POSSIBLE LSND EFFECT AS A SMALL PERTURBATION OF THE BIMAXIMAL TEXTURE FOR THREE ACTIVE NEUTRINOS *

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A particular form of mixing matrix for three active and one sterile neutrinos is proposed. Its 3×3 part describing three active neutrinos arises from the popular bimaximal mixing matrix that works satisfactorily in solar and atmospheric experiments if the LSND effect is ignored. Then, the sterile neutrino, effective in the fourth row and fourth column of the proposed mixing matrix, is responsible for the possible LSND effect by inducing one extra neutrino mass state to exist actively. The LSND effect, if it exists, turns out to reveal its *perturbative* nature related to small mixing of three active neutrinos with their sterile partner.

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Recent experimental results for atmospheric ν_μ's as well as solar ν_e's favour excluding the hypothetical sterile neutrinos from neutrino oscillations [1]. However, the problem of the third neutrino mass-square difference, related to the possible LSND effect for accelerator ν_μ's, still exists [2], stimulating a further discussion about mixing of three active neutrinos with their sterile counterparts. In the present note we contribute to this discussion by constructing a particular 4×4 texture of three active and one sterile neutrinos, ν_e , ν_μ , ν_τ and ν_s, whose 3×3 part describing three active neutrinos arises from the popular bimaximal texture that works in a satisfactory way in solar and atmospheric experiments if the LSND effect is ignored [3].

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In this popular 3×3 texture the mixing matrix has the form

$$U^{(3)} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{1}$$

where $c_{12} \simeq 1/\sqrt{2} \simeq s_{12}$. Such a form corresponds to $c_{23} = 1/\sqrt{2} = s_{23}$ and $s_{13} = 0$ in the notation usual for a general Cabibbo–Kobayashi–Maskawa-type matrix [4] (if the LSND effect is ignored, the upper limit $|s_{13}| \lesssim 0.1$ follows from the negative result of Chooz reactor experiment [5]). Then, in the 4×4 texture we proposed the following mixing matrix:

$$U = \begin{pmatrix} c_{14}c_{12} & s_{12} & 0 & s_{14}c_{12} \\ -\frac{c_{14}s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{s_{14}s_{12}}{\sqrt{2}} \\ \frac{c_{14}s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{s_{14}s_{12}}{\sqrt{2}} \\ -s_{14} & 0 & 0 & c_{14} \end{pmatrix} \tag{2}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. Here, $U = (U_{\alpha i})$, $\alpha = e\,,\, \mu\,,\, \tau\,,\, s$ and $i = 1, 2, 3, 4$, while the unitary transformation describing the mixing of four neutrinos $\nu_e\,,\, \nu_\mu\,,\, \nu_\tau$ and ν_s is inverse to the form

$$\nu_\alpha = \sum_i U_{\alpha i}\nu_i\,, \tag{3}$$

where $\nu_1\,,\, \nu_2\,,\, \nu_3$ and ν_4 denote the massive neutrinos carrying the masses $m_1\,,\, m_2\,,\, m_3$ and m_4. Of course, $U^\dagger = U^{-1}$ and $U^* = U$. Note that

$$U = \begin{pmatrix} & & & 0 \\ & U^{(3)} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + O(s_{14}) \tag{4}$$

in the limiting case of $|s_{14}| \ll |c_{14}|$. Since in the limit of $s_{14} = 0$ there is no LSND effect, Eq. (4) suggests that this possible effect has a *perturbative* character, consistent with its small estimated amplitude $\sin^2 2\theta_{\rm LSND} \sim 10^{-2}$.

