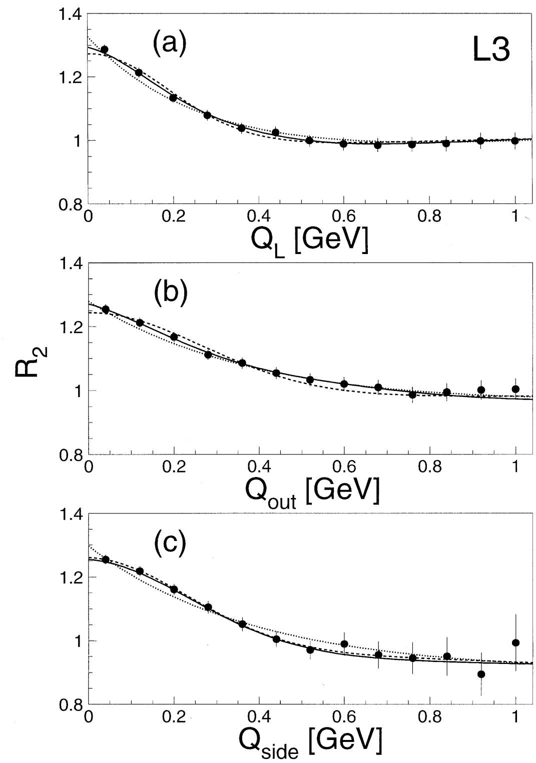

Fig. 1. Projections of the double ratio (1) from the data of the L3 collaboration on the three axes $Q_{\rm L}$, $Q_{\rm out}$ and $Q_{\rm side}$.

3. Asymmetric effects from symmetric models

The geometric interpretation of data requires a comparison with the results from the standard MC procedures modelling the BE effect. In the L3 paper such an analysis is given for the standard LUBOEI procedure built into the JETSET Monte Carlo generator. This procedure modifies the final state by a shift of momenta for each pair of identical pions. The shift is calculated to enhance low values of Q^2 and to reproduce the experimental ratio in this variable. The function defining this shift is

$$f\left(Q^2\right) = 1 + \lambda_{\rm in} \exp\left(-R_0^2 Q^2\right) \, . \tag{3}$$

The superposition of the procedure for all the pairs and subsequent rescaling (restoring the energy conservation) makes the connection between the parameters of the shift $\lambda_{\rm in}$, R_0 and the parameters describing the resulting double ratio in Q^2

$$R_2\left(Q^2\right) = \frac{\frac{\rho_2}{\rho_2^{\mathrm{mix}}}}{\frac{\rho_2^{\mathrm{MC}}}{\rho_2^{\mathrm{mix,MC}}}} \tag{4}$$

(which may be parametrised analogously to (3)) rather indirect.

Using the JETSET parameters adjusted to all the L3 data and the LUBOEI parameters fitted to describe the BE ratio in Q^2 the authors of the L3 paper calculated the same quantities as measured in the experiment. The projections of R_2 are qualitatively very similar to the experimental ones. However, the fit to the 3-dimensional distribution gives results different from data. The ratio $R_{\mathrm{side}}/R_{\mathrm{L}}$ is not smaller but greater than one; the fitted values (in fm) are:

$$R_{\mathrm{L}} = 0.71 \pm 0.01\,, \quad R_{\mathrm{out}} = 0.58 \pm 0.01\,, \quad R_{\mathrm{side}} = 0.75 \pm 0.01\,.$$

We confirmed these numbers in our calculations. We found also that the results are sensitive to the JETSET parameters. Using the default values instead of the L3 values we obtained a significantly smaller value of R_{out} (below 0.5) and significantly smaller λ. Other values are less affected and $R_{\mathrm{side}}/R_{\mathrm{L}}$ is still bigger than 1.