It is interesting to consider a 6×6 texture of three active and three sterile neutrinos which may be the active and conventional–sterile Majorana neutrinos, $\nu_\alpha^{(a)} \equiv \nu_{\alpha L} + (\nu_{\alpha L})^c$ and $\nu_\alpha^{(s)} \equiv \nu_{\alpha R} + (\nu_{\alpha R})^c$, $\alpha = e\,,\, \mu\,,\, \tau$. Define in this texture the following mixing matrix

$$\begin{aligned} U^{(6)} &= \begin{pmatrix} U^{(3)} & 0^{(3)} \\ 0^{(3)} & 1^{(3)} \end{pmatrix} \begin{pmatrix} C & S \\ -S & C \end{pmatrix}, \\ C &= \begin{pmatrix} c_{14} & 0 & 0 \\ 0 & c_{25} & 0 \\ 0 & 0 & c_{36} \end{pmatrix}, \quad S = \begin{pmatrix} s_{14} & 0 & 0 \\ 0 & s_{25} & 0 \\ 0 & 0 & s_{36} \end{pmatrix}. \end{aligned} \tag{5}$$

Then, it is easy to discover that

$$U^{(6)} = \left(\begin{array}{cccccc} & & & & 0 & 0 \\ & & & & 0 & 0 \\ & & U & & 0 & 0 \\ & & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right) + O(s_{25}) + O(s_{36}) \tag{6}$$

in the limiting case of $|s_{25}| \ll |c_{25}|$ and $|s_{36}| \ll |c_{36}|$. In this case, two sterile neutrinos $\nu_\mu^{(s)}$ and $\nu_\tau^{(s)}$ become decoupled from three active neutrinos $\nu_e^{(a)}$, $\nu_\mu^{(a)}$, $\nu_\tau^{(a)}$ and from one sterile neutrino $\nu_e^{(s)}$, if our 6×6 texture is realized indeed with the use of three active and three conventional–sterile Majorana neutrinos. Then, four neutrinos $\nu_e^{(a)}$, $\nu_\mu^{(a)}$, $\nu_\tau^{(a)}$ and $\nu_e^{(s)}$ mix through the matrix inverse to U given in Eq. (2).

In the representation, where the mass matrix of three charged leptons e^-, μ^-, τ^- is diagonal, the 4×4 neutrino mixing matrix U is at the same time the diagonalizing matrix for the $4{\times}4$ neutrino mass matrix $M = (M_{\alpha\beta})$:

$$U^\dagger M U = \mathrm{diag}(m_1\,, m_2\,, m_3\,, m_4)\,. \tag{7}$$

Here, by definition $m_1^2 \le m_2^2 \le m_3^2$ and either $m_3^2 \le m_4^2$ or $m_4^2 \le m_1^2$. Then, evidently $M_{\alpha\beta} = \sum_i U_{\alpha i} m_i U^*_{\beta i}$, and hence with the use of proposal (2) we obtain

$$\begin{aligned}
M_{ee} &= c_{12}^2\left(c_{14}^2 m_1 + s_{14}^2 m_4\right) + s_{12}^2 m_2\,, \\
M_{e\mu} &= -M_{e\tau} = -\frac{1}{\sqrt{2}}\, c_{12}\, s_{12}\left(c_{14}^2 m_1 + s_{14}^2 m_4 - m_2\right)\,, \\
M_{es} &= -c_{12}\, c_{14}\, s_{14}\,(m_1 - m_4)\,, \\
M_{\mu\mu} &= M_{\tau\tau} = \frac{1}{2}\left[s_{12}^2\left(c_{14}^2 m_1 + s_{14}^2 m_4\right) + c_{12}^2 m_2 + m_3\right]\,, \\
M_{\mu\tau} &= -\frac{1}{2}\left[s_{12}^2\left(c_{14}^2 m_1 + s_{14}^2 m_4\right) + c_{12}^2 m_2 - m_3\right]\,, \\
M_{\mu s} &= -M_{\tau s} = \frac{1}{\sqrt{2}} s_{12}\, c_{14}\, s_{14}\,(m_1 - m_4)\,, \\
M_{ss} &= s_{14}^2 m_1 + c_{14}^2 m_4\,,
\end{aligned} \tag{8}$$

where $c_{12} \simeq 1/\sqrt{2} \simeq s_{12}$ *i.e.* $\theta_{12} \simeq \pi/4$. Of course, $M^\dagger = M$ and $M^* = M$.