We have also checked how the results depend on the source radius R_0 and on the incoherence parameter λ_{in} assumed in the LUBOEI input function (3). In all cases we get $R_{\mathrm{side}} > R_{\mathrm{L}} > R_{\mathrm{out}}$, although the input function was obviously symmetric. The values of R_{side} and R_{L} are proportional to R_0, whereas R_{out} changes much less; the dependence on λ_{in} is very weak. The output value of λ decreases quite strongly with increasing R_0 and increases with λ_{in}. No choice of input parameters gives the values of R_i compatible with data. This is shown in Fig. 2(a).

Another interesting observation is that to fit the L3 data one needs $\lambda = 1.5$, which is beyond the physically acceptable value of 1. This supports our doubts about usefulness of the LUBOEI procedure in understanding the experimental results (although certainly it is the most practical description of data).

In fact, there is one more degree of freedom in the prescription for modelling the BE effect: the definition of direct pions. Since the decay products of long-living resonances and of particles decaying by electroweak interactions are born far from the original collision point, their effective source size is much bigger than that for direct pions. Thus they contribute to the BE effect for momentum differences much below the experimental resolution and should not be taken into account.

In the LUBOEI procedure this distinction is made by the decay width of unstable particles: only pions from the decay of particles with the width

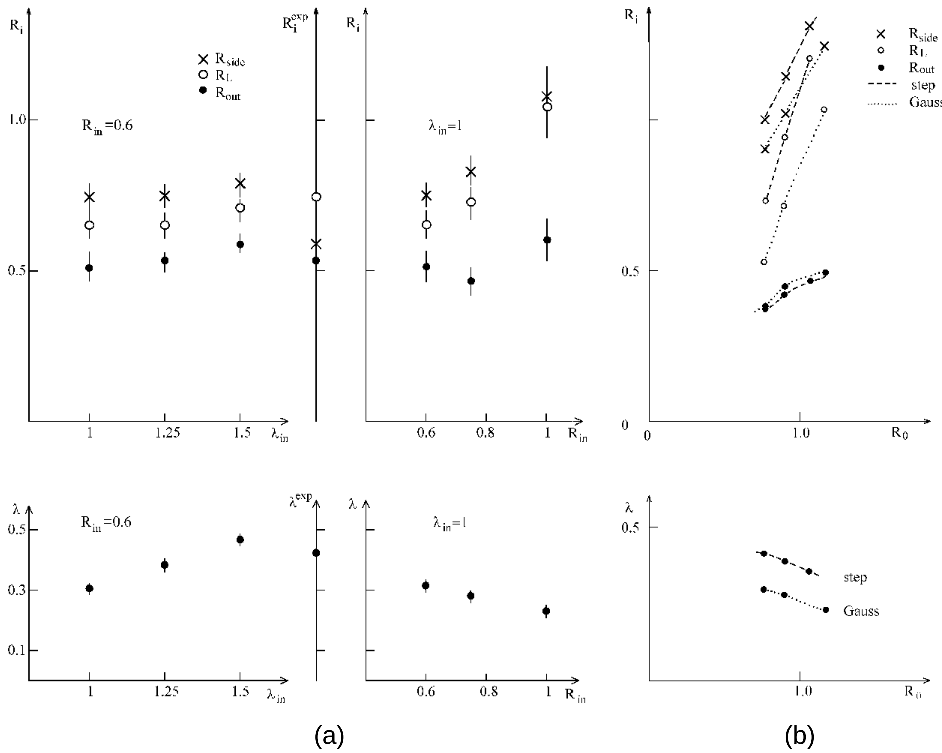

Fig. 2. Fit parameters λ and R_i as functions of the input parameters (a) for the LUBOEI procedure, (b) for the weight method. Experimental values are shown on a separate vertical axis.

above 20 MeV and the direct ones are included in the momentum shifting procedure. Obviously, this is just a rough prescription which may be changed, and the values of fit parameters may change then quite strongly. The user of the procedure should be aware that (according to author's warning) it works properly only when called from LUEXEC; if LUBOEI is called directly from the master program, all pions are regarded as the direct ones.