Due to mixing of four neutrino fields described by Eq. (3), neutrino states mix according to the relation

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle\,. \tag{9}$$

This implies the following familiar formulae for probabilities of neutrino oscillations $\nu_\alpha \to \nu_\beta$ on the energy shell:

$$P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta | \mathrm{e}^{iPL} | \nu_\alpha \rangle|^2 = \delta_{\beta\alpha} - 4 \sum_{j>i} U^*_{\beta j} U_{\beta i} U_{\alpha j} U^*_{\alpha i} \sin^2 x_{ji} \,, \tag{10}$$

being valid if the quartic product $U^*_{\beta j} U_{\beta i} U_{\alpha j} U^*_{\alpha i}$ is real, what is certainly true when the tiny CP violation is ignored. Here,

$$x_{ji} = 1.27 \frac{\Delta m^2_{ji} L}{E} \,, \quad \Delta m^2_{ji} = m^2_j - m^2_i \tag{11}$$

with Δm^2_{ji}, L and E measured in eV^2, km and GeV, respectively (L and E denote the experimental baseline and neutrino energy, while $p_i = \sqrt{E^2 - m^2_i} \simeq E - m^2_i/2E$ are eigenvalues of the neutrino momentum P).

With the use of proposal (2) for the 4×4 neutrino mixing matrix the oscillation formulae (10) lead to the probabilities

$$\begin{aligned}
P(\nu_e \to \nu_e) &\simeq 1 - (2c_{12}s_{12})^2 c^2_{14} \sin^2 x_{21} - 4(1 - c^2_{12}s^2_{14}) c^2_{12} s^2_{14} \sin^2 x_{41} \,, \\
P(\nu_\mu \to \nu_\mu) &= P(\nu_\tau \to \nu_\tau) \simeq 1 - (c_{12}s_{12})^2 c^2_{14} \sin^2 x_{21} \\
&\quad - (1 - s^2_{12}s^2_{14}) \left(\sin^2 x_{32} + s^2_{12}s^2_{14} \sin^2 x_{41} \right) - s^2_{12}s^2_{14} \sin^2 x_{43} \,, \\
P(\nu_\mu \to \nu_e) &= P(\nu_\tau \to \nu_e) \simeq 2(c_{12}s_{12})^2 \left(c^2_{14} \sin^2 x_{21} + s^4_{14} \sin^2 x_{41} \right) , \\
P(\nu_\mu \to \nu_\tau) &\simeq -(c_{12}s_{12})^2 c^2_{14} \sin^2 x_{21} + (1 - s^2_{12}s^2_{14}) \left(\sin^2 x_{32} - s^2_{12}s^2_{14} \sin^2 x_{41} \right) \\
&\quad + s^2_{12}s^2_{14} \sin^2 x_{43}
\end{aligned} \tag{12}$$

in the approximation where $m^2_1 \simeq m^2_2$ (and both are much different from m^2_3 and m^2_4), and also to the probabilities involving the sterile neutrino ν_s

$$\begin{aligned}
P(\nu_\mu \to \nu_s) &= P(\nu_\tau \to \nu_s) = 2s^2_{12}(c_{14}s_{14})^2 \sin^2 x_{41} \,, \\
P(\nu_e \to \nu_s) &= 4c^2_{12}(c_{14}s_{14})^2 \sin^2 x_{41} \,, \\
P(\nu_s \to \nu_s) &= 1 - 4(c_{14}s_{14})^2 \sin^2 x_{41} \,,
\end{aligned} \tag{13}$$

where only m^2_1 and m^2_4 participate.

If $|\Delta m^2_{21}| \ll |\Delta m^2_{41}|$ (*i.e.*, $|x_{21}| \ll |x_{41}|$) and

$$|\Delta m^2_{21}| = \Delta m^2_{\mathrm{sol}} \sim (10^{-5} \ \text{ or } \ 10^{-7} \ \text{ or } \ 10^{-10})\ \mathrm{eV}^2 \tag{14}$$

for LMA or LOW or VAC solution, respectively [1,6], then under the conditions of solar experiments the first Eq. (12) with $c_{12} \simeq 1/\sqrt{2} \simeq s_{12}$ gives

$$\begin{aligned}
P(\nu_e \to \nu_e)_{\mathrm{sol}} &\simeq 1 - c^2_{14} \sin^2 (x_{21})_{\mathrm{sol}} - \frac{(1 + c^2_{14}) s^2_{14}}{2} \,, \\
c^2_{14} &= \sin^2 2\theta_{\mathrm{sol}} \sim 0.8 \text{ or } 0.9 \text{ or } 0.7 \,.
\end{aligned} \tag{15}$$

If $|\Delta m^2_{21}| \ll |\Delta m^2_{32}| \ll |\Delta m^2_{41}|\,,\, |\Delta m^2_{43}|$ (*i.e.*, $|x_{21}| \ll |x_{32}| \ll |x_{41}|\,,\; |x_{43}|$) and

$$|\Delta m^2_{32}| = \Delta m^2_{\rm atm} \sim 3 \times 10^{-3}\ {\rm eV}^2\,, \tag{16}$$

then for atmospheric experiments the second Eq. (12) with $c_{12} \simeq 1/\sqrt{2} \simeq s_{12}$ leads to

$$\begin{aligned} P(\nu_\mu \to \nu_\mu)_{\rm atm} &\simeq 1 - \frac{1 + c^2_{14}}{2} \sin^2 (x_{32})_{\rm atm} - \frac{(3 + c^2_{14}) s^2_{14}}{8}\,, \\ \frac{1 + c^2_{14}}{2} &= \sin^2 2\theta_{\rm atm} \sim 1\,. \end{aligned} \tag{17}$$

Eventually, if $|\Delta m^2_{21}| \ll |\Delta m^2_{41}|$ and

$$|\Delta m^2_{41}| = \Delta m^2_{\rm LSND} \sim 1\ {\rm eV}^2\ \ (e.g.)\,, \tag{18}$$

then in the LSND experiment the third Eq. (12) with $c_{12} \simeq 1/\sqrt{2} \simeq s_{12}$ implies

$$P(\nu_\mu \to \nu_e)_{\rm LSND} \simeq \frac{s^4_{14}}{2} \sin^2 (x_{41})_{\rm LSND}\,, \quad \frac{s^4_{14}}{2} = \sin^2 2\theta_{\rm LSND} \sim 10^{-2}\ \ (e.g.). \tag{19}$$

Thus,

$$\begin{aligned} s^2_{14} &\sim 0.14\,, \quad c^2_{14} \sim 0.86\,, \quad \frac{1 + c^2_{14}}{2} \sim 0.93\,, \\ \frac{(1 + c^2_{14}) s^2_{14}}{2} &\sim 0.13\,, \quad \frac{(3 + c^2_{14}) s^2_{14}}{8} \sim 0.068\,, \end{aligned} \tag{20}$$

if the LSND effect really exists and develops the amplitude $s^4_{14}/2 \sim 10^{-2}$. Through Eq. (19) the LSND effect (if it exists) reveals its *perturbative* nature related to the small constant $s^4_{14}/2 \sim 10^{-2}$ that measures coupling of ν_1 with ν_4 in the process of $\nu_\mu \to \nu_e$ oscillations at LSND.

If *e.g.* the LOW solar solution $\sin^2 2\theta_{\rm sol} \sim 0.9$ and $\Delta m^2_{\rm sol} \sim 10^{-7}\ {\rm eV}^2$ is *accepted*, then $c^2_{14} \sim 0.9$, what *predicts* that $\sin^2 2\theta_{\rm atm} = (1 + c^2_{14})/2 \sim 0.95$ and $\sin^2 \theta_{\rm LSND} = s^4_{14}/2 \sim 5 \times 10^{-3}$.

Concluding, we can say that Eqs. (15), (17) and (19) are not inconsistent with solar, atmospheric and LSND experiments, respectively. Note that in Eqs. (15) and (17) there are constant terms that modify moderately the usual two-flavor formulae. Any LSND-type accelerator experiment, in contrast to the solar and atmospheric projects, investigates a small $\nu_\mu \to \nu_e$ oscillation effect caused possibly by the sterile neutrino. So, this effect (if it exists) plays the role of a small *perturbation* of the basic bimaximal texture for three active neutrinos.