The problems of LUBOEI procedure in describing the asymmetry of experimental distributions are not the first ones noted in applications to describe various data. It has been already indicated that the procedure with parameters fitted to the two-particle data fails to reproduce the three-particle spectra [22] and the semi-inclusive data [23]. Moreover, as already noted, the fitted values of parameters needed in the input function (3) are quite different from the values one would get fitting the resulting double ratio (4) to the same form [24, 25]. Thus, it seems to be difficult to learn something reliable on the space–time structure of the source from the values of the fit parameters in this procedure.

All this led to a revival of weight methods, known for quite a long time [26], but plagued also with many practical problems. The method is clearly justified with in the formalism of the Wigner functions, which allows one to represent (after some simplifying assumptions) any distribution with the BE effect built in as a product of the original distribution and the weight factor, depending on the final state momenta [13]. With an extra assumption on factorisation in momentum space we may write the weight factor for a final state with n identical bosons as

$$W(p_1, \ldots p_n) = \sum \prod_{i=1}^{n} w_2(p_i, p_{P(i)}) \,, \tag{5}$$

where the sum extends over all permutations $P_n(i)$ of n elements, and $w_2(p_i, p_k)$ is a two-particle weight factor reflecting the effective source size. Problems with an enormous number of possible terms in this sum may be cured by a proper clustering procedure [14]. A reasonable description of the effect in Q^2 is obtained with a simple Gaussian form of the weight factor

$$w_2(p_1, p_2) = \exp\left[-(p_1 - p_2)^2 \frac{R_0^2}{2}\right] \,, \tag{6}$$

or, even simpler, a step function form with $w_2 = 1$ for some range of $-(p_1 - p_2)^2 < 1/R_0^2$ and $w_2 = 0$ outside [27].

In this method we may repeat the same calculation as done for the LUBOEI procedure. Obviously the weights may be calculated for the events generated by any MC generator, but here we restrict ourselves to the results from the same PYTHIA/JETSET code which was used above. The resulting double ratios are not that smooth and monotonically decreasing as in the data or from the LUBOEI procedure (which is the usual drawback of the weight methods). However, the major features are surprisingly similar: with weight factors depending only on Q^2 we get different values of fitted R_i parameters. Moreover, the hierarchy of parameters is the same: $R_{\text{side}} > R_{\text{L}} > R_{\text{out}}$. This suggests that the asymmetry is generated by the jet-like structure of final states and not by any specific features of the procedure modelling the BE effect. In Fig. 2(b) we show the values of the fit parameters as functions of R_0 for a Gaussian as well as the θ-like weight factors. Again, no choice of the input parameters allows to describe the data.

The comparison of two methods is not straightforward. In particular, one should make sure that the same definition of "direct" pions is used. The weights are calculated after the event was fully generated (and all the decays of unstable particles occurred). Therefore, one should define the pions which are counted as direct ones. We did it by enumerating particles which contribute significantly to the pion production and live too long for their decay products to produce a visible BE effect (using the same limit for decay width as in LUBOEI). If one enumerates the short-living resonances and adds their decay products to the direct pions, one should remember that this list is different in various options of JETSET (*e.g.* the option used by the L3 collaboration takes into account mesons built from quarks with non-zero orbital momentum, which are neglected in the default version).

The results presented in this section suggest that one should be careful with the geometric interpretation of the data. If one gets asymmetric distributions from the generator without assuming explicitly space asymmetry of the source, it is not clear how the assumed asymmetry will be reflected in the results.