The final equations (15), (17) and (19) follow from the first three oscillation formulae (12), if either

$$m_1^2 \simeq m_2^2 \ll m_3^2 \ll m_4^2 \tag{21}$$

with

$$\begin{aligned} m_3^2 \;&\ll\; 1\,\mathrm{eV}^2\,,\;\; m_4^2 \sim 1\,\mathrm{eV}^2\,, \\ \Delta m_{21}^2 \;&\sim\; (10^{-5} - 10^{-10})\,\mathrm{eV}^2 \ll \Delta m_{32}^2 \sim 10^{-3}\,\mathrm{eV}^2 \end{aligned} \tag{22}$$

or

$$m_1^2 \simeq m_2^2 \simeq m_3^2 \gg m_4^2 \tag{23}$$

with

$$\begin{aligned} m_1^2 \;&\sim\; 1\,\mathrm{eV}^2\,,\;\; m_4^2 \ll 1\,\mathrm{eV}^2\,, \\ \Delta m_{21}^2 \;&\sim\; (10^{-5} - 10^{-10})\,\mathrm{eV}^2 \ll \Delta m_{32}^2 \sim 10^{-3}\,\mathrm{eV}^2. \end{aligned} \tag{24}$$

Indeed, when either $m_1^2 \simeq m_2^2 \ll m_3^2 \ll m_4^2 \sim 1$ eV^2 or $m_4^2 \ll m_1^2 \simeq m_2^2 \simeq m_3^2 \sim 1$ eV^2, we may obtain $\Delta m_{21}^2 \ll \Delta m_{32}^2 \ll |\Delta m_{41}^2| \sim 1$ eV^2. The second case of $m_4^2 \ll m_1^2 \sim 1$ eV^2, where the neutrino mass state $i = 4$ induced by the sterile neutrino ν_s gets a vanishing mass, seems to be more natural than the first case of $m_3^2 \ll m_4^2 \sim 1$ eV^2, where such a state gains a considerable amount of mass "for nothing". This is so, unless one believes in the liberal maxim "whatever is not forbidden is allowed": the Majorana righthanded mass is not forbidden by the electroweak SU(2)×U(1) symmetry, in contrast to Majorana lefthanded and Dirac masses requiring this symmetry to be broken (for the active Majorana neutrinos), say, by a Higgs mechanism that becomes then the origin of these masses. Here, the active Majorana neutrinos are $\nu_\alpha^{(a)} \equiv \nu_{\alpha L} + (\nu_{\alpha L})^c\,,\ \alpha = e\,,\,\mu\,,\,\tau$, while the sterile Majorana neutrino is $\nu_s \equiv \nu_{sR} + (\nu_{sR})^c$ with $\nu_{sL} = (\nu_{sR})^c = (\nu_s^c)_L$ (implying effectively the Dirac-type 3×1 mass matrix and the Dirac-type transposed 1×3 mass matrix as well as the Majorana righthanded trivial 1×1 mass matrix; of course, the Majorana lefthanded mass matrix is 3×3). Possibly $\nu_s = \nu_e^{(s)}$ [*cf.* the comment to Eq. (6)]; then $\nu_{sR} = \nu_{eR}$.

In the approximation used before to derive Eqs. (15), (17) and (19) there are true also the relations

$$\begin{aligned} P(\nu_e \to \nu_e)_{\mathrm{sol}} \;&\simeq\; 1 - P(\nu_e \to \nu_\mu)_{\mathrm{sol}} - P(\nu_e \to \nu_\tau)_{\mathrm{sol}} - (c_{14}s_{14})^2, \\ (c_{14}s_{14})^2 \;&\sim\; 0.12\,, \end{aligned}$$

$$P(\nu_\mu \to \nu_\mu)_{\rm atm} \simeq 1 - P(\nu_\mu \to \nu_\tau)_{\rm atm} - \frac{(1+c_{14}^2)s_{14}^2}{4}\,, \qquad \frac{(1+c_{14}^2)s_{14}^2}{4} \sim 0.065\,, \tag{25}$$

as well as

$$P(\nu_\mu \to \nu_e)_{\rm LSND} \simeq \frac{1}{2}\left(\frac{s_{14}}{c_{14}}\right)^2 P(\nu_\mu \to \nu_s)_{\rm LSND}\,, \quad \frac{1}{2}\left(\frac{s_{14}}{c_{14}}\right)^2 \sim 0.082\,. \tag{26}$$