4. Asymmetric weights

One may get more information on the problem of asymmetric BE effect in MC generators using the asymmetric weight method, *i.e.* introducing weight factors which depend in a different way on $Q_{\rm L} = |p_{1{\rm L}} - p_{2{\rm L}}|$, $Q_{\rm side} = |p_{1{\rm side}} - p_{2{\rm side}}|$ and $Q_{\rm out} = |p_{1{\rm out}} - p_{2{\rm out}}|$, where the indices denote the components defined in the previous section. We have used two such generalisations of a Gaussian weight factor (6)

$$w_2(Q_{\rm L}, Q_{\rm out}, Q_{\rm side}) \;=\; \exp\frac{-Q_{\rm L}^2(R_{\rm L}^{\rm in})^2 - Q_{\rm out}^2(R_{\rm out}^{\rm in})^2 - Q_{\rm side}^2(R_{\rm side}^{\rm in})^2}{2} \qquad (7)$$

and

$$w_2(Q_{\rm L}, Q_{\rm out}, Q_{\rm side}) \;=\; \exp\frac{-Q_{\rm L}^2(R_{\rm L}^{\rm in})^2 - (1-\beta^2)Q_{\rm out}^2(R_{\rm out}^{\rm in})^2 - Q_{\rm side}^2(R_{\rm side}^{\rm in})^2}{2}, \qquad (8)$$

where β is defined as

$$\beta = \frac{p_{\rm out1} + p_{\rm out2}}{E_1 + E_2}\,. \qquad (9)$$

The weight factor (8) reduces to the symmetric weight factor (6) when $R_{\rm L}^{\rm in} = R_{\rm out}^{\rm in} = R_{\rm side}^{\rm in} = R_0$. The formula (8) gives nearly the same results as the formula (7) when $R_{\rm out}^{\rm in}$ is multiplied by 2. We have used both forms finding no definite preference for any of them.

Fluctuations in the weight values are large and the resulting fluctuations in the values of double ratios describing the BE effect are bigger than for the momentum shifting method. Therefore, it is necessary to use large samples of generated events. We found that for the samples of 5 million events, the fluctuations visible in the plots of projections of double ratios on components of Q are comparable with those seen in the experimental data shown in Fig. 1. In fact, the plots obtained for the weight method with the input radii around 0.5 fm are visually similar to those of experimental data. However, the fitted values of the parameters from formula (2) are different.

Since for the symmetric weights the resulting fitted values of $R_{\rm side}$ are bigger than the values of $R_{\rm L}$ (contrary to the inequality seen in the data), it seemed natural to take the input value of $R^{\rm in}_{\rm side}$ smaller than $R^{\rm in}_{\rm L}$. Indeed decreasing $R^{\rm in}_{\rm side}$ one reduces the resulting fitted value of $R_{\rm side}$ but this dependence is not linear and saturates for $R^{\rm in}_{\rm side}$ around 0.3 fm. Moreover, the fitted values of other parameters change as well although their input values were not changed. Therefore, looking for the best set of input parameters in the formula for weights is a rather involved procedure.

Let us add two more remarks. A replacement of the products of Gaussians by the proper products of step functions in the formulae for weights (7), (8) leads to even bigger fluctuations in the resulting distributions and we do not advocate such parametrisations. Finally, there is some ambiguity concerning the use of weights for the calculations of double ratio (1). If we use the weights only for the two-particle distributions, the two denominators cancel and we calculate effectively just the ratio of two-particle distributions with- and without weights. It seems, however, that the justification for the weight method [13] requires using weights both for the single- and two-particle distributions. We have looked for the best set of parameters with this prescription, using a Gaussian form without the "β-factor" (7). The best set we found is

$$R^{\rm in}_{\rm L} = 0.9 \text{ fm}\,, \quad R^{\rm in}_{\rm out} = 0.3 \text{ fm}\,, \quad R^{\rm in}_{\rm side} = 0.4 \text{ fm}\,. \tag{10}$$

The resulting projections of the double ratios are shown in Fig. 3. The fitted values of parameter we get in formula (2) are

$$R_{\rm L} = 0.73 \text{ fm}\,, \quad R_{\rm out} = 0.54 \text{ fm}\,, \quad R_{\rm side} = 0.65 \text{ fm}\,. \tag{11}$$

Obviously, it is now possible to reproduce the experimental hierarchy of the radii. The fitted value of λ is smaller than in data (0.35 instead of 0.41), but the difference is well within the systematic errors of the fit to the experimental data. Note that we are not showing the errors in Fig. 3 (nor quoting them in the values of parameters listed above), since these errors result mainly from the fluctuations in weights. Some estimate is obtained by comparing the results for 1 and 5 million events samples; in Fig. 3 the differences are of the order of size of the points.