The second relation (25) demonstrates a leading role of the appearance mode $\nu_\mu \to \nu_\tau$ in the disappearance process of atmospheric ν_μ's, while the relation (26) indicates a direct interplay of the appearance modes $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_s$. In the case of the first relation (25), both appearance modes $\nu_e \to \nu_\mu$ and $\nu_e \to \nu_\tau$ contribute equally to the disappearance process of solar ν_e's, and the role of the appearance mode $\nu_e \to \nu_s$ (responsible for the constant term) is also considerable.

Finally, for the Chooz experiment [5], where it happens that $(x_{ji})_{\rm Chooz} \simeq (x_{ji})_{\rm atm}$, the first Eq. (12) predicts

$$P(\bar{\nu}_e \to \bar{\nu}_e)_{\rm Chooz} \simeq P(\bar{\nu}_e \to \bar{\nu}_e)_{\rm atm} \simeq 1 - \frac{(1+c_{14}^2)s_{14}^2}{2}\,, \quad \frac{(1+c_{14}^2)s_{14}^2}{2} \sim 0.13\,, \tag{27}$$

if there is the LSND effect with the amplitude $s_{14}^4/2 \sim 10^{-2}$ as written in Eq. (19). Here, $(1+c_{14}^2)s_{14}^2 \sin^2(x_{41})_{\rm Chooz} \simeq (1+c_{14}^2)s_{14}^2/2$. In terms of the usual two-flavor formula, the negative result of Chooz reactor experiment excludes the disappearance process of reactor $\bar{\nu}_e$'s for moving $\sin^2 2\theta_{\rm Chooz} \gtrsim 0.1$, when the range of moving $\Delta m^2_{\rm Chooz} \gtrsim 3 \times 10^{-3}$ eV^2 is considered. In our case $\sin^2 2\theta_{\rm Chooz} \sim (1+c_{14}^2)s_{14}^2/2$ for $\sin^2 x_{\rm Chooz} \sim 1$. Thus, the Chooz effect for reactor $\bar{\nu}_e$'s may appear at the edge (if the LSND effect really exists).

From the neutrinoless double β decay, not observed so far, the experimental bound $\overline{M}_{ee} \equiv \left|\sum_i U_{ei}^2 m_i\right| < [0.4(0.2)$ — $1.0\ (0.6)]$ eV follows [7]. On the other hand, with $c_{12} \simeq 1/\sqrt{2} \simeq s_{12}$ and the values (20) the first Eq. (8) gives

$$\overline{M}_{ee} = |M_{ee}| \sim \tfrac{1}{2}\left|0.86 m_1 + 0.14 m_4 + m_2\right|\,, \tag{28}$$

what in the case of Eq. (21) with $m_4^2 \sim 1$ eV^2 or Eq. (23) with $m_1^2 \sim 1$ eV^2 leads to the estimate $\overline{M}_{ee} \sim 0.07|m_4| \sim 0.07$ eV or $\overline{M}_{ee} \sim 0.9 m_1 \sim 0.9$ eV, respectively (the second estimate follows if $m_1 \simeq m_2 \sim 1$ eV; if $-m_1 \simeq m_2 \sim 1$ eV this estimate becomes $\overline{M}_{ee} \sim 0.07 m_2 \sim 0.07$ eV). Of course, the value $(m_4^2$ or $m_1^2) \simeq \Delta m^2_{\rm LSND} \sim 1$ eV^2 in Eq. (18) is only an example, and may turn out to be smaller.

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