There is a striking difference between the input values of the radii (10) assumed in the weight factors and the resulting best fit values (11) from the double ratio calculated with these weights. Although the hierarchy $R_{\rm L} > R_{\rm side} > R_{\rm out}$ is the same in both cases, the fitted values differ by

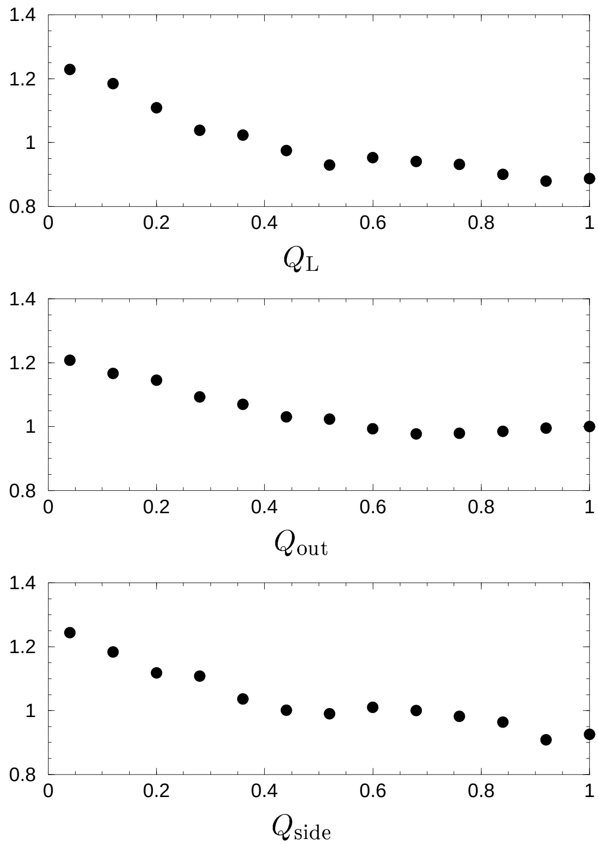

Fig. 3. Projections of the double ratio (1) from the PYTHIA/JETSET MC generator with the asymmetric weight method for parameters (11) on the three axes $Q_{\rm L}$, $Q_{\rm out}$ and $Q_{\rm side}$.

less than 25 %, whereas there is a difference by more than a factor of two between the input values.

Moreover, further decrease of the values of $R_{\rm out}^{\rm in}$ and $R_{\rm side}^{\rm in}$ hardly affects the resulting double ratio and fitted values of R_i. This seems to be the inherent property of the JETSET generator, which yields a rather strong suppression of large values of Q_i and Q^2 even without any procedure imitating the BE effect. Apparently this suppression dominates over the weak enhancement of low values of Q_i induced by the weight factors with small values of R_i. For small $R_i^{\rm in}$ there is no simple correspondence between the input and output values of radii. This looks analogous to the effect noted already for a symmetric BE effect described by the LUBOEI procedure [25].

Therefore, any direct interpretation of the fit values for BE double ratios in terms of the different radii of the asymmetric source is a rather delicate matter.

5. Conclusions

In this note we present the results of our investigation concerning the asymmetry of the BE effect in two procedures imitating this effect in the Monte Carlo generators. A comparison with the data at Z^0 peak is presented. We found that both the momentum shifting method and the weight method with weights depending on Q^2 only give different distributions in different components of Q^2. However, the hierarchy of the radii parametrising these distributions is different from the experimental one. Introducing weights which depend in different ways on different components of Q^2 we are able to reproduce the experimental data.

